Assessment of the muzzle velocity decrease and the barrel bore lifetime

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Abstract: The muzzle velocity is an important characteristic of the tank weapon system. As the barrel bore does wear during service, the muzzle velocity decreases to its limit when the barrel lifetime ends. The prediction of reaching the limit is being done empirically or is absent in the conditions of Czech Armed Forces at the time. The paper offers mathematical prediction method of number of shots left to the end of barrel lifetime based on interior ballistics modeling and barrel bore profile measurement and analysis. This prediction could be made more precise continuously and could be beneficial to military material management.

Key-Words: bore, wear, muzzle velocity, interior ballistics, barrel lifetime, military material management

1 Introduction
Muzzle (or more precisely initial) velocity $v_0$ of the projectile shot from the barrel weapon system is the crucial input of the exterior ballistics model and necessary quantity utilized when the problem of hitting the target is being solved. The more barrel is used the more muzzle velocity decreases. It is the result of the barrel bore wear (the forcing cone part in particular) leading to enlargement of the initial combustion volume $c_0$ (the effect of leading part of the barrel bore wear and the powder gases increased leakage are negligible compared to the main cause). The scale of the wear is measured before the deployment of the weapon system and calculated muzzle velocity deviation is inserted into the fire control system (ballistic computer). Erroneous determination of the real muzzle velocity could cause missing the target. Therefore measurement results are monitored and evaluated by responsible material manager and if the muzzle velocity decrease reaches the value of the velocity decrease limit $VDL$, the barrel is put out of service.

In the paper we are focused on the weapon system of the tank T-72M4CZ, specifically on smooth bore cannon 2A46 of calibre 125 mm, which is in service not only in the Czech Armed Forces. The projectiles used in service are of types APFSDS (in several modifications), HE and HEAT. Considering the APFSDS projectiles the muzzle velocity of 1800 m/s is produced when typical initial combustion volume is of $13,1\cdot10^{-3}$ m$^3$. In the case of HE projectiles $v_0=906$ m/s, in the case of HEAT projectiles $v_0=852$ m/s, as both are characteristic with the initial combustion volume of $c_0=10.87\cdot10^{-3}$ m$^3$. The lifetime of the barrel bore is limited by the condition of muzzle velocity drop by 4 percent ($VDL=0.04$).

![Fig.1 – Comparison of projectile velocity numerically modelled development along new barrel bore (solid line) to extremely worn barrel bore (dotted line), the muzzle velocity decreased to 1480 m/s as a result of initial combustion volume enlargement to 24.9 $\cdot10^{-3}$ m$^3$.](image)
2 Problem Formulation
Current method of barrel bore wear diagnostics is based on the measurement by the PKI-26 bore gauge instrument. It is prescribed to realize one measurement of the inside diameter of barrel bore at one place of certain depth (10 mm behind the forcing cone), the precision of the measurement is relatively low (standard deviation equals approximately 0.05 mm). The value obtained by described measurement is converted to the relative decrease of the muzzle velocity by the empirically established tables (see Table 1). The mathematical model of the lifetime prediction is absent presently, as the barrel lifetime left guess is dependent on the commander’s or material manager’s experience only.

<table>
<thead>
<tr>
<th>APFSDS</th>
<th>HE and HEAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δd [mm]</td>
<td>-Δν₀/ν₀</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td>2.0</td>
<td>1.4</td>
</tr>
<tr>
<td>2.5</td>
<td>1.7</td>
</tr>
<tr>
<td>3.0</td>
<td>2.5</td>
</tr>
<tr>
<td>3.4</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table 1 – Conversion tables for measured diameter increase Δd and relative decrease of muzzle velocity for different types of ammunition [3].

There will be established sufficient conditions for the more precise prediction of both the muzzle velocity drop and the barrel bore lifetime left, if the real barrel bore profile is mapped in further detail (the measurement in more places along the barrel bore at fine intervals is needed as well as more measurement of inside barrel bore diameter at one place to eliminate barrel bore ovality) using the more precise instrumentation (e.g. BG CG-D160 or BG-20 Mk.II) and the suitable model of interior ballistics (acceleration of the projectile in the barrel bore) is placed. That implies the requirement of ascertainment of continuous function

\[ v₀ = fₜ (c₀) \] (1)

defined on the enough width domain (from the start of lifetime to the moment of putting out of service) as well as the requirement of setting up method resulting into sufficiently precise determination of argument \( c₀ \) in the equation (1).

3 Problem Solution
3.1 Wear limit assessment
The functional dependency of the muzzle velocity on the size of the initial combustion volume could be established by the numerical integration of the mathematical model of interior ballistic action, when the entire solution problem is transferred on the basis of gunfire adjustment theory into adjustment relation

\[ \frac{\partial ν₀}{ν₀} = l_c \frac{\partial c₀}{c₀}, \] (2)

where adjustment coefficient \( l_c \) presents percentual change of \( ν₀ \) induced by change of \( c₀ \) by 1 percent. The value (or better dependency) of adjustment coefficient could be gained from the measurement results analysis or by repeatedly solved mathematical model of the interior ballistics action applying small changes of \( c₀ \). Considering the second way the notation of relation (2) will change into

\[ \frac{Δν₀}{ν₀} = l_c \frac{Δc₀}{c₀}. \] (3)

The value of adjustment coefficient \( l_c \) for approximate estimates could be picked from the approximate adjustment relations derived in [3], on which basis is given the value -0.33.

As possible method suggestion we have chosen the numerical solving of the problem using the results of complex interior ballistics model. The computation of interior ballistics was done by...
implementing solving the following system of equations

\[
\psi = \kappa' z + \kappa \lambda z^2 + \kappa \mu z^3
\]

\[
f \omega \psi - \frac{1}{2} \Theta \varphi m_q v^2 = p
\]

\[
p = \frac{\varphi m_q d_v}{s (l_p + l)} + p_z
\]

\[
\frac{d}{dt} \left( \frac{m + u_1 (p + p_z)}{l_k} \right)^\nu
\]

\[
\frac{dz}{dt} = \frac{m + u_1 (p + p_z)}{l_k}
\]

\[
l_p = l_0 \left[ 1 - \frac{\Delta}{\delta} - \Delta \psi \left( \alpha - \frac{1}{\delta} \right) \right]
\]

\[
\frac{d}{dt} \left[ 1 - \frac{\Delta}{\delta} - \Delta \psi \left( \alpha - \frac{1}{\delta} \right) \right]
\]

where \(\psi\) stands for the relative quantity of burnt-out powder, \(\kappa, \lambda, \mu\) are geometric characteristics of the powder grain, \(z\) is the relative burnt thickness of the powder grain, \(p\) is the ballistic pressure and \(p_z\) is the primer pressure, \(f\) is the specific energy of propellant, \(\varphi\) is the fictivity coefficient of the projectile, \(s\) is the cross-section area of bore, \(m, u_1, \nu\) are rate of burning coefficients, \(l_k\) is the pressure impulse of ballistic pressure, \(l\) is the projectile travel as \(l_p\) is the relative length of initial combustion volume, \(\Theta\) is the heat parameter of powder expansion, \(\Delta\) stands for the loading density, \(\delta\) for the powder mass density and \(\alpha\) is the covolume of powder gases.

We have used the Runge-Kutta fourth order method. Resulting dependency is illustrated in comparison in the following figure.

If the inverse function to (1) is determined and the value of the velocity decrease limit \(V_{DL}\) is substituted we obtain the wear limit (in terms of the limit of change of initial combustion volume \(\Delta c_0\) which dictates the moment of putting the barrel out of service:

\[
\Delta c_0 = f_w^{-1} \left[ v_0 (1 - V_{DL}) \right].
\]

| Wear limit assessment \([10^{-3} \text{ m}^3]\) |
|-----------------|-----------------|-----------------|
| Empiria         | Adjustment theory | Numerical model |
| 1,60            | 1,57            | 1,85           |

Table 2 – Comparison of wear limit values assessed by different methods.

3.2 Initial combustion volume measurement

The barrel bore profile has to be analyzed prior to volume value assessment. We carried out several measurements and analysis of barrel bore profiles on cannons in service just as the cannon barrels at Military Technical and Testing Institute Zahorie (Slovakia) [1]. Inside diameters were measured by the BG-20 Mk.II instrument (the standard deviation was approximately 0.01 mm). At one place were measured both horizontal diameter \(d_h\) and vertical diameter \(d_v\), relevant location of the measuring head was measured from the breech barrel end by the laser rangefinder Leica Disto (the standard deviation was approximately 0.4 mm). The simplified diagram is illustrated in following figure.

The initial combustion volume enlargement could be explained by two main causes. Firstly the wear (erosion) of the combustion chamber itself is considered. The second cause is the vacation of the certain volume by the projectile, which is indented more deeply in the barrel bore as a result of the forcing cone extension and shift (the place is located
where the diameter of bore is equal to indentation diameter of projectile’s obturator, at analyzed profile the location is marked \( n_{ob} \). The size of the effect was determined as the difference between the typical volume of new combustion chamber \( V_{new} \) and volume resulting from the barrel bore profile measurement \( V_{meas} \):

\[
\Delta c_0 = V_{meas} - V_{new}.
\]  

(6)

We calculated the size of the volume \( V_{meas} \) as a volume of measured barrel bore profile when simplification of the oval cross-section and linear interpolation between each two neighbouring points was applied up to the place of indentation \( n_{ob} \). Real (measured) volume of initial combustion space equals to

\[
V_{meas} = \sum_{i=1}^{n_{ob}} \frac{\pi}{12} (l_{i+1} - l_i)(\overline{d}_i^2 + \overline{d}_{i+1}^2 + \overline{d}_i \overline{d}_{i+1}),
\]

(7)

where \( \overline{d} = \sqrt{d_h d_v} \) is geometric mean of the horizontal and vertical inside diameter of barrel bore.

3.2 Bore lifetime assessment

The intensity of barrel bore wear varies with the different types of ammunition used. The most significant wear intensity could be observed in the case of APFSDS projectiles, intensity of others is lower. As a result of carried out measurement and with use of records of ‘books of the gun’ is possible to establish relative intensity of barrel bore wear according to the ammunition type. This relative projectile type coefficients \( k \) are also recommended in [4].

<table>
<thead>
<tr>
<th>Projectile type coefficients</th>
<th>APDSFS</th>
<th>HE</th>
<th>HEAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.167</td>
<td>0.250</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 – Recommended relative projectile type coefficients.

Therefore the real number of shots \( n \) could be converted into equivalent number of shots by the APFSDS projectiles \( n_e \) with use of relation

\[
n_e = n_{APFSDS} + k_{HE} n_{HE} + k_{HEAT} n_{HEAT}.
\]

(8)

The increment of the wear is the exponential function of number of shots [1], since the change of the initial combustion volume could be expressed by the equation

\[
\Delta c_0 = e^{Kn_e} - 1,
\]

(9)

where \( K \) is the coefficient of the wear progress.

The value of the coefficient \( K \) could be recommended for given type of weapon system, however it is expedient to determine the value individually for each piece of weapon system on the basis of carried out measurement and calculation of \( \Delta c_0 \) in compliance with (6). When only one measurement is considered, then by the modifying and solving equations (6) and (9) we can obtain

\[
K = \frac{\ln (V_{meas} - V_{new} + 1)}{n_e}.
\]

(10)

When the number of measurements is higher, it is suitable to use approximative method in order to determine the value of wear progress coefficient; we have applied the least square method. The advatage of more measurements completed is the higher reliability of wear development determination.

Fig. 5 – Measured data and data aproximation (solid line, \( K=0.039 \)) together with coefficient K development (dotted line).

Fig. 6 – The lifetime prediction development when additional measurement are completed.

The main interest of the commander (responsible material manager) is the number of shots left to the end of barrel bore lifetime \( n_z \). After the barrel bore profile is measured (and the volume \( V_{meas} \) is calculated consequently), the knowledge of number
of shots of each type of ammunition and the barrel bore wear limit in the form of the velocity decrease limit are incorporated, then the desired number could be obtained as a product of the relation resulting from solution of mentioned equations:

$$n_c = n_e \left( \ln \left( \frac{(1 - V_{DL})v_0 - V_{new} + 1}{\ln (v_{meas} - V_{new} + 1)} \right) - 1 \right). \quad (11)$$

The number of shots left is given in the terms of the equivalent number of APFSDS shots. The real number of shots left is dependent on intended combination of types of ammunition to fire. If only the most frequent APFSDS and HEAT ammunition is considered, the possible combinations of real number of shots could be read from the diagram below easily.

![Diagram showing possible combinations of real numbers of HEAT and APFSDS projectiles shots left to the end of the barrel bore lifetime.](image)

**4 Conclusion**

Under the conditions of Czech Army there is not used any real system of barrel bore wear prediction. The given rules and procedures allows only the phase of the wear determination at the time. The described way of bore wear evaluation requires

- the analysis of profile of the barrel bore before the first use in order to eliminate tolerances of the dimensions;
- the determination of the method, places and intervals of the profile measurement with respect to the type of used ammunition;
- reliable records of types and quantity of used ammunition and
- reliable records of the firing conditions (particularly the firing regime and gunfire conditions),

then will be possible to predict the bore wear development and this way the assessment of the barrel bore lifetime left in the numbers of shots including the possible combinations of shots by different types of projectiles. This prediction could be made more precise continuously by the additional measurement and further supported by the muzzle (initial) velocity measurement and could play a key role concerning the evaluation of the technical readiness of the weapon systems of the units and during the maintenance planning.

**References:**


