Uncertainty Assessment for Material Selection at Conceptual Design: Complementarities of Probabilistic and Evidence Approaches

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Abstract: - A major issue in conceptual design of composite material used in space exploration vehicles is the unavailability of operational data for uncertainty assessments. Expert judgment methodologies using probabilistic approaches have been utilized in prior studies to quantify uncertainty as probability distributions. For this study, a complimentary probabilistic and evidence theory approach is utilized to enhance uncertainty assessments in the area of critical safety characteristics of composites during conceptual design. This combined approach has been applied to a composite materials used for space missions evaluates uncertainty assessment for a space transportation system. Uncertainty estimates are presented bounded by belief and plausibility functions. The results may provide additional information to the decision makers in critical system safety and uncertainty assessments.

Key-Words: - Uncertainty; Expert judgment elicitation; Evidence theory

1 Introduction

In conceptual design of composite material used in space exploration vehicles, quantifying operational uncertainty and performing risk analysis is a challenging task mainly due to lack of data. Asking disciplinary experts for their "best expert judgment" may sometimes be the only option available. Expert judgment (EJ) methodologies were utilized in prior studies for quantifying input parameter uncertainty as probability distributions so that probabilistic risk analysis studies can be conducted [1]. Data obtained utilizing EJ can in many cases provide a basis for analysis and interpretation of significance of risk [2]. Through the use of EJ, prior studies introduced an approach to quantify critical system design parameter uncertainty as probability distributions [3]; however, there is significant uncertainty in these judgments themselves, and a probabilistic assessment alone may not be sufficient.

This study explores a complimentary probabilistic and non-probabilistic approach for uncertainty assessment in conceptual design. The extension of the efforts to define the development of a more robust approach for uncertainty assessment is explored through evidence theory [4]-[16]. Evidence theory provides a promising addition to current probabilistic uncertainty assessment practices, and the combination of the approaches may allow for a more realistic representation of the implications of uncertainty, given the complex nature of real world problems.

In this study, comparison of two theories was conducted: the probability theory and evidence theory. Application used as an example was the composite material for a Space Exploration Vehicle (SEV) to assess the level of uncertainty using expert judgment elicitation and the combined methods of probabilistic and non-probabilistic approach.

2 Probabilistic Approach

Probability theory provides a mathematical structure traditionally used in the representation of aleatory (i.e., random) uncertainty for well known systems. Aleatory uncertainties are typically modelled as random variables described by probability distributions. A probability in this case refers to the number of times an event occurs divided by the total number of trials. For instance, the flipping of a truly fair coin would have a probability of landing on heads of 0.5, indicating that for every N trials, the coin would land heads up, 0.5*N times. In order to attain the actual probability for an event, an experiment would have to be repeated an infinite number of times. Since this is impossible, decision makers typically make assumptions about the characteristics of the probabilities (i.e. the mean and variances). Given the lack of operational data in conceptual design for one-of-a-kind systems, one may have to rely on expert judgment data obtained by a probability elicitation method to quantify CDFs in representing uncertainty. The use of expert judgment or opinion to aid in decision-making is well known. The knowledge of subject matter experts (SMEs) has been "mined" in many disciplines (such as medicine, weather forecasting, and military tactics) to provide estimates for parameters associated with yet-to-be-developed systems (such as advanced space launch vehicles) [2][3].

A probability elicitation method may be any aid that is used to acquire a probability from an expert [18]. Generally, a distinction is made between direct and indirect methods. With direct methods, experts are asked to directly express their degree of belief as a number, be it a probability, a frequency or an odds ratio. For expressing probabilities, however, people prefer to express their beliefs linguistically rather than numerically. This is likely because the ambiguity of words captures the uncertainty they feel about their probability assessment; the use of numerical probabilities can produce considerable discomfort and resistance among those not used to it [19]. In addition, since directly assessed numbers tend to be biased, various indirect elicitation methods have been developed to quantify parameters of a CDF for uncertainty [11].

However, probabilistic approaches to uncertainty assessment have been criticized for lacking the capability of capturing epistemic uncertainty [12]. Klir notes that as a consequence of this criticism, supporting theories have been developed and categorized into the "fuzzy measure theory" [13]. One such approach, evidence theory, takes into account aleatory and epistemic uncertainty that is bounded by the belief and plausibility functions [Bel(Ai) , Pl(Aj)] and is found without any assumptions made on the information obtained from the experts [13]. Evidence theory is discussed in further detail in the following section.

3 Evidence Theory

Evidence theory originated with Arthur Dempster in the 1960's and was expanded by Glen Shafer in the 1970's [4][5][6]. In evidence theory, uncertainty is separated in Belief (*Bel*) and Plausibility (*Pl*), whereas traditional probability theory uses only the probability of an event to analyze uncertainty [11]. Belief and plausibility provide bounds on probability. In special cases, they converge on a single value, probability. In other cases, such as in the evidence theory representation of uncertainty, they represent a range of potential values for a given parameter, without specifying that any value within the range is more or less likely than any other. The Dempster-Shafer evidence theory has three important functions: the basic probability assignment function (BPA or m), the Belief function (*Bel*), and the Plausibility function (*Pl*) [11]. These three functions can be viewed as alternate representations of uncertainty regarding the same parameter x [11].

The basic probability assignment (BPA) is a primitive of evidence theory. BPA does not refer to probability in a classical sense; rather, it defines a mapping of the power set to an interval between 0 and 1. The value of the BPA for a given set A (represented as $m(A)$), expresses the proportion of all relevant and available evidence that supports the claim that a particular element of X (the universal set) belongs to the set A but to no particular subset of A [4]-[7]. From the basic probability assignment, the upper and lower bounds of an interval can be defined [14]. This interval contains the precise probability of a set of interest (in the classical sense) and is bounded by two non additive continuous measures called Belief and Plausibility. In addition to deriving these measures from the basic probability assignment (m), these two measures can be derived from each other. For example, Plausibility can be derived from Belief in the following way:

$$Pl(A) = 1 - Bel(\bar{A}) \qquad (1)$$

Where A is the classical complement of subset A [4][5][6].

The probability is uniquely determined if $Bel(A) = Pl(A)$. Otherwise, $Bel(A)$ and $Pl(A)$ and may be viewed as lower and upper bounds on probabilities respectively, where the actual probability is contained in the interval described by the bounds [8]. Upper and lower probabilities derived by the other frameworks in generalized information theory cannot be directly interpreted as Belief and Plausibility functions [9]. In other words, the basic belief assignment is not a probability, but just a belief in a particular proposition irrespective of other propositions. This structure gives the flexibility to express belief for possible propositions with partial and insufficient evidence [10][11]. According to Belief and Plausibility Functions, the likelihood for Event A lies in the interval [$Bel(A)$, $Pl(A)$] and may be shown as in Figure 1 [10].

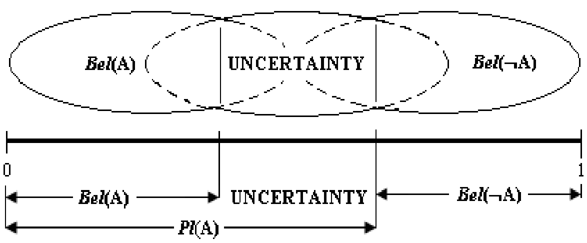

Figure 1. Belief (Bel) and plausibility (PL) relationship

Dempster-Shafer [4][5][6] methods of Evidence Theory may be applied by identifying the upper limit of uncertainty called Cumulative Plausibility Function (CPF) and lower limit of uncertainty called Cumulative Belief Function (CBF). Both probability theory and evidence theory are applied to a case study in the following section.

Figure 2 shows a graphical representation [17].

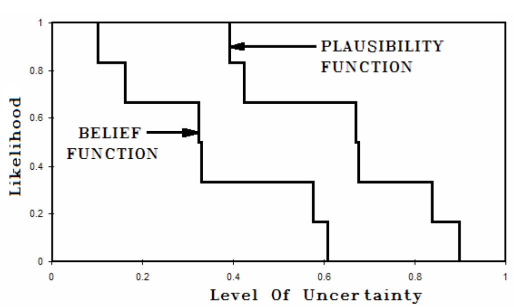

Figure 2. Graphical Representation of CPF and CBF

4 Example Application: Using FRP as Space Materials

This research extends the current studies on composite and fiber reinforced plastic (FRP) material and their viable application as space materials. A design for composite materials was selected to incorporate an uncertainty assessment using expert judgment elicitation through a combined probabilistic and non-probabilistic, evidence theory approach. Three variables were chosen that would lead to a critical subsystem failure of the material during its lifecycle. It was thought that critical subsystem failures may be a function of Construction (production), Installation (delamination) and Operations (such as, debris damage at lift-off that causes burn through). These failures were:

1) Construction anomalies that can occur during composite material production,
2) Installation anomalies that may lead to the possibility of delamination of FRP,
3) Operations anomalies that may result from debris damage at any time during mission, and
4) All combinations of the three anomalies.

Figure 3 shows the process that was followed in analyzing the composite material.

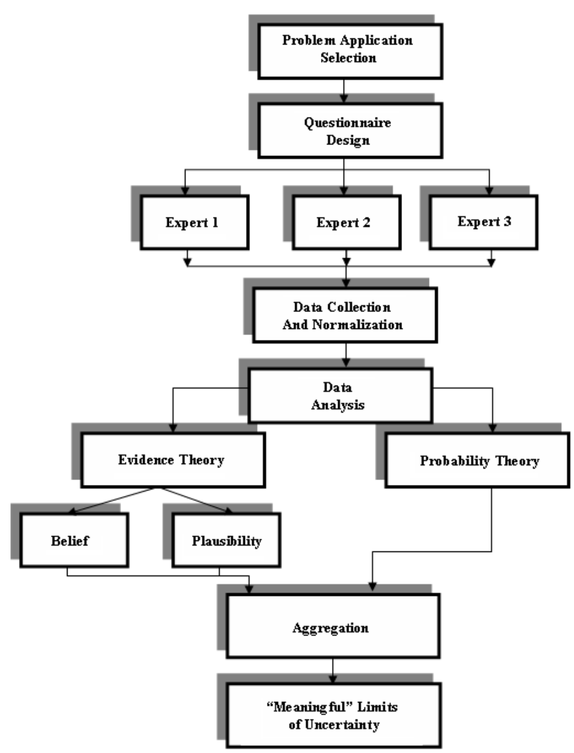

Figure 3. Combined approach for uncertainty assessment

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A pre-selected panel of three NASA systems engineers with significant knowledge of such a system agreed to participate in this study. This pre-selected team of experts played a key role in the design and structural testing needed during the past and future development of composite material. An expert judgment elicitation questionnaire was developed to quantify potential anomalies for this study [1][2][3]. Once the data was collected, a normalization factor was applied to each expert's input to comply with Evidence theory. The results from each expert were then utilized in constructing cumulative distribution functions (CDF) and determining belief and plausibility measures.

The questionnaire followed a combination of various methodologies [1][2][3]. The experts were asked to consider the input parameters and select an option representing the believed assessment based on the given selection of anomalies and the nominal values. Through the questionnaire, each expert was asked the likelihood of each scenario. The experts provided low, moderate and high likelihood values for each anomaly.

The experts also provided their personal opinion as to which of the values is most likely to occur. The answers of the questionnaire were used to develop the basic assignment of each expert in an additive manner to compute the unions of belief and plausibility measures. Then the aggregated results were input into a Monte Carlo simulation using @RISK® software [20], in order to generate distribution data for the experts' input parameters.

Finally, limits of uncertainty were derived and conveyed in a graphical representation that may potentially enable decision makers to better assess uncertainty levels presented by multiple experts in high-risk environments. The questionnaire was designed specifically to serve as a means of dual analysis: First, probability theory is utilized to addresses the probability of the occurrence of an event (system failure due to an anomaly) and second, evidence theory is used to addresses the degree of uncertainty of whether an event will occur. The bounds provided by evidence theory provide more accurate estimates of the uncertainty presented in these real world environments than the point estimates provided by traditional probability theory.

4.1 Probabilistic Risk Assessment

The questionnaire was used to collect each expert's assessments of possible percentage of anomalies that can lead to critical failure of the composite material due to problems in construction (C), installation (I), operations (O), and their possible combinations [construction union Installation (CUI), construction union operations (CUO), installation union operations (IUO), and due to construction union installation union operations (CUIUO)].

Throughout the questionnaire, each expert was asked the likelihood of each scenario and was also asked to provide low, moderate and high likelihood values of anomaly (in percentage) that can result in a critical system failure. The experts also provided their personal opinion as to which of the values is most likely to occur. Figure 4 presents each expert's assessments for construction, installation, operations, and the unions in minimum, most likely, and maximum likelihood numbers.

PROBABILITY TO PRODUCE CDF					
Expert 1		Expert 2		Expert 3	
Construction		Construction		Construction	
Min	0.0750	Min	0.0500	Min	0.0500
Most Likely	0.2000	Most Likely	0.2000	Most Likely	0.1500
Max	0.5000	Max	0.5000	Max	0.5000
Installation		Installation		Installation	
Min	0.0750	Min	0.1000	Min	0.0500
Most Likely	0.2000	Most Likely	0.5000	Most Likely	0.1500
Max	0.5000	Max	1.0000	Max	0.5000
Operations		Operations		Operations	
Min	0.0500	Min	0.0010	Min	0.0500
Most Likely	0.1500	Most Likely	0.0500	Most Likely	0.1500
Max	0.3000	Max	0.0900	Max	0.5000
C U I		C U I		C U I	
Min	0.0750	Min	0.1000	Min	0.0500
Most Likely	0.2000	Most Likely	0.5000	Most Likely	0.1500
Max	0.4000	Max	1.0000	Max	0.5000
C U O		C U O		C U O	
Min	0.0750	Min	0.0500	Min	0.0500
Most Likely	0.1500	Most Likely	0.2000	Most Likely	0.1500
Max	0.3000	Max	0.5000	Max	0.5000
I U O		I U O		I U O	
Min	0.0750	Min	0.1000	Min	0.0500
Most Likely	0.2000	Most Likely	0.5000	Most Likely	0.1500
Max	0.4000	Max	1.0000	Max	0.5000
C U I U O		C U I U O		C U I U O	
Min	0.1000	Min	0.1333	Min	0.0500
Most Likely	0.2000	Most Likely	0.4666	Most Likely	0.1500
Max	0.4000	Max	1.0000	Max	0.5000

Figure 4. Expert Assessment for likelihood of Anomalies

Triangular distributions were constructed from this expert assessment data in terms of minimum (a), most likely (c) and maximum values (b). Next, a Monte Carlo simulation was performed by sampling from these triangular distributions to determine the overall likelihood of any critical failure. This operation was done using the @RISK® software

[20]. The CDF curves for the overall likelihood of a critical failure were constructed for each expert.

4.2 Non-Probabilistic Risk Assessment Using Evidence Theory

The expert assessments from the questionnaire were also incorporated into the basic probability assignment (m) of the Evidence theory for the computation of the Belief (lower) and Plausibility (upper) limits of uncertainty; however, before beginning the computations, the basic probability assignment must be normalized such that summation of all inputs (Failure Causes) equal to:

$$\sum_{all\ A \in P_x} m(A) = 1 \qquad (2)$$

The next step is to substitute the normalized basic assignments into m1 basic assignment column. Figure 5 lists the possible failure causes based on Dempster-Shafer's Belief and Plausibility functions as follows:

- The first three failure causes (C, I, & O) or subsets are directly mapped into the belief column.
- The values of CUI are the additive values of C, plus I, plus CUI.
- The values of CUO are the additive values of C, plus O, plus CUO.
- The values of IUO are the additive values of I, plus O, plus IUO.
- The assignment of CUIUO was computed based on the equation shown, to obtain a total of one for the assignments provided by each expert.

The belief and plausibility measures were computed based on the following equations for any set Ai∈Px:

$$Bel(A_j) = \sum_{all\ A_j \subseteq A_i} m(A_i)$$

$$Pl(A_i) = \sum_{all\ A_j \cap Ai \neq \varnothing} m(A_j) \qquad (3)$$

BELIEF COMPUTATIONS						
SUBSET*	EXPERT 1		EXPERT 2		COMBINED JUDGMENT 1,2	
Failure Cause	m_1	Bel_1	m_2	Bel_2	$m_{1,2}$	$Bel_{1,2}$
C = Construction Error	0.17	0.17	0.02	0.02	0.15	0.15
I = Installation Error	0.17	0.17	0.18	0.18	0.35	0.35
O = Operations Error	0.04	0.04	0.00	0.00	0.05	0.05
C U I	0.11	0.45	0.18	0.39	0.13	0.62
C U O	0.09	0.30	0.02	0.04	0.05	0.25
I U O	0.17	0.38	0.18	0.37	0.16	0.56
C U I U O	0.25	1.00	0.41	1.00	0.11	1.00
TOTAL	1.00		1.00		1.00	
PLAUSIBILITY COMPUTATIONS						
SUBSET*	EXPERT 1		EXPERT 2		COMBINED JUDGMENT 1,2	
Failure Cause	m_1	Pl_1	m_2	Pl_2	$m_{1,2}$	$Pl_{1,2}$
C = Construction Error	0.17	0.63	0.02	0.63	0.15	0.44
I = Installation Error	0.17	0.70	0.18	0.96	0.35	0.75
O = Operations Error	0.04	0.55	0.00	0.61	0.05	0.38
C U I	0.11	0.96	0.18	1.00	0.13	0.95
C U O	0.09	0.83	0.02	0.82	0.05	0.65
I U O	0.17	0.83	0.18	0.98	0.16	0.85
C U I U O	0.25	1.00	0.41	1.00	0.11	1.00
TOTAL	1.00		1.00		1.00	

Figure 5. Dempster-Shafer's Belief and Plausibility for Experts 1 and 2

As an example, Figure 5 shows that belief for Failure Cause C for Expert 1 is 0.17 and plausibility is 0.63. These numbers indicate a measure of the lower and upper limits of uncertainty for Expert 1 as expressed by the expert. A similar operation is repeated for Expert 2. A modification of the Dempster-Shafer combination rule was used to combine the assessments of Expert 1 and Expert 2 [4]. Since there were three experts in this study, Yager's rule of combination was used to expand the number of experts from two to three [8].

The combined judgment generated by Experts 1 and 2 is transferred into Figure 6 and the third expert's basic assignment is computed. The results produce the combined judgments of all three experts.

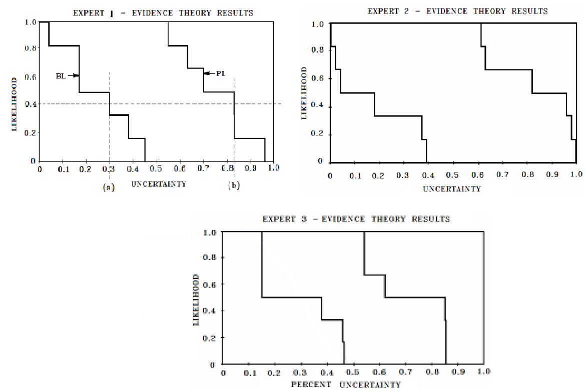
BELIEF COMPUTATIONS						
SUBSET*	EXPERT 1,2		EXPERT 3		COMBINED JUDGMENT 1,2,3	
Failure Cause	$m_{1,2}$	$Bel_{1,2}$	m_3	Bel_3	$m_{1,2,3}$	$Bel_{1,2,3}$
C = Construction Error	0.15	0.15	0.15	0.15	0.21	0.21
I = Installation Error	0.35	0.35	0.15	0.15	0.40	0.40
O = Operations Error	0.05	0.05	0.15	0.15	0.17	0.17
C U I	0.13	0.62	0.07	0.38	0.05	0.66
C U O	0.05	0.25	0.15	0.46	0.05	0.42
I U O	0.16	0.56	0.15	0.46	0.10	0.66
C U I U O	0.11	1.00	0.15	1.00	0.03	1.00
TOTAL	1.00		1.00		1.00	
PLAUSIBILITY COMPUTATIONS						
SUBSET*	EXPERT 1,2		EXPERT 3		COMBINED JUDGMENT 1,2,3	
Failure Cause	$m_{1,2}$	$Pl_{1,2}$	m_3	Pl_3	$m_{1,2,3}$	$Pl_{1,2,3}$
C = Construction Error	0.15	0.44	0.15	0.54	0.21	0.34
I = Installation Error	0.35	0.75	0.15	0.54	0.40	0.58
O = Operations Error	0.05	0.38	0.15	0.62	0.17	0.34
C U I	0.13	0.95	0.07	0.85	0.05	0.83
C U O	0.05	0.65	0.15	0.85	0.05	0.60
I U O	0.16	0.85	0.15	0.85	0.10	0.79
C U I U O	0.11	1.00	0.15	1.00	0.03	1.00
TOTAL	1.00		1.00		1.00	

Figure 6. Yager's Rule Belief and Plausibility for Experts 1, 2 and 3

Lastly, these bounds or values are converted to a cumulative graphic form for each expert, through Monte Carlo simulation. The lower bounds or minimum value is called Belief and the upper bounds or maximum value is called Plausibility. In these graphs:

- the y-axis represents the expert's assessment of the likelihood of composite material system failure, and
- The x-axis represents the range of the expert's estimated confidence interval or the level of uncertainty.

Figures 7, 8, and 9 are graphical representation of uncertainty based upon the total combined evidence obtained from all 3 Experts during the elicitation process. The graphs illustrate the boundaries of belief and plausibility of this expert's hypothesis with regard to the unknown parameter. This unknown parameter is the likelihood of system failure due to the pre-defined anomalies and the various unions. The upper and lower limits shown in this graph are indicators of a conservative, minimum risk taking expert with equal levels of certainty and uncertainty, as evidenced by the wide uncertainty bounds present in the figure.



Figures 7, 8 and 9. Evidence theory Graphical Results for Experts 1, 2 and 3

Evidence theory allows the decision maker to assess the values of the belief (minimum) and plausibility (maximum) of an extended cumulative distribution function. If the separating distance between minimum and maximum values is as great as shown in Figure 8, then the level of uncertainty is larger; meaning, that additional data may be required before a decision is made.

Figure 9 is the graphical representation of uncertainty based upon the total combined evidence obtained from Expert 3 illustrating the boundaries of belief and plausibility of this expert's hypothesis with regard to the unknown parameter. Figure 9 indicates Expert 3 expressing less variance between upper and lower limits of uncertainty than Experts 1 and 2. The separating distance between minimum and maximum values in this figure is much narrower than is seen in Figure 8. This indicates that the level of uncertainty for this expert is smaller by comparison, or the expert has more confidence in his/her judgment. These narrower bounds do not necessarily make the results more dependable or usable than the other experts but the variation in results provides an understanding of the difficulty of the problem at hand.

5 Conclusion

In this study a combined probabilistic and evidence approach was utilized. This research provided more exploration into the failure modes necessary to utilize FRP and composites to their fullest potential in an effort to enhance uncertainty assessments in critical safety assessments for composite materials during conceptual design. Uncertainty estimates obtained from a panel of experts were presented bounded by belief and plausibility functions as well as probability distributions. The results suggest that this combined probabilistic and evidence approach may provide additional information to the decision maker in critical system safety and uncertainty assessments.

To elicit inputs, a questionnaire was utilized for uncertainty assessments for anomalies in a thermal protection system that can lead to a critical system failure. The resulting data was utilized to conduct a probabilistic and evidence theory based analysis. Using a graphical approach, this study provided various visual representations of the experts' uncertainty assessments. The methodology demonstrated in this study enabled the capturing of expert confidence in uncertainty assessments for complex systems. A probabilistic analysis alone may lead to conclusions that may be misleading without further investigation, while the Evidence approach does not provide a concrete non-probabilistic assessment; rather it provides an enhancement of probabilistic analysis.

The literature seems to be in concurrence that the use of Evidence theory is not fully developed and is yet to have widespread applications in the engineering field [11]. During this study, Probability theory is utilized to address the probability of the occurrence of a safety event (critical system failure due to an anomaly) while Evidence theory is used to addresses the degree of uncertainty of the results. The results suggest that the assessment of uncertainty of experts in high-risk environments may be better conveyed to decision makers by using both probabilistic and non-probabilistic theories.

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