The Modelling and Simulation of Gas Pipeline Dynamics

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Abstract:- Gas transportation via long pipelines is considered. Distributed parameter modelling with series and shunt energy dissipation and gas stream capacitance and inductance effects are incorporated. Hybrid analysis methods, wherein both the distributed and the lumped, concentrated elements of the pipeline system are included in the overall model, are advocated. An illustrative application study is outlined validating thereby the analytical procedures employed.

Key-Words: gas, pipeline, transient, modelling, response

1. Introduction
The transportation of gas over long distances by pipelines will be considered in this contribution. This method of supply is a relatively safe, reliable and cost-effective form of conveying natural gas which is universally employed.

Constructing and the installation of gas pipelines is an expensive, labour intensive and politically sensitive operation. These networks often span remote regions, cross national boundaries and ecologically protected areas resulting in delicate, protracted negotiations.

Beyond this the running cost associated with gas pipelines is substantial. Owing to the frictional energy dissipation arising from the internal pipeline roughness, welds, joints, bends and discontinuities, there is a continuous reduction in the gas stream pressure and hence, volume gas steam flow rate. To counter this effect, centrifugal or axial flow compressors are installed, at strategic locations along the pipeline restoring thereby the pressure loss and gas volume flow rate.

Due to the length of gas pipelines and the proportional pressure loss, the compressors employed operate continuously. Consequently, the running cost, maintenance and refit charges associated with this requirement, are substantial.

This problem is exacerbated by the remote locations, monitoring and operation of the compressor drive systems and gas coolers. These active devices must also respond to varying load demands with the requirement for constant delivery pressure and supply rates.

As with all continuously operated systems, any operational economy is translated into significant savings owing to the accumulating reduction in running, maintenance and delivery costs. However, to assess the energy efficiency of the system modelling procedures have to be derived enabling the dynamic system response to be investigated.

The classical theory for spatially dispersed pipeline systems results in irrational, multivariable, input-output models which are incomplete in the Laplace transform variable, see for example [1], [2]. Theoretically, it is possible to obtain the predicted, system responses from these models. However, the procedures involved do not provide simple, usable results which can be incorporated into design, analysis or optimisation investigations.

Finite element techniques may be used to assess the pipeline dynamics. With this procedure, large matrix models arise from the modelling procedure attracting thereby the possibility of computational
errors. Considerable speculation surrounding the computed pipeline performance may also be encountered in that the number and composition of the elements employed is unspecified.

2. Long Pipeline Modelling Methods

When considering pipelines of 1-10 km in length, the matrix models, derived from finite element methods, would be dimensionally very large [3]. Consequently, in addition to the analytical disadvantages cited above, numerical computational errors which would further contaminate the results would be encountered [4].

Alternatively, with the employment of hybrid, distributed- lumped modelling an accurate modelling method is available [5],[6]. This procedure allows pipeline elements which are clearly distributed, to be modelled using distributed parameter methods. Otherwise, relatively pointwise components and sub-assemblies such as valves, compressors, bends and restrictions may be represented using lumped analysis methods without too much loss of accuracy. This allows engineering judgment to be exercised in selecting the appropriate modelling method, for each system element.

3. Series and Parallel Representations

Elements comprising an overall pipeline system may be assembled in series, parallel or in series parallel form where in each case, the steady state volume flow would be inversely proportional to the pipeline input impedance.

For the connecting elements, where there is energy storage effects, modelling using two port network analogues which contain inductance and capacitance elements, as shown in [7], could be employed.

4. Distributed Parameter Modelling

Pioneering work, as detailed in [8], [9] and [10], showed that theoretically derived, first order, perturbed, one dimensional approximations for the Navier-Stokes equations were available. These modelling restrictions included zero bulk modulus and radiant heat transfer. A continuous, homogenous medium was also assumed with no radial or axial heat transfer effects.

Further work within this framework was undertaken in [11] where a general discrete, distributed-lumped parameter representation, for linear systems was presented. Low temperature application studies using this approach were proposed in [12] where all of these methods related to the perturbed, pressure variation dynamics, relative to steady state, equilibrium conditions.

Extending this work, this contribution focuses on the distributed parameter system model shown in [1], [2], where \( L_j, C_j, r_j \) and \( g_j \) are the pipeline system, equivalent distributed inductance, capacitance and series (longitudinal) and shunt (radial) flow resistance and conductance, per metre length of pipeline, respectively. The governing equations for this type of element, for the \( j \)th pipeline section are:

\[
\frac{\partial p_j}{\partial x}(t,x) = -L_j \frac{\partial q_j}{\partial t} - r_j q_j(t,x) \quad (1)
\]

and

\[
\frac{\partial q_j}{\partial x}(t,x) = -C_j \frac{\partial p_j}{\partial t} - g_j p_j(t,x). \quad (2)
\]

Following Laplace transformation, with zero initial conditions, equations 1 and 2 yield the solution, for the \( j \)th distributed parameter model of a system of \( m \) elements, of:

\[
\begin{bmatrix}
\zeta_j(s) \omega_j(s) \\
\zeta_j(s) \omega_j(s-1)
\end{bmatrix}
\begin{bmatrix}
P_{j(x,s)} \\
P_{j(x,s-1)}
\end{bmatrix}
\begin{bmatrix}
\zeta_j(s) \omega_j(s) \\
\zeta_j(s) \omega_j(s-1)
\end{bmatrix}
\begin{bmatrix}
Q_{j(s)} \\
Q_{j(s-1)}
\end{bmatrix}
\]

(3)

where: \( j = 2k+1, \ k = 0,1,\cdots m - 1 \),
\( (m = \text{ number of distributed parameter elements}) \),
\( \zeta_j(s) = \left[ \left( L_j s + r_j \right) / \left( C_j s + g_j \right) \right]^{\frac{1}{2}} \),
\( \omega_j(s) = \left( e^{2\Gamma_j(s)} s^j + 1 \right) / \left( e^{2\Gamma_j(s)} s^j - 1 \right) \)
and:
\( \Gamma_j(s) = \left[ \left( L_j s + r_j \right) \left( C_j s + g_j \right) \right]^{\frac{1}{2}} \)
Consequently, even with all of the constraints mentioned earlier the input-output relationship for a typical distributed parameter, gas pipeline network model is multivariable, irrational and is incomplete in the Laplace variable $s$. This difficulty effectively masks any correspondence between the actual system performance and the governing equations so that extracting information from this representation is markedly impaired, see for example [13].

In this regard, of interest here, is the nature of the series impedance and shunt admittance of the infinitesimal pipeline element, shown in equations 4.1 and 4.2. The series impedance frictional drag $r_j$, for example, represents the effect of the gas flow on the pressure gradient arising from shear action, at the pipeline wall, boundary layer. Contrasting this, the shunt $g_j$, admittance or conductance arises from compressibility effects, as shown in [14] where the frictional drag arises from varying gas path compliance, owing to turbulence and molecular friction.

In pipeline systems, the flow impedance is principally due to the entrance/exit losses and to the $r_j$ and $g_j$, the distributed pipeline frictional resistance effects where:

$$\frac{1}{g_j} > r_j$$  \hspace{1cm} (4)

In dimensionally “long” pipelines both the series $r_j$ and the shunt $g_j$, frictional factors, contribute to the overall pressure drop and diminishing volume flow characteristics. The analysis herein also confirms that the inclusion of both these dissipation mechanism is mandatory.

The per unit length energy storage parameters, as shown in [15] and [16], for a circular pipeline, diameter $2a_j$, are the gas path capacitance and inductance of:

$$C_j = \frac{\pi a_j^2}{\gamma R_k \theta_j}$$  \hspace{1cm} (5)

and

$$L_j = \frac{1}{\pi a_j^2},$$ respectively, where:

$$L_j >> C_j$$  \hspace{1cm} (7)

for engineering applications.

In view of the inequalities of equation 4 and 7 rationality may be recovered by equating:

$$\left( \frac{C_j}{r_j} + 1 \right) \prod_{k=1}^{L} \left( \frac{T_{jk}s + 1}{\tau_{jk}s + 1} \right) \equiv \left( \frac{L_j}{r_j} + 1 \right)$$  \hspace{1cm} (8)

It should be noted that for the $j$th section, with an appropriate choice of $T_{jk}$ and $\tau_{jk}$, the approximation of equation 8, with $s = j\omega$, is accurate at:

- low frequencies $\omega < r_j/L_j$
- high frequencies $\omega > g_j/C_j$

and at $(2L-1)$ selected intermediate frequencies $\omega^*$, where $r_j/L_j < \omega^* < g_j/C_j$.

Then from equation 3 since:

$$\Gamma_j(s) = \left[ r_j \left( \frac{L_j}{r_j} + 1 \right) \left( \frac{C_j}{r_j} + 1 \right) g_j \right]^{1/2}$$  \hspace{1cm} (9)

equation 9 becomes, following the substitution shown in equation 8:

$$\Gamma_j(s) = \alpha_j \left( \frac{\left( \frac{T_{jk}s + 1}{\tau_{jk}s + 1} \right) \left( \frac{C_j}{r_j} + 1 \right)}{\left( \frac{T_{jk}s + 1}{\tau_{jk}s + 1} \right) \left( \frac{C_j}{r_j} + 1 \right)} \frac{C_j}{r_j} \right)$$  \hspace{1cm} (10)

where: $\alpha_j = \sqrt{r_j/g_j}$

Equally, since:

$$\zeta_j(s) = \left( \frac{L_j + r_j}{C_j + g_j} \right)$$  \hspace{1cm} (11)

then equation 11, with the substitution of equation 8, continuing with $L=2$ for illustration purposes, is:

$$\zeta_j(s) = \sigma_j \left( \frac{T_{jk}s + 1}{\tau_{jk}s + 1} \frac{C_j}{r_j} \right)$$  \hspace{1cm} (12)

where $\sigma_j = \frac{r_j}{\sqrt{g_j}}$. 

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The remaining important function of equation 2 is \( w_j(s) \). Since:
\[
w_j(s) = \left( e^{2j\beta \tau_s} + 1 \right) / \left( e^{2j\beta \tau_s} - 1 \right)
\] (13)
then with \( \Gamma_j(s) \) from equation 10:
\[
w_j(s) = \left( e^{2j\beta \tau_s} + 1 \right) / \left( e^{2j\beta \tau_s} - 1 \right)
\] (14)

Consequently, upon substituting for equation 14:
\[
\left( w_j^2(s) - 1 \right) / \left( w_j^2(s) + 1 \right) = 2e^{2j\beta \tau_s} / \left( e^{2j\beta \tau_s} - 1 \right)
\] (15)

where in equation 15:
\[
\chi_j(s) = \frac{\left( T_{1,j} + T_{2,j} \right) + \zeta_j}{g_j}
\] (16)
where \( \chi_j(s) \) is given by:
\[
\chi_j(s) \cong a_j + b_j
\] (17)
and in equation 17, expanding \( \chi_j(s) \) for low frequencies from equation 16 gives:
\[
a_j = \frac{\zeta_j}{g_j} \left( T_{1,j} + T_{2,j} \right) - \left( \tau_{1,j} + \tau_{2,j} \right)
\]

It is evident from equations 13, through to equation 17, that the distributed parameter model is now in an attractive form. The functions comprising equation 3 are free from origin branch points with each component \( \zeta_j(s) \), \( w_j(s) \) and \( \left( w_j^2(s) - 1 \right) / \left( w_j^2(s) + 1 \right) \) being single valued and complete, in the Laplace variable \( s \) with simple steady state values of:
\[
\zeta_j(0) = \pi_j, \quad w_j(0) = \frac{e^{2j\beta \tau_s \pi_j} + 1}{e^{2j\beta \tau_s \pi_j} - 1} \quad \text{and} \quad \left( w_j^2(0) - 1 \right) / \left( w_j^2(0) + 1 \right) = \frac{2e^{2j\beta \tau_s \pi_j}}{e^{2j\beta \tau_s \pi_j} - 1}
\]

5. An Overall Pipeline System Model

An overall model structure can now be derived enabling the assembly of the system matrix. For purposes of illustration, if a distributed- lumped configuration is assumed, then an appropriate system matrix can be constructed by adding consecutive distributed or lumped system descriptions and in so doing, eliminate all intermediate variables.

6. Single Pipeline and Compressor

In this application a model representing a single, long pipeline and a compressor will be considered.

From the theory of section 5, the system equation 5.1 is relevant, since there is only a single distributed parameter section and a single termination, lumped resistance element.

Hence:
\[
\begin{bmatrix}
P_1(s) \\
P_2(s)
\end{bmatrix} =
\begin{bmatrix}
\zeta(s)w(s) & -\zeta(s)(w^2(s) - 1)^{1/2} \\
\zeta(s)(w^2(s) - 1)^{1/2} & -\zeta(s)w(s)
\end{bmatrix}
\begin{bmatrix}
P_1(s) \\
Q_1(s)
\end{bmatrix}
\] (18)

where in equation 18, the termination relationship between the transformed pressure change \( P_2(s) \) and the transformed airflow change \( Q_1(s) \) is simply:
\[
P_2(s) = RQ_1(s)
\]

Consequently, following inversion equation 18 becomes:
\[
\begin{bmatrix}
P_1(s) \\
Q_1(s)
\end{bmatrix} =
\begin{bmatrix}
\zeta(s)w(s) & -\zeta(s)(w^2(s) - 1)^{1/2} \\
\zeta(s)(w^2(s) - 1)^{1/2} & -\zeta(s)w(s)
\end{bmatrix}^{-1}
\begin{bmatrix}
P_1(s) \\
Q_1(s)
\end{bmatrix}
\] (19)

If the compressor unit is assumed to be relatively lumped, in comparison to the pipeline, comprising rotors and bearings, then:
\[
P_1(s) = k_{fj}U(s)/\left( \tau_{fj}(s) + 1 \right)
\]

where: \( k_{fj} \) is the gain and \( \tau_{fj} \) is the compressor time constant and \( U(s) \) is the applied voltage for the electrical drive. For this particular application, the parameters are:
From equations 5 and 6 the gas capacitance and inductance per meter length of pipeline are:

\[ C = \pi a^2 / \gamma R_s \Theta = 0.643 \times 10^4 \text{ m}^2 \]
and: \( L = 1/\pi a^2 = 1.2835 \text{ m}^2 \), respectively.

From equations 10 and 12, and 11, respectively:

\[ \alpha = \sqrt{r/g} = 0.7745 \]
and \( \zeta(s) = \sqrt{T_i / \tau_i} = \sqrt{(T_{s,i} + 1)(\tau_{s,i} + 1)} \)

Also equation 8 requires that for a single pipeline section where for illustration purposes, \( L = 2 \) and the section subscripts \( j \) have been dropped:

\[ \left( \frac{T_{s} + 1}{\tau_{s} + 1} \right)^2 \left( \frac{T_{s,j} + 1}{\tau_{s,j} + 1} \right) = \frac{L_{s,j} + 1}{C_{s,j} + 1} \]

where here: \( L/r = 2.1392 \times 10^4 \text{ sec} \) \( (20) \)
and \( C/g = 0.0643 \text{ sec} \)

From the Bode diagram shown in Fig. 1, where the frequency response curves for:

\[ \frac{(L_i / r + 1)}{(C / g + 1)} \]
and \( \frac{(T_i / \omega + 1)(T_{i,j} / \omega + 1)}{(\tau_{i,j} + 1)(\tau_{i,j} + 1)} \)

are shown in full and dotted lines, respectively, for the series resistance of: \( r = 0.6 \times 10^4 \text{ Nsec/m}^3 \) with a shunt admittance of \( g = 10^{-2} \text{ m}^2 / \text{Nsec} \).

The break frequencies selected for the Bode characteristics for \( \frac{1}{T_i} \) and \( \frac{1}{\tau_i} \) are presented below, where:

\[ r = 0.6 \times 10^4 \text{ Nsec/m}^3, \quad \frac{1}{T_i} = 0.467 \times 10^4 \text{ rad/sec}, \]
\[ \frac{1}{T_s} = 15.552 \text{ rad/sec}, \quad \frac{1}{\tau_i} = 6.0 \times 10^4 \text{ rad/sec}, \]
\[ \frac{1}{\tau_s} = 0.05 \text{ rad/sec} \]
and \( 1/\tau_s = 6.02 \text{ rad/sec with } g = 10^4 \text{ m}^2 / \text{Nsec} \).

It is evident, from Fig. 1, that the approximations has the mid-range frequency intersection and exact low and high frequency correspondence, as stated earlier.

If now, as in equation 16,

\[ \chi(s) = \frac{(T_{s,i} + 1)(T_{s,j} + 1)}{(\tau_{s,i} + 1)(\tau_{s,j} + 1)} \left( \frac{C_{s,j} + 1}{g} \right) \]

is evaluated, then \( w(s) \) and \( \zeta(s) \), in equation 18, are fully defined and outputs may now be computed from this equation, where:

\[ \frac{Q(s)}{P(s)} = \frac{(\zeta(s)w(s) + R)}{(\zeta(s)(w(s)R + \zeta(s)))} \]

\[ \frac{Q_i(s)}{P_i(s)} = \frac{(w^2(s) - 1)^{1/2}}{(w(s)R + \zeta(s))} \]

where in delay form \( w(s) = \left( 1 + e^{-2\pi l \omega s} \right) / \left( 1 - e^{-2\pi l \omega s} \right) \)
and \( \tilde{w}(s) = \left( w^2(s) - 1 \right)^{1/2} = \frac{2e^{-\pi l \omega s}}{\left( 1 - e^{-2\pi l \omega s} \right)} \)

Alternatively, commensurate with the pipeline system topology, equations 22 and 23 may be written as:

\[ \frac{Q(s)}{P(s)} = \frac{(\zeta(s)w(s) + R)}{\zeta(s)(w(s)R + \zeta(s))} \]

and

\[ \frac{Q_i(s)}{P_i(s)} = \frac{(w^2(s) - 1)^{1/2}}{(\zeta(s)(w(s) + R))} \]
The block diagram for the representation given by equations 24 and 25 is as shown in Fig. 2, which is in series form whereas equations 22 and 23 provide the parallel equivalent realisation, for this system model.

To simplify the simulation process it would be prudent to construct sub-system, blocks for \( \hat{w}(s) \) and \( \hat{\dot{w}}(s) \). Since, from equation 17:

\[ \chi(s) = as + b \]

where the low frequency approximation is:

\[ a = \frac{C}{g} + \left( \tau_i + \tau_j \right) \quad \text{and} \quad b = 1 \]  
(26)

so that:

\[ w(s) = \left( 1 + e^{-2\pi(a+b)} \right) \]  
(27)

so that the output following any arbitrary finite input change would be stable since:

\[ b > 0 \]

From the geometry of the approximation given by equation 8, evidently:

\[ (\tau_i + \tau_j) > (\tau_i + \tau_j) \quad \text{so that} \quad a > 0 \]

resulting in the finite time delay \( e^{-2\pi a} \)

Also, for \( \hat{\dot{w}}(s) \), in delay form is:

\[ \hat{\dot{w}}(s) = \frac{2e^{-2\pi(a+b)}}{1-e^{-2\pi(a+b)}} \]  
(28)

and this produces a stable output response since again: \( b > 0 \) and \( a > 0 \) gives a finite time delay \( e^{-2\pi a} \) and attenuation \( e^{-2\pi a b} \).

In this case, in accordance with the break frequency for \( \frac{1}{\sqrt{L/r}} \) etc.

then following division the low frequency approximation from equation 26 is:

\[ \chi(s) = 1 + 1648.9s \quad \text{and} \quad \chi(s) = \left[ \frac{1666.6s + 1}{20s + 1} \right] \left( \frac{1.5s + 1}{0.166s + 1} \right) \]

As the pipeline length varies, the characteristic impedance \( \chi(s) \), the exit resistance \( R \) and the compressor model remain invariant. Consequently, only \( w(s) \) and \( \hat{\dot{w}}(s) \) need to be adjusted, in the simulation model, to obtain the gas flow characteristics for any length of pipeline with the same diameter and per unit length resistance values. In this regard, substituting for \( \chi(s) \) in the equations for \( w(s) \) and \( \hat{\dot{w}}(s) \) are given by equations 27 and 28. Hence for the 1,000 m pipeline:

\[ w(s) = \left( \frac{1 + 0.8555e^{-7.7062\tau}}{1 - 0.8555e^{-7.7062\tau}} \right) \]

\[ \hat{\dot{w}}(s) = \left( \frac{1.8508e^{-38.5813\tau}}{1 - 0.8555e^{-7.7062\tau}} \right) \]

for the 5,000 m pipeline:

\[ w(s) = \left( \frac{1 + 0.4609e^{-385.3\tau}}{1 - 0.4609e^{-385.3\tau}} \right) \]

\[ \hat{\dot{w}}(s) = \left( \frac{1.3578e^{-192.5794\tau}}{1 - 0.4609e^{-385.3\tau}} \right) \]

and for the 10,000 m pipeline:

\[ w(s) = \left( \frac{1 + 0.2125e^{-770.62\tau}}{1 - 0.2125e^{-770.62\tau}} \right) \]

\[ \hat{\dot{w}}(s) = \left( \frac{0.9218e^{-385.15\tau}}{1 - 0.2125e^{-770.62\tau}} \right) \]

The distributed – lumped parameter block representation for the series configuration including the compressor unit, given by:

\[ P(s) = k \frac{1}{\tau s + 1} \]

is shown in Fig. 2.
Following unit step changes on the compressor motor voltage input, the changes in the volume flow at \( Q_1(t) \) and \( Q_2(t) \) are shown in Fig. 3 in dotted and bold lines, respectively. This figure is initially for a 1000 m long, 1.0 m diameter, distributed parameter pipeline model, with \( r = 0.6 \times 10^{-4} \text{Nsec/m}^5 \) and \( g = 10^{-3} \text{m}^3 / \text{Nsec} \), with the remaining parameters given earlier. Upon increasing the pipeline length to 5000 m and then 10,000 m the responses, following a 1% change in the compressor motor voltage, are as shown in this figure again with dotted and bold traces for the inlet and exit volume flow transients, respectively.

![Fig. 3, Percentage Changes in Inlet and Exit Pipeline Volume Flow Rates for 1m Diameter Gas Pipeline with an Exit Resistance of 1 Nsec/m^5](image)

7. Conclusion

In this contribution, the theory for modelling long pipelines was presented. It was shown that accurate, unambiguous, simple models could be easily constructed for pipeline – compressor configurations with the model-simulation block diagram requiring no more than 4 basic sub-system models for the compressor, and \( w(s), \left( w^2(s) - 1 \right)^{1/2} \) and \( \zeta(s) \) for the complete system representation.

The distributed parameter theory involved was also uncomplicated with the incorporation of the continuous series and shunt resistances and the gas stream capacitance and inductance, energy storage effects. Consequently, the perturbed transient volume flow responses following any input variation are readily available, from the theoretical development and simulation process, established herein.

The transient response representations for pipelines up to 10km were obtained from the application study. Owing to the generality of the theory, pipelines with a variety of physical features could be accommodated.

References