Algebraic 1DoF Control of a Heating Process with Internal Delays

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Abstract: - This paper is focused on algebraic control of a circuit heating laboratory plant with internal (state) delays using the simple feedback control structure known as One-Degree-of-Freedom (1DoF). The existence of delayed feedbacks in the controlled system brings about inconveniences in controller design and it deteriorates the control performance, namely feedback system stability and periodicity. Algebraic notions such as rings, fields or modules proved to be suitable tools for handling with these problems. In our contribution, a recently developed and revised ring of retarded meromorphic functions is used. A mathematical model of the plant was derived in our previous works. The final controller is verified by simulations in Matlab-Simulink first. Since the plant is controlled via a discrete-time program in PC, a simple controller discretization is proposed. Finally, the control algorithm is verified on a real process and compared with simulations. The results prove the usability of the controller design methodology.

Key-Words: - Time-delay systems, Algebraic control, 1DoF control system, Matlab-Simulink, Discretization, Heating plant

1 Introduction
Circuit heating plants and processes are typical systems where internal delays can be easily imagined and modeled [1] – [5]. Unlike traditional models including delays in input-output relations, internal delays bring about problems in controller design and tuning due to the fact that they yields an infinite-dimensional controlled system itself. Although some authors suggested finite-dimensional control structures via state feedbacks, see e.g. [6] – [8], many other – mainly input-output – control methodologies result in an infinite-dimensional control system as well. Naturally, the task to assign or place the infinite spectrum appropriately to meet stability and performance requirements can be a tough proposition to be solved. Algebraic tools and their parlance, in the state or input-output formulation, proved to be suitable in an effort to overcome these obstacles.

State (physical) models of time-delay systems (TDS) apply both integrators and delay elements on the left-hand side of a differential equation, either in a lumped or distributed form, yielding functional differential equations. Using the Laplace transform, TDS can be represented by a ratio of so called quasipolynomials or a matrix of them in one complex variable $s$, formed as linear combinations of products of $s$-powers and exponential terms, rather than polynomials. Hence, the Laplace transform of LTI-TDS is no longer rational and so called meromorphic functions have to be introduced. The transfer function denominator decides about the systems stability as usual, except cases of input-output or internal distributed delays [9].

However, a meromorphic transfer function representation as a fraction of quasipolynomials is not suitable in an endeavour to meet some control requirements, such as internal stability, reference tracking, controller properness etc. [7], [9]. One of a possible algebraic structure, namely a ring, that can be used here, is the ring of stable and proper quasipolynomial meromorphic functions ($R_{MS}$). Although the original definition of the structure, [10], is convenient for the objectives of this contribution, it has some drawbacks and had to be revised and redefined [11].

The laboratory heating plant used to be controlled in this paper was assembled at the Faculty of Applied Informatics of Tomas Bata University in Zlin in order to test control algorithms for systems with dead time. The original description of the apparatus and its electronic circuits can be found in [12]. Although the plant was originally intended to test control algorithms for input delays only, it has been shown that it contains internal delays as well [13].

The paper is organized as follows. As first, a mathematical (anisochronic) model of the laboratory...
heating plant is introduced. As second, $R_{MS}$ ring is defined and 1DoF structure and basic control requirements are presented. Then, the controller structure using the ring and its discretization is derived. Description of a simple Matlab-Simulink user interface follows. A comparison of simulated (continuous and discretized) and real (laboratory) control responses finishes this contribution.

2 Circuit Heating Plant Model

The description of the circuit heating plant to be controlled according to [12] follows. A photo and a sketch of the scheme of it are displayed in Fig. 1.

![Circuit heating plant](image)

Fig. 1 – Circuit heating plant

The heat transferring fluid (namely distilled water) is transported using a continuously controllable DC pump (6) into a flow heater (1) with maximum power $P(t)$ of 750 W. The temperature of a fluid at the heater output is measured by a platinum thermometer giving value of $\vartheta_{HO}(t)$. Warmed liquid then goes through a 15 meters long insulated coiled pipeline (2) which causes the significant delay in the system. The air-water heat exchanger (cooler) (3) with two cooling fans (4, 5) represents a heat-consuming appliance. The speed of the first fan can be continuously adjusted, whereas the second one is of on/off type. Input and output temperatures of the cooler are measured again by platinum thermometers giving $\vartheta_{CI}(t)$ and $\vartheta_{CO}(t)$, respectively. The expansion tank (7) compensates for the expansion effect of the water.

A detailed mathematical model was presented in [13]. Although there are three continuous-time manipulated inputs ($P(t)$, voltage input to the pump $u_p(t)$ and voltage input to the cooler $u_c(t)$) and three measured outputs ($\vartheta_{HO}(t), \vartheta_{CI}(t), \vartheta_{CO}(t)$), the intention is to control $\vartheta_{CO}(t)$ only by $P(t)$. For this relation, it was derived the following transfer function

$$G(s) = \frac{\vartheta_{CO}(s)}{P(s)} = \frac{b_{0D} \exp(-\tau_0 s) + b_0}{s^3 + a_2 s^2 + a_1 s + a_0 + a_{0D} \exp(-\vartheta s)}$$

(1)

where all real parameters in the model are complex algebraic functions of physical quantities in the circuit and of input and output steady states. It was determined that for the working point

$$[u_p, u_c, \vartheta_{HO}, \vartheta_{CI}, \vartheta_{CO}] = [5 V, 3 V, 300 W, 44.1^\circ C, 43.8^\circ C, 36.0^\circ C]$$

(2)

that the parameters in (1) are

$$b_{0D} = 2.334 \cdot 10^{-6}, b_0 = -2.146 \cdot 10^{-7},$$
$$a_2 = 0.1767, a_1 = 0.009, a_0 = 1.413 \cdot 10^{-4},$$
$$a_{0D} = -7.624 \cdot 10^{-5}, \tau_0 = 1.5, \tau = 131, \vartheta = 143$$

(3)

For more details, the reader is referred to [13].

3 Controller Design in $R_{MS}$ Ring

The definition of the ring and a controller design procedure for 1DoF structure using the ring are aims of this section.

3.1 Revised ring definition

The original definition of the ring of proper and stable (retarded) quasipolynomial (RQ) meromorphic functions, $R_{MS}$, was introduced in [10]. However, the original definition of $R_{MS}$ has some drawbacks; in particular, it does not constitute a ring, hence, a revisited definition was proposed [11].

**Definition 1 ($R_{MS}$ ring).** An element $\mathcal{T}(s)$ of $R_{MS}$ ring can be expressed as a fraction $y(s)/x(s)$ where

$$y(s) = \mathcal{Y}(s) \exp(-\tau s)$$

(4)

where $\mathcal{Y}(s)$ is a (quasi)polynomial of degree $l$ and $\tau \geq 0$. Moreover, $T(s)$ lies in the Hardy space $H_\infty(\mathbb{C}^+)$, i.e. it is analytic and bounded in $\mathbb{C}^+$. Particularly, there is no pole $s_0$ such that $\text{Re}s_0 \geq 0$ for a retarded denominator. For a neutral one (the difference between these two types of
quasipolynomials can be found e.g. in [9], [14]), $T(s)$ is formally stable (the definition of formal stability was presented e.g. in [15]), i.e. there is no pole $s_0$ with $\Re s_0 \geq -\varepsilon, \varepsilon > 0$. If the term includes distributed delays, all roots of $x(s)$ in $\mathbb{C}^-$ must be those of $y(s)$ (i.e. they constitute removable singularities). In addition, the ratio is proper, i.e. $l \leq n$. Alternatively, the properness can be given as follows: There exists a real positive number $R$ for which holds that

$$\sup_{\Re s_0 \leq R} |T(s)| < \infty$$

(5)

see [16].

### 3.2 Control design in $R_{MS}$ for 1DoF

A concise description of controller design using $R_{MS}$ with a simple negative feedback loop depicted in Fig. 2 was presented e.g. in [17]. Although an unrevised conception of $R_{MS}$ was used there, it will do nicely for the task of this paper.

Fig. 2 – 1DoF control structure

In the figure, $W(s)$ is the Laplace transform of the reference signal, $D(s)$ stands for that of the load disturbance, $E(s)$ is transformed control error, $U_0(s)$ expresses the controller output (control action), $U(s)$ represents the plant input affected by a load disturbance, and $Y(s)$ is the plant output controlled signal in the Laplace transform. The first step is to subject all external inputs and the plant transfer function to a coprime factorization. Let the plant be initially described by the transfer function

$$G(s) = \frac{B(s)}{A(s)}$$

(7)

where $A(s), B(s) \in R_{MS}$ are coprime, i.e. there does not exist a non-trivial (non-unit) common factor of both elements and, moreover, they are Bézout coprime, see [15].

Similarly, let reference and load disturbance signals, respectively, be of forms

$$W(s) = \frac{H_w(s)}{F_w(s)}, D(s) = \frac{H_d(s)}{F_d(s)}$$

(8)

where $H_w(s), H_d(s), F_w(s), F_d(s) \in R_{MS}$.

A transfer functions analysis in 1DoF yields these requirements: The control system is stable in the sense that all transfer functions are from $R_{MS}$ if and only if the controller $G_n(s) = Q(s)/P(s)$ is given by a coprime pair satisfying the Bézout identity

$$A(s)P(s) + B(s)Q(s) = 1$$

(9)

a particular stabilizing solution which, say $P_0(s), Q_0(s)$, can be parameterized as

$$P(s) = P_0(s) \pm B(s)Z(s) \neq 0$$
$$Q(s) = Q_0(s) \mp A(s)Z(s)$$

(10)

where $Z(s) \in R_{MS}$.

Then, the reference signal is tracked if $Z(s)$ is chosen so that $F_w(s)$ divides $A(s)P(s)$. Analogously, it can be proved that load disturbance is asymptotically attenuated if $F_d(s)$ divides $B(s)P(s)$. Details about divisibility in $R_{MS}$ can be found in [11].

### 4 Plant Controller Derivation and Discretization

#### 4.1 Derivation of the controller for the heating process

Consider the plant described by the transfer function (1) and let the external inputs be from the class of linearwise functions, i.e.
where \( m_w(s) \) and \( m_d(s) \) are arbitrary stable (quasi)polynomials of degree one, say, \( m_w(s) = m_d(s) = s + m_0, m_0 > 0 \), for the simplicity, and \( w_0 \) and \( d_0 \) are real constants.

The plant transfer function is factorized as

\[
G(s) = \frac{b_0 \exp(-\tau_0 s) + b_0}{s^3 + a_2 s^2 + a_1 s + a_0 + a_0D \exp(-\delta s)}
\]  

where \( m(s) \) is a stable (quasi)polynomial of degree three, for instance, \( m(s) = (s + m_0)^3 \) for the simplicity.

If we take \( Q_0(s) = 1 \) in (9), it is obtained a particular stabilizing solution

\[
P_o(s) = \frac{(s + m_0)^3 - b_0 \exp(-\tau_0 s) + b_0}{s^3 + a_2 s^2 + a_1 s + a_0 + a_0D \exp(-\delta s)}
\]  

(13)

For reference tracking and disturbance rejection, here, \( P(s) \) must include at least one zero root. Thus, try to choose

\[
Z(s) = \frac{(s + m_0)^3}{s^3 + a_2 s^2 + a_1 s + a_0 + a_0D \exp(-\delta s)} Z_0
\]  

(14)

in (10) to have a simple form of \( P(s) \). Condition \( P(0) = 0 \) results in

\[
Z_o = \frac{m_0^3}{b_0D + b_0} - 1
\]  

(15)

The final controller structure by inserting (14), (15) into (10) reads

\[
G_D(s) = \frac{m_0 \left[ s^3 + a_2 s^2 + a_1 s + a_0 + a_0D \exp(-\delta s) \right]}{(b_{oD} + b_0)(s + m_0) - m_0 \left[ b_{oD} \exp(-\tau_0 s) + b_0 \exp(-\alpha s) \right]}
\]  

(16)

The controller owns only one selectable (free) parameter \( m_0 \) and it has a rather complex anisochronic structure [1], yet simply realizable by integrators and delay elements.

### 4.2 Controller discretization

In order to implement the controller on a discrete-time computers or PLC, it is usually inevitable to discretize its algorithm. There were investigated a large number of discretization approaches, mainly for the spectrum estimation, e.g. [18] – [20]. In our paper, a simple yet sufficient input-output method based on delta models and linear delay interpolation is used [21] – [22].

The idea rests on the introduction of variable \( \gamma \) associated with the delta operator \( \delta \) defined as

\[
\gamma = \frac{z - 1}{\alpha T_s z + (1 - \alpha) T_s}
\]  

(17)

where \( z \) is the variable from the \( z \)-transform, \( \alpha \in [0, 1] \) represents a weighting parameter and \( T_s \) means a sampling period. The choice of \( \alpha \) enables to obtain different first order models, such as forward \( \alpha = 0 \), backward \( \alpha = 1 \) or Tustin \( \alpha = 0.5 \) one. The substitution \( s \to \gamma \) in the transfer function system model results in a discrete-time model in \( z \) associated with the shifting operator \( q \). However, this substitution is applied to \( s \)-powers expressing derivatives only, whereas delay exponential terms are subjected to a natural transformation

\[
\exp(-\eta s) X(s) \to x(t - \eta)
\]  

(18)

followed by (linear) interpolation

\[
x(t - \eta) = (1 - \alpha_i) x(t - \tau_{di}) + \alpha_i x(t - \tau_{di+1})
\]  

(19)

where \( \tau_{di} = \left[ \eta_i / T_s \right], \tau_{di+1} = d_i T_s, \tau_{di+1} = (d_i + 1) T_s \), \( \tau_{di} \leq \eta_i \leq \tau_{di+1} \) and a weighting coefficient

\[
\alpha_i = (\eta_i - \tau_{di}) / T_s \in [0, 1].
\]

Then, finally

\[
x(t - kT_s) \to z^{-k} X(z)
\]  

(19)

The Tustin (trapezoidal) method was utilized in this paper. The final discrete rule is quite complex and, hence, it is not displayed here due to a limited space.
5 Simulations and Real Experiment
A comparison of continuous-time and discrete-time simulations in Matlab-Simulink with a real experiment is the objective of this section.

As first, control and output responses as differences from the operating point, $\Delta P(t)$ and $\Delta \vartheta(t)$, respectively, for three different values of $m_0$ are displayed in Fig. 3. Note that a step load disturbance $d_P(t) = -10$ enters at $t = 2000$.

![Fig. 3 – Continuous-time control responses](image1)

The value $m_0 = 0.012$ was selected for real experiments (notice that the maximum feasible value of the heat power is $\Delta P_{MAX}(t) = 450$ W).

As second, compare continuous and discrete-time responses as in Fig. 4. Obviously, they are nearly identical.

![Fig. 4 – Continuous-time vs. discrete-time responses](image2)

Finally, the discrete-time form of the controller law was used to test real control performance on the laboratory appliance (without the impact of the load disturbance), as can be seen in Fig. 5, and compared with simulations.

![Fig. 5 – Real vs. simulated responses](image3)

For the verification of the controller ability to asymptotically reject a simple “stepwise” disturbance, the cooler input voltage was abruptly changed to $U_c = 9$ V. The reaction to this disturbance is depicted in Fig. 6.

![Fig. 6 – Step disturbance rejection verification](image4)

6 Conclusion
The presented paper has introduced control of a circuit heating system using a simple feedback loop (i.e. 1DoF) via algebraic tools of the $R_{MS}$ ring.

The laboratory appliance together with its model of a relation between one manipulated input and a measured output were presented first. Then, the definition of the $R_{MS}$ structure was given to the reader. A brief general description of controller design in this ring with 1DoF system has followed. The next part of this contribution has bought the particular controller derivation for the laboratory plant ensuring asymptotical reference tracking and (stepwise) load disturbance rejection. A sketch of a simple discretization procedure has been presented as well. Finally, simulation verification of both controller rules, i.e. continuous-time and discrete-time ones, together with comparison with real laboratory control responses have been brought out. The results confirm the usability of the presented controller design approach.

In the future research, one can suggest a controller tuning ideas or to use other (more complex) control systems (2DoF, TFC, etc.).

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