Estimating CES Aggregate Production Functions of China and India

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Abstract: - We have observed remarkably different patterns in the development processes of China and India. For example, data from our sample show that, comparing with India, China has considerably higher rates of physical capital formation as well as much higher ratios of measured physical to human capital. Motivated by these empirical observations, we estimate the key technological parameters of a CES production function with physical and human capital using panel data at the provincial level for China and at the state level for India, respectively. Our estimation results on the distributional parameter and the elasticity of substitution between the input factors yield implications that match broadly with the above stylized factor accumulation patterns in China and India.

Key-Words: - CES production function, Physical capital, Human capital, Economic development

1 Introduction

China and India, the world’s two most populous countries, have both featured prominently in the recent decades as the world’s most successful stories of economic development. While casual observers might be struck by the similar economic successes that China and India have enjoyed, many researchers have pointed out that the economic transformations taking place in these two economies are in fact accompanied by markedly different development patterns along various dimensions. For instance, among others, China’s remarkable growth has been accompanied by significant rates of savings and investment, while India has achieved its reasonably high rate of growth with a much lower rate of investment and an almost stagnant industry sector share of GDP (see Figure 1). Furthermore, while China seems to rely heavily on accumulating large quantities of physical capital relative to that of human capital, India appears to depend less on physical capital deepening but rather rely more on the human-capital-intensive service sector to fuel its growth (see Figure 2).1

Motivated by the above observations, the current paper aims at estimating the distribution parameter and the substitution parameter for both China and India in a general CES aggregate production function with physical and human capital.2 Following the same reasons and technique as in Duffy and Papageorgiou (2000), we estimate a log-linearized approximation of CES specification by using panel data (at province level for China and at state level for India) models. Our estimation results show that, like many previous studies on this issue, the Cobb-Douglas specification of the aggregate production function is rejected for both China and India. Consistent with the estimation results in Duffy and Papageorgiou (2000) based on a panel data of 82 countries, our provincial-level panel data for China and state-level panel data for India yield estimations of the elasticity of substitution between capital and labor (adjusted for human capital) that are significantly above unity. Furthermore, our panel model estimations also yield

1 For calculating this $K/H$ ratio in Figure 2, other than obtaining the education data for both China and India from Barro and Lee (2001), we obtain the Chinese capital stock data from China Research Database 2006 compiled by Kui-Wai. Li, the rest of the data, i.e., the Indian capital stock, population figures for both China and India, and exchange rates, from CEIC dataset.

2 While the Cobb-Douglas production function, which corresponds to the special case of CES function with the elasticity of substitution of one, has been the most prevalent formulation in the growth literature (see, Lucas 1988, Barro 1990, and Jones 1995), however, the Cobb-Douglas formulation has not been backed up by some recent empirical estimations of the elasticity parameter (see, for example, Duffy and Papageorgiou 2000, and Klump et al. 2007).
a considerably higher distribution parameter in the aggregate CES production function for China than for India. We argue that these estimates corroborate well with the aforementioned stylized facts about the growth experiences in these two countries.

The rest of the paper proceeds as follows. We describe the construction of the panel data that we use for estimation in Section 2, followed by presenting our estimation method and results in Section 3. Finally, Section 4 contains some concluding remarks.

2 Data Construction
Due to the short time series of the relevant data at the national level, we adopt the panel model estimation strategy as in Duffy and Papageorgiou (2000) by using the provincial-level data for China and the state-level data for India. In the following we will describe the sources of our data and the construction of variables used in the estimation.

Our data on China at the province level are from the latest version of the China Research Database (2006), which is compiled by Dr. Kui Wai Li. The raw data used in constructing capital stock come from Statistical Yearbook of China and the Comprehensive Statistical Data and Materials in 50 Years of New China. In constructing the human capital variable $H_{jt} = E_{jt} \cdot L_{jt}$, the perpetual inventory approach (see Barro and Lee 2001) is used to estimate the years of education $E_{jt}$, while the raw labor input $L_{jt}$ will be measured by either employment or population data. This dataset covers 30 Chinese provinces for the period 1985-2006 (see Li 2003 for details).

Data on India are primarily drawn from the database in Ghate and Wright (2011). This database covers the period 1993-2004 and contains state level data for most of the variables we need for our empirical analysis. Due to unavailability of data and data quality concerns for some states, we use data for only 23 out of a total of 31 states.

As for human capital measures, there is no state level data on years of education for the population/labor force in Ghate and Wright’s database, and we are not able to obtain such information from other sources. Given such data constraint, we adopt the following method to construct a proxy for human capital at the state level. We first use the national mean year of schooling of individuals in the labor force from the Barro & Lee Education Index (see Barro and Lee 2001), which provides data every five years. To compute data in off years (those for which data are not available) we use the method of interpolation assuming that mean years of schooling grow at a constant rate within each of the five-year intervals. We then construct our state-level mean years of schooling by using the state level data on education expenditure as follows:

$$E_{jt} = E_t \frac{e_{jt}}{e_t},$$

where $E_{jt}$ is the state $j$ mean years of schooling of, $E_t$ is the national level of mean years of schooling, and $e_{jt}$ and $e_t$, respectively, are the state $j$ and the national level per capita education expenditure constructed as $e_t = \sum_j e_{jt} / 23$. Again, the state-level human capital measures are obtained from $H_{jt} = E_{jt} \cdot L_{jt}$, where we use either the employment (labor force) data or population data as the measure for $L_{jt}$.

Because of the space limitation, we do the include here the summary statistics, such as means and standard deviations, of the data for both China and India, but they are available upon request.

3 Estimation of Production Function
In the following, we opt to estimate the intensive form of a CES production function

$$y_t = A(a_k \rho^p + (1 - \alpha) h_t \rho^p)^{\frac{1}{\rho}}, \quad (1)$$

where $y_t$, $k_t$, and $h_t$ represent per-capita output, physical capital, and human capital, respectively. The distributional parameter $\alpha$ measures the relative marginal productivity of physical capital. While the distributional parameter $\alpha$ in the CES production function in general does not equal to the capital income share (except for the case of the Cobb-Douglas production function), it correlates positively with the true capital income share which can be shown as $s_k = \frac{a_k \rho}{a_k \rho + (1 - \alpha) h_t \rho}$. Therefore, a higher $\alpha$ means a higher capital income share $s_k$, which encourages relatively more physical capital accumulation for any level of the elasticity of substitution. In addition, the production function in (1) exhibits a constant elasticity of substitution between physical and human capital measured by $\sigma = 1/(1 - \rho)$.

From the property of constant returns to scale, we can rewrite the intensive form of the CES production function in (1) as:

$$\tilde{y}_{jt} = A[a_k \tilde{k}_{jt}^p + (1 - \alpha)]^{\frac{1}{\rho}}$$

where $\tilde{k}_{jt} = k_{jt}/h_{jt}$, and $\tilde{y}_{jt} = y_{jt}/h_{jt}$, are province-level physical capital and output normalized by human capital. Following Duffy and
Papageorgiou (2000), we proceed to estimate a log-linearized approximation of the above intensive form of the CES production function with panel data. Adding the subscript \( j \) to represent province/state, we arrive at the following linearized specification

\[
\log \bar{y}_{jt} = \log A + \beta_1 \log \bar{k}_{jt} + \beta_2 [\log k_{jt}]^2 + \epsilon_{jt}
\]

(2)

where

\[
\rho = \frac{2\beta_2}{\beta_1 (1 - \beta_2)}, \quad \alpha = \beta_1,
\]

and we incorporate time and province fixed effects in \( \epsilon_{jt} \) in order to control for province heterogeneity and year-specific shocks. The advantage of estimating the above linear model instead of its nonlinear counter is that it is more stable numerically.

Based on the estimates of \( \rho \) and \( \alpha \), we can also obtain an estimate of the income share of capital as

\[
s_k = \frac{ak^\rho}{ak^\rho + (1 - \alpha)\rho^P}.
\]

In the empirical analysis, we evaluate \( s_k \) at the mean of capital and labor. In the estimating (2), we have also experimented with using instrumental variables because it is suggested that physical capital \( \bar{k}_{jt} \) is an endogenous variable in the output equation. We used lagged variables \( k_{jt-1} \) and \( k_{jt-2} \) as instruments for estimation. However, as indicated by our Hausman test results, it turns out that endogeneity is not found in \( \bar{k}_{jt} \). Because of this, we report only the two-way fixed effects results and use them for our examination for the validity of our theoretical model’s implications in matching with actual observations. The estimation results for the key parameters are reported in Tables 1 and 2, with t-statistics in parentheses and p-values in square brackets.

We see from these tables that no matter whether we use education-adjusted population or education-adjusted labor force as a measure of human capital, the results are similar in magnitude. This is especially surprising for developing countries like China and India given the sizes of their informal sectors.

For China, the estimates of \( \rho \) (0.0994 and 0.1537) are somewhat larger than previous studies of China’s production function using province level data. For example, using a two-stage method, He et al. (2007) obtain estimates of 0.003 (coastal provinces) and -0.001 (inland provinces) using province level data, and their estimates are statistically insignificant. On the other hand, our results on \( \alpha \) and hence \( s_k \) are consistent with those obtained by He et al. (2007) whose estimation of a Cobb-Douglas production yielded estimates of capital’s share of income to be 0.619 for coastal provinces, 0.419 for inland province, and 0.592 for all provinces. Similarly, by estimating of a Cobb-Douglas production, Chow and Li (2002) obtained an estimate of the capital income share of 0.628. These estimates are comparable with our estimates of \( s_k \) of 0.5450 when human capital is proxied by population adjusted for education.

For India, our estimation yields estimates of 0.2232 and 0.1721 for \( \rho \). Since we cannot find any recent studies estimating the substitution parameter in the aggregate production function for India, especially those using state level data, we are not able to benchmark our estimates for comparison purpose. Our estimates of the income share of capital \( s_k \) are 0.4028 and 0.6675, respectively, with education adjusted population and education adjusted labor force as measures of human capital. These figures are quite close to several previous studies on the income share of capital for India. For example, according to Asian Productivity Organization (2004), the capital income share is estimated on average to be 0.4073 for the formal sector and 0.3052 for the informal sector for the period 1976-2001. Sivasubramonian (2004) also has obtained estimates of the capital income share for India ranging from 45.85% to 50.46% in 1950-2000. In addition, the estimates in Gollin (2002) range from 0.162 (with adjustment for self-employment and proprietors) to 0.309 (without adjustment) for India for 1980, based on an accounting approach. In particular, our estimate of \( s_k = 0.4028 \) from using population adjusted for education as human capital corroborates very closely with this group of previous estimates. However, our estimates of on \( \alpha \) and hence \( s_k \) for India based on employment data resulted in an estimate for capital income share of 0.6675, which is a bit too high comparing with all previous estimates. We interpret this as suggesting the employment data of India are less reliable than population data as measurement for labor input, again, perhaps due to the well-recognized problem with informal sector.

4 Conclusion

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3 Our estimate of \( s_k = 0.4298 \) when using human capital as labor force adjusted for education, though still not far away from the previous estimates, reinforce our concern of using employment data to measure labor input due to the undocumented informal sector for developing countries.
To summarize, our estimation has yielded reasonable estimates for the key model parameters of α and ρ for both China and India. Focusing on the more reliable set of results with measuring human capital by population adjusted for education, our estimates suggest that the distribution parameter α as well as the capital income share of s_k for China are higher than the respective Indian counterparts. In particular, the distribution parameter α is greater than 0.5 for China and less than 0.5 for India, implying that, to the extent possible, China would like to substitute physical capital for human capital because of the relatively high marginal product of physical capital while India would like to do just the opposite. Meanwhile, although the estimates of ρ are somewhat larger for India than those for China, even based on the estimates of ρ = 0.0994 and ρ = 0.2232 for China and India, respectively, the implied elasticity of substitution is 1.11 for China and 1.28 for India, which are quite comparable in magnitude. Furthermore, the estimates of the elasticity parameter ρ for China and India are both significantly different from zero, hence rejecting the Cobb-Douglas specification for both countries. These estimated values of α and ρ would lead to predictions of China’s having a greater physical capital to human capital ratio in its economic development process than India does. These predictions accord well with the evidence depicted in Figures 1 and 2.

References:


Figure 1
Gross Capital Formation

Figure 2
Physical Capital/Human Capital Ratio
Table 1: Estimation Results for China

<table>
<thead>
<tr>
<th></th>
<th>Education adjusted population as human capital</th>
<th>Education adjusted labor force as human capital</th>
</tr>
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<tbody>
<tr>
<td>$\rho$</td>
<td>0.0994 (1.90)</td>
<td>0.1537 (3.32)</td>
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<tr>
<td>$\alpha$</td>
<td>0.6057 (17.86)</td>
<td>0.5520 (13.42)</td>
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<tr>
<td>$s_k$ (Income share of capital)</td>
<td>0.5450 (8.72)</td>
<td>0.4298 (5.97)</td>
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<tr>
<td>$R^2$</td>
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<td>0.86</td>
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<td>F-Test for Fixed Effects</td>
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<td>Observations</td>
<td>600</td>
<td>660</td>
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Table 2: Estimation Results for India

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<th>Education adjusted population as human capital</th>
<th>Education adjusted labor force as human capital</th>
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<tbody>
<tr>
<td>$\rho$</td>
<td>0.2232 (4.77)</td>
<td>0.1721 (5.00)</td>
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<tr>
<td>$\alpha$</td>
<td>0.2562 (1.83)</td>
<td>0.3973 (3.30)</td>
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<tr>
<td>$s_k$ (Income share of capital)</td>
<td>0.4028 (1.97)</td>
<td>0.6675 (2.68)</td>
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<td>$R^2$</td>
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