Abstract: - The paper deals with the numerical solution of a conveyer drive with a hydraulic motor. The gearbox is equipped with a Maltese cross having the variable transmission ratio. The purpose of this article is to show of the conveyer kinematical characteristics drive with a variable workload and variable of the mass moment of inertia.

Key-Words: - shell conveyer, Maltese cross, variable transmission, reduced mass moment of inertia, hydraulic capacity, flow, pressure, hydraulic drive, hydraulic resistance.

1 Introduction
Before the shot it is necessary to load the cartridge into the cartridge chamber of the barrel. This loading influences the rapidity of fire of guns and it is also one of the most difficult operations, see [8], [9], [10], and [11]. This difficulty results mainly from:
- great weight and length of the artillery cartridges,
- great length of the displacement at the cartridge ramming,
- necessity to ram the cartridge into the barrel by the velocity ensuring the pressing of the shell ring into the forcing cone,
- compression of ejectors spring,
- storing of the ammunition far from the elevating parts,
- necessity of the short time of ramming,
- complicated working condition of the crew at towed guns and especially for self-propelled guns.

Therefore it is very difficult to ram the artillery cartridges into the barrel by hands of members of the gun crew and its mechanization is useful. Let us explain main parts of the loading system for the separated ammunition, because in comparison with the fixed ammunition it is more complicated. Such a heavy gun loading system in Fig. 1, (see [1], [2], [3], [4], [5], and [7]), consists of:
- shell storage system,
- case storage system,
- shell feeding device,
- case feeding device,
- ramming device,
- device for removing of cartridge cases from the loading system,
- control system.

Two storage systems are used in the loading system for separate ammunition: for shells and for cases. Each system includes the magazines (the box for shells and the box for cases with propellant charge) and the conveyers (also for two components of the cartridge). Modern self-propelled guns use the placing of the ammunition in the rotating traverse parts mostly.

Fig. 1 Loading system of heavy gun scheme
In the Czech 152mm self-propelled cannon-howitzer M77 there are both the shells and the cases placed vertically in four rows conveyers in the left and right cabin mounted on rotating parts of the weapon, see Fig. 2. The capacity of conveyers satisfies the firing of 30 rounds without completing of ammunition. Each component of the cartridge has a bed and a fixation at its disposal. Individual beds are connected into the belt by means of chains. Their guiding is provided with rollers to decrease the resistance against the motion, see [7], and [16].

2 Problem Formulation
The drive consists of the MA-2 rotary hydraulic motor and the gearbox connected with driven chains via chain wheels. The kinematical scheme is represented in Fig. 3, see [7], and [17]. An emergency drive by hand (with angular velocity $\omega_0$) is possible when the hydraulic drive does not operate. This shell conveyer is interesting with an application of the Maltese cross in the gearbox which causes a variable transmission $i_3$, variable reduced mass moment of inertia and variable static workload. It makes different behaviour of the whole system with respect to the conveyors having the constant transmission. The Maltese cross ensures an intermittent motion in two phases. The Fig. 4 shows two positions of the Maltese cross - during meshing and no meshing. The kinematical relations in the Maltese cross are apparent from scheme in Fig. 5. The Maltese cross is being replaced to a link mechanism in course of cross meshing with a carrier. The distance $a = O_O{MK}$ between the carrier rotation axis and the Maltese cross rotation axis is constant value and $r$ is the length from the centre roller meshing with the cross to the carrier axis rotation.

Then we can write the relation between angles $\alpha$ (the carrier turning) and $\beta$ (the Maltese cross steering angle)

$$\tg \beta = \frac{r \sin \alpha}{a - r \cos \alpha}.$$  
(1)

The transmission ratio of Maltese cross is

$$i_3 = \frac{\mu \cos \alpha - 1}{\mu^2 - 2 \mu \cos \alpha + 1},$$  
(2)
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where
\[ \mu = \frac{a}{r}. \]
The angular velocity of the Maltese cross is
\[ \dot{\beta} = \dot{\alpha} \dot{i}_3. \]  
(3)
The first derivation of transmission ratio is necessary to determine because the acceleration of the Maltese cross depends on it as well, see next formulae
\[ \ddot{i}_3 = \left[ \mu \sin \alpha \left(1 - \mu^2\right) \right] \left[ \mu^2 - 2 \mu \cos \alpha + 1 \right]. \]  
(4)
The angular acceleration of the Maltese cross is hence
\[ \ddot{\beta} = i_3 \ddot{\alpha} + \dot{i}_3 \dot{\alpha}. \]  
(5)
Then the velocity and acceleration of the shell conveyer may be determined with respect to the total constant transmission ratio \( i_{\text{const}} \)
\[ v_{\text{core}} = i_3 \dot{i}_{\text{const}} \dot{\phi}_1, \]  
(6)
and
\[ a_{\text{core}} = i_3 \left( \dot{i}_{\text{const}} \dot{\phi}_1 \right) + i_3 i_{\text{const}} \ddot{\phi}_1, \]  
(7)
where
\( \dot{\phi}_1, \ddot{i}_{\text{const}} \) – angular velocity and angular acceleration of the hydraulic motor.

After the kinematical analysis we can approach to the dynamic solution of the drive. The equation of motion is described with the following formulae, see [17]:
\[ I_{\text{m}} \ddot{\phi}_1 + 0.5 \dot{\phi}_1^2 \frac{dI_{\text{m}}}{d\phi_1} = M_{\text{m}} - M_{\text{z}} - M_{\text{d}}, \]  
(8)
where
\( I_{\text{m}} \) – reduced mass moment of inertia of the whole system,
\( \phi_1 \) – angular displacement of the hydraulic motor shaft,
\( M_{\text{m}} \) – driving torque,
\( M_{\text{z}} \) – reduced moment of workload,
\( M_{\text{d}} \) – damping moment.
The mass moment of inertia of the whole system is determined from the system kinetic energy:
\[ E_{\text{k}} = 0.5 \omega_0^2 I_{\text{M}}, \]  
(9)
\[ I_{\text{M}} = \left\{ \begin{align*} & I_1 + I_2 \frac{\dot{\omega}_0^2}{\dot{\omega}_0^2} + I_3 \frac{\dot{\omega}_0^2}{\dot{\omega}_0^2} + I_4 \frac{\dot{\omega}_0^2}{\dot{\omega}_0^2} + I_5 \frac{\dot{\omega}_0^2}{\dot{\omega}_0^2} + \\
& I_6 \frac{\dot{\omega}_0^2}{\dot{\omega}_0^2} + I_7 \frac{\dot{\omega}_0^2}{\dot{\omega}_0^2} + I_8 \frac{\dot{\omega}_0^2}{\dot{\omega}_0^2} + I_9 \frac{\dot{\omega}_0^2}{\dot{\omega}_0^2} + m_v \dot{v}_l^2 \end{align*} \right\}, \]  
(10)
\( I_i \) – shaft mass moment of inertia with its gear wheel or wheels,
\( m_v \) – mass of chain with shells beds,
\( v_l \) – chain velocity (i.e. conveyer velocity).
The Maltese cross transmission ratio to the angular displacement of the hydraulic motor shaft is written with formulae
\[ i_3 = \frac{\dot{\alpha}_3}{\dot{\alpha}}. \]  
(6)
It means that the mass moment of inertia \( I_M \) can be divided onto the constant part \( I_{\text{M1}} \) and the variable part \( I_{\text{M2}} \). The variability of the \( I_{\text{M2}} \) mass moment of inertia is caused with the variable transmission ratio \( i_3 \).

Then after arrangements and putting outside bracket \( i_3 \) we can write
\[ I_{\text{M}} = I_{\text{M1}} + i_3^2 I_{\text{M2}}, \]  
(11)

After differentiation with respect to \( \phi_1 \) we get
\[ \frac{dI_{\text{m}}}{d\phi_1} = 2 I_{\text{M2}} \frac{di_3}{d\phi_1} i_3, \]  
(12)
where
\( i_3 \) – constant transmission ratio between the Maltese cross and the hydraulic motor shaft.
The hydraulic motor driving torque is given, see [6], and [18], for example,
\[ M_{\text{M}} = \frac{V_0}{2\pi} \left( p_1 - p_2 \right), \]  
(13)
where
\( V_0 \) is the geometrical volume of the hydraulic motor.
The hydraulic equations for determining input and output pressures can be introduced in a way following from the hydraulic circuit in Fig. 6, as well, see [6]:
\[ \frac{dp_1}{dt} = \frac{Q_1 - \frac{V_0}{2\pi} \frac{\phi_3}{Z_1} p_1}{C_1}, \]  
(14)
\[ \frac{dp_2}{dt} = \frac{Q_2 - \frac{V_0}{2\pi} \frac{\phi_3}{Z_2} p_2}{C_2}, \]  
(15)
\[ Q_1 = \text{sgn}(p_g - p_h) \sqrt{\frac{|p_g - p_h|}{R_1}}, \]  
(16)
\[ Q_2 = \text{sgn}(p_2 - p_0) \sqrt{\frac{|p_2 - p_0|}{R_2}}, \]  
(17)
The parameters used in equations above are:
\( Q_1 \) – input flow of hydraulic motor,
\( Q_2 \) – output flow of hydraulic motor,
\( C_i \) – \( i \)th input hydraulic capacity,
\[ C_2 = \beta_2 V_{02} \] - output hydraulic capacity,
\[ V_{01} \] - input liquid volume in the pipe, leading from distributor to hydraulic motor,
\[ V_{02} \] - output liquid volume in the pipe, leading from hydraulic motor to distributor,
\[ \beta_1 \] - liquid volume compressibility factor, in this case is equal to \(6.8 \times 10^{-10} \text{ Pa}^{-1}\),
\[ R_1 \] - input hydraulic resistance,
\[ R_2 \] - output hydraulic resistance,
\[ p_0 \] - waste pressure taken \(0.6\text{MPa}\),
\[ p_G \] - source pressure, given as
\[ p_G = 4.4 + 0.1\sin(30\pi t) \text{[MPa]}. \quad (18) \]
The source pressure has decreased from \(4.4\text{MPa}\) to \(4.1\text{MPa}\) as it was determined from the technical experiments in [13] during the conveyer operation. Damping coefficient \(b_0 = 0.036 \text{N} \cdot \text{m} \cdot \text{s} \cdot \text{rad}^{-1}\) has been determined by measurement in the course of steady-state motion of the system under off-load conditions, see [4]. The hydraulic resistances \(R_1 \) \((2.8 \times 10^{13} \text{ Pa} \cdot \text{s}^2 \cdot \text{m}^6)\) and \(R_2 \) \((17.35 \times 10^{12} \text{ Pa} \cdot \text{s}^2 \cdot \text{m}^6)\) have been chosen from measured pressures and angular velocity of the hydraulic motor output shaft, see [13]. These values of hydraulic resistances have been considered when the electro-hydraulic distributor is fully closed. The opening time has been considered \(40\text{ms}\). Afterwards the hydraulic resistances have been \(R_1 = 9.3 \times 10^{12} \text{ Pa} \cdot \text{s}^2 \cdot \text{m}^6\) and \(R_2 = 2.45 \times 10^{12} \text{ Pa} \cdot \text{s}^2 \cdot \text{m}^6\).

Both hydraulic losses \(Z_1, Z_2\) have been used in the same values \(1 \times 10^{12} \text{ m}^3 \cdot \text{s}^{-1} \cdot \text{Pa}^{-1}\) from experiences and comparing with similar hydraulic motors. The comparing calculations have been carried out using the following input values of the hydraulic capacities, by way published for example in [6], and [13]:
\[ C_1 = 8 \times 10^{12} \text{ m}^3 \cdot \text{Pa}^{-1}, \text{ and } C_2 = 6 \times 10^{12} \text{ m}^3 \cdot \text{Pa}^{-1}. \]
The reduced moment of workload has been determined from the equation
\[ M_Z = -F_N f r_0 i_x, \quad (19) \]
where
\[ F_N \] - the normal force in guiding from the weight or from the conveyer inclination,
\[ f \] - friction coefficient in the conveyer guiding,
\[ r_0 \] - pitch diameter of the driving chain wheel,
\[ i_x \] - total transfer ratio from the conveyer to the hydraulic motor.
The friction force \(f F_N\) varies depending on the conveyer loading from 100N to 4250N.

### 3 Results of calculations
The mass moment of inertia is depending on it if the conveyer is full or empty. Its value for full conveyer is four times greater than the conveyer without the shells, for example. The Fig. 7 shows the system inertia mass moment - see the equation (10) - and the workload for the full conveyer from the equation (19).
initial angular displacement $\varphi_1$. The initial integration step has been used $\Delta t = 0.0001$ s.
The kinematical values, the conveyer displacement $x_{\text{conv}}$, the conveyer velocity $v_{\text{conv}}$ and the conveyer acceleration $a_{\text{conv}}$, are displayed in Fig. 8. From this figure follows that the Maltese cross enables the smooth rising of the velocity from zero to the maximum value and back. It means that the velocity is controlled by the mechanical transmission. The angular velocity of the hydraulic motor shaft represents the Fig. 9.

Its course depends on the system workload and it varies in the other way around than the input pressure $p_1$ and in the same direction as the output pressure $p_2$. Their curves are plotted in Fig. 10. When the conveyer is empty the pressures are smaller, the maximal value of the input pressure $p_1$ is approximately 2.0 MPa. Both pressures depend on the instantaneous values of the source pressure $p_0$ and the waste pressure $p_0$ as it has been presented on the real system in [13].

**4 Conclusion**

Experiences have shown possibilities of the mechanical systems modelling having the variable reduced mass moment of inertia. The results correspond with measured values mainly in course of steady-state conditions. In the future is supposed to improve the dynamic model by means of the electro-hydraulic distributor modelling, hydraulic generator with variable output pressure and output flow by implementation of flexible binding as it is shown in Fig. 11. This system is described by two equations of motion.
Fig. 11 Flexible binding in dynamic model of the shell conveyer.

The first part solves the system from the hydraulic motor to the shaft having the $I_3$ mass moment of inertia and the second one is from this to the rest of the conveyer as it is explained in Fig. 3. The shaft $I_3$ has been chosen due to the stiffness of torsion is the least as it is given in [16]. On the other hand the solution described there gives results which are satisfied comparing with the technical experiments in [13]. Mainly the beginning and the end of periods have to be improved to obtain the more accurate results. The results given in the figures reflect a good coincidence with the real piece which was explored according to presented theory. The theory has been verified on the Czech 152mm self-propelled cannon-howitzer M77. The procedure used in this article has been applied in the Czech research institutes and in the University of Defence in Brno as additional teaching material for students of weapons and ammunition branch.

References:
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