The Mathematical Model of the Order Realizing System

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Abstract: - The paper highlights the problem of mathematical modelling of the serial logistic manufacturing system in which each work stand performs autonomously independent number of operations in sequence on the order vector elements. It is assumed that machines in each work stand are identical and equipped with the same set of various tools which can be arranged optionally to carry required operations on defined elements. The specification presented hereby and its subsequent model illustrate the behaviour of the discussed system. A sample control method is based on the criteria approach implementing adequate manufacturing strategies as well as heuristic algorithms. The solution satisfying the set criterion is sought for.

Key-Words: - Heuristic algorithm, discrete event simulation, production system, logistic system, mathematical modelling, optimization criteria, manufacturing strategies.

1 Introduction

Manufacturing companies are currently facing very strong pressures in terms of cost, quality, flexibility, customization and time to market. Manufacturing or production systems that transform raw materials into high quality and highly reliable products are being developed and improved to address these needs. The manufacturing system is defined as being the ensemble of machining systems which are used for realization of a certain product. Each of these machining systems is made up of machine-tool/ tools, apparatus, parts, an operator and it executes one of the manufacturing operations [1]. Manufacturing is performed on the basis of customer orders and each order can be unique. Naturally, the through put times of the components may differ from one another. The production systems have to be flexible and able to react to changing production capacity requirements. All this planning and management of production networks a complex task. Manufacturing process design and scheduling process are critical areas. They are primarily focused on how to improve line efficiency. Manufacturing system design involves a number of interrelated subjects, e.g. the tooling strategy, allocation of buffer storage structures with certain capacities between stations, material-handling system, system size, process flow configuration, flexibility needed for future engineering changes or capacity adjustment and space strategy [2]. Production scheduling has long been a hot research direction in these fields such as automation, industrial engineering, management engineering and so on. The production scheduling problem is related to the constraints of resources and processes [3].

One of the most useful tools in the arsenal of an operations research (industrial engineering) management science analyst consists in computer simulation. Frequently, simulation is considered to be synonymous with Discrete Event Simulation (DES). A simulation is simply an imitation of the operation of a real-world system for purposes of evaluating that system [4]. Simulation involves creating a model which imitates the behaviors of interest; experimenting with the model to generate observations of these behaviors; and attempting to understand, summarize, and generalize these behaviors. Simulation has been used to study such wide ranging topics as urban systems, buildings design [5], social systems, transportation systems, health care delivery systems, logistic systems, production or manufacturing systems [6], e-commerce systems [7] and many more. Simulations are often used to analyze systems which are too complicated to attack via analytic methods such as calculus, standard probability and statistics, or queuing theory. Moreover, simulation is the most widely-used management science and operation research technique employed by industry and government. Manufacturing is one of the earliest simulation application areas [8] , and today it remains one of the most popular application areas. Over the past four decades a large amount of research has been devoted to the analysis and modelling of manufacturing systems. The tutorial [9] introduces manufacturing applications of simulation through three illustrative example applications. Discrete-event simulation can be used in the design, operation and continuous improvement of complex manufacturing and logistical systems [10]. Many articles and many software programs focus on manufacturing scheduling problems. Simulation-based planning and scheduling systems have proven to be very successful in this area. Development of scheduling algorithm is a fundamental and important problem for realizing flexible manufacturing systems. It is a problem of combinatorial optimization and includes difficulties such as complicated constraints and many locally optimal solutions. Several approaches have been investigated to overcome the difficulties. The papers [11] and [12] direct the attention to a genetic algorithm (GA) which aims at obtaining a suboptimal solution by a skilful combination of random search with heuristic method. The design of production systems, and also management of annual production targets, requires determining optimal buffer storage allocation on the line. Analysis and synthesis of systems with optimal buffer stocks is thus of both theoretical and considerable practical interest. For example, the paper [13] solves modeling of the logistic system with shared interoperation buffer stores or a simulation model which was proposed in [14] defines the optimal dimension of the buffer with regard to the maintenance policy. In discrete manufacturing processes such as stamping, assembly, or machining processes, product quality, often defined in terms of the dimensional integrity of work pieces, is jointly affected by multiple process variables. During the production phase, the states of tooling components, which are measured by adjustable process variables, are subject to possible random continuous drifts in their means & variances. These drifts of component states may significantly deteriorate product quality during production. Therefore, maintenance of the tooling components with consideration of both their continuous state drifts as well as catastrophic failures is crucial in assuring desired product quality and productivity. In contrast to traditional maintenance models where product quality has not been well addressed, especially for discrete manufacturing processes, a general quality oriented maintenance methodology is proposed to minimize the overall production costs [15]. In this research, the total production cost includes product quality loss due to process drifts, productivity loss due to catastrophic failures, and maintenance costs.

One of the most important issues for managers of manufacturing companies to decide on is what production control system would be the most appropriate for their companies. The choice is a matter of research and investigation but choosing the right system is a very important competitive advantage for the manufacturing companies. Optimal control of a substitutable inventory system, structured assemble to order systems and the impact of advance demand information on various production inventory control mechanisms are the key factors which must be taken into account while planning order realization procedures [16]. Developing solutions with heuristic tools offer two major advantages: shortened development time and more robust systems [17]. Evolutionary design of intelligent systems is gaining much popularity due to its capabilities in handling several real world problems involving optimization, complexity, noisy and non-stationary environment, imprecision, uncertainty and vagueness [18].

One feature of simulation is that one can change the parameters of a simulation model easily and try to observe the system performance under different sets of parameters. Therefore, it is natural to try to find the set of parameters which optimizes the system performance and is understood as optimization via simulation or simulation optimization [19]. Simulation optimization is an extremely valuable technique for investigating the behaviour of many business processes. The high abstraction level of the concept of discrete event simulation means that its application potential is extremely wide-ranging. Some common application areas of discrete event simulation or simulation optimization are service stations such as airports [20], call centers and supermarkets; road and rail traffic; industrial production lines [21], [23] or technological process and logistical operations like warehousing and distribution [22]. For example the paper [23] treats the optimization of production batches by computer simulation in a manufacturing company producing electric and pneumatic actuators.

2 General assumption

Let us assume that the discussed manufacturing system consists of I production stands equipped with the machines used to manufacture the elements of the order vector Z. The order vector consists of N elements. Each work stand is equipped with exactly the same machine. There is only one machine in each work stand. Each machine has some of the M tools. Each tool performs a defined operation. At one moment only one operation can be carried out in each work stand. This system requires K stages to realize the order vector elements. The vector of orders at the kth stage is considered in the form (1). The stage k, $k=1,\ldots,K$ is the moment at which the manufacturing process at any production stand begins.

$$Z^k = \left[z_n^k\right] \ , \ n = 1,\ldots,N \ , \ k = 1,\ldots,K \qquad (1)$$

The order vector is modified after every decision about production accordance with the specification (2).

$$z_n^k = \begin{cases} z_n^{k-1} - x_n^k & \text{if the number of units } x_n^k \text{ of the } n\text{th order is realized at the } k \text{ stage,} \\ z_n^{k-1} & \text{otherwise.} \end{cases} \qquad (2)$$

Let us assume that charge materials are represented by the vector of charges in the form (3) where: ω_w is the wth charge material.

$$\Omega = \left[\omega_w\right] \ , w = 1,\ldots,W \qquad (3)$$

Now we can introduce the matrix of charge material allocation to products in the form (4), where $\theta_{w,n}$ is the wth charge material allocation to the product of the nth order and take the values pursuant to (5).

$$\Theta = \left[\theta_{w,n}\right] \ , w = 1,\ldots,W \ , \ n = 1,\ldots,N \qquad (4)$$

$$\theta_{w,n} = \begin{cases} 1 & \text{if the product of the } n\text{th order is made from the } w\text{th charge,} \\ 0 & \text{otherwise.} \end{cases} \qquad (5)$$

In each ith work stand the machine includes tools which get worn out and are subject to replacement or regeneration. In such cases the machine must be stopped for the defined period of time and either the replacement or regeneration process is carried out. It is assumed that the regeneration process can be carried out only a certain number of times and after exceeding this number the used up tool has to be excluded from the manufacturing process.

Let the matrix in the form (6) be the matrix of operations performed by tools on the order vector elements. The elements of this matrix take the values pursuant to (7), where: $\lambda_{i(m,n)}$ is the operation carried out with the mth tool on the product of the nth order in the ith work stand.

$$\Lambda_i = \left[\lambda_{i(m,n)}\right] \ , i=1,\ldots,I; \ n=1,\ldots,N; \ m=1,\ldots,M \qquad (6)$$

$$\lambda_{i(m,n)} = \begin{cases} 1 & \text{if the operation is carried out on the } n\text{th order vector element with the use of the } m\text{th tool in the } i\text{th workstand,} \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

Let $G_{i,m}(n) = \left[g_{i,m}(n)\right]$, $i=1,\ldots,I;$ $m=1,\ldots,M;$ $n=1,\ldots,N;$ be the life matrix of tools in the manufacturing system in case of producing the nth product where $g_{i,m}(n)$ is the number of units of the nth order vector element which can be realized by the mth tool in the ith production stand before the need for replacement or regeneration arises.

Let $S_{i,m}^k(n) = \left[s_{i,m}^k(n)\right]$, $i=1,\ldots,I;$ $m=1,\ldots,M;$ $n=1,\ldots,N;$ $k=1,\ldots,K;$ be the state matrix of tools in the manufacturing system in case of producing the nth product where $s_{i,m}^k(n)$ is the number of units of the nth order vector element already realized by the mth tool in the ith production stand by the kth stage.

If the ζ th tool is to be replaced with a new one in the υ th work stand, where $1 \le \upsilon \le I$, $1 \le \zeta \le M$, then the state of tools changes according to (8).

$$s(n)_{i,m}^k = \begin{cases} 0 & \text{if } i = \upsilon \text{ and } m = \zeta \text{ at the stage } k, \\ s(n)_{i,m}^{k-1} & \text{otherwise,} \end{cases} \qquad (8)$$

Let $P_{i,m}^k(n) = \left[P_{i,m}^k(n)\right]$, $i=1,\ldots,I;$ $m=1,\ldots,M;$ $n=1,\ldots,N;$ $k=1,\ldots,K;$ be the flow capacity matrix of tools in the manufacturing system in case of producing the nth product where $p_{i,m}^k(n)$ is the number of units of the nth order vector element

which still can be realized by the *m*th tool in the *i*th production stand at the *k*th stage.
The number of units of the *n*th order vector element which still can be realized by the *m*th tool in the *i*th production stand at the *k*th stage can be calculated from the equation (9).

$$p_{i,m}^{k}(n) = g_{i,m}(n) - s_{i,m}^{k}(n) \tag{9}$$

3 Manufacturing time

Let $T_n^{pr} = \left[\tau_{n(i,m)}^{pr}\right]$, *n=1,...,N; i=1,...,I; m=1,...,M;* be the matrix of production times on the *n*th product with the use of *m*th tools in *i*th work stands. If the machine tool is not used for carrying out an operation on the product of *n*th order, then $\tau_{n(i,m)}^{pr} = 0$.

Let $T^{repl} = \left[\tau_m^{repl}\right]$, *n=1,...,N;* be the vector of replacement times of tools in work stands where τ_m^{repl} is the *m*th tool replacement time.

Let $T^{reg} = \left[\tau_m^{reg}\right]$, *n=1,...,N;* be the vector of regeneration times of tools in work stands where τ_m^{reg} is the *m*th tool regeneration time. If the machine tool is not subject to regeneration and should be replaced by a new one after exhaustion of the operating life, then $\tau_m^{reg} = 0$.

The *n*th product manufacturing time in the *i*th working stand is calculated by means of the formula (10), where $\Delta\tau_{n,i}^{reg}$ is the time throughout which the production process of the *n*th order vector element in the *i*th work stand is in a standstill mode. The variable y_{repl}^{k} takes the values pursuant to (11).

$$T_{n,i} = \sum_{m=1}^{M} \tau_{n(i,m)}^{pr} + \sum_{k=1}^{K}\sum_{m=1}^{M} y_{repl}^{k}\tau_{m}^{repl} + \Delta\tau_{n,i}^{reg} \tag{10}$$

$$y_{repl}^{k} = \begin{cases} 1 & \text{if the decision about the replacement is made at the kth stage,} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

A charge is passed subsequently through work stands and finds its final location meaning realizing the order. Semi-products relocation times are given in the matrix of relocation times in the form (12), where $\tau_{n,i\to i+1}$ is the time of passing the semi-product of the *n*th order to the subsequent work stand.

$$T_{reloc} = \left[\tau_{n,i\to i+1}\right],\ n=1,\dots,N;\ i=0,\dots,I \tag{12}$$

In a specific case:
$\tau_{n,0\to i=1}$ - the time of moving the charge of the *n*th order to the first work stand e_i,
$\tau_{n,I\to I+1}$ - the time of moving the product of the *n*th order from the last work stand e_i to its proper location.

To calculate the total manufacturing time of the *n*th order realization we need to use the formula (13), where τ_i^k is the time of awaiting for completing the manufacturing process in the *i*th work stand at the *k*th stage.

$$T_n = \sum_{i=1}^{I} T_{n,i} + (I+1)\tau_{n,i\to i+1} + \sum_{k=1}^{K}\sum_{i=1}^{I}\tau_i^k \tag{13}$$

There are no buffer stores in the system so manufacturing process in the stand *i+1* blocks passing order vector elements to it and they remain in the preceding work stand *i* till the moment when the stand *i+1* is ready to accept it.

4 System control

The presented system requires either the strategy on the basis of which the whole logistic process will be run or the criterion which will be responsible for setting the right sequence of products or, finally, the heuristic approach to determine the *n*th order to be realized.
Let us assume that Q_φ, $\varphi = 1,...,\Phi$ is the manufacturing criterion, Ξ_β, $\beta = 1,...,B$ is the production strategy, H_α, $\alpha = 1,...,A$ is the heuristic algorithm responsible for choosing the order vector element for production and Ψ_φ is the simulation result of the simulation process concerning the φth criterion. The sample control solution is proposed in the form of the diagram shown in the Fig. 1.

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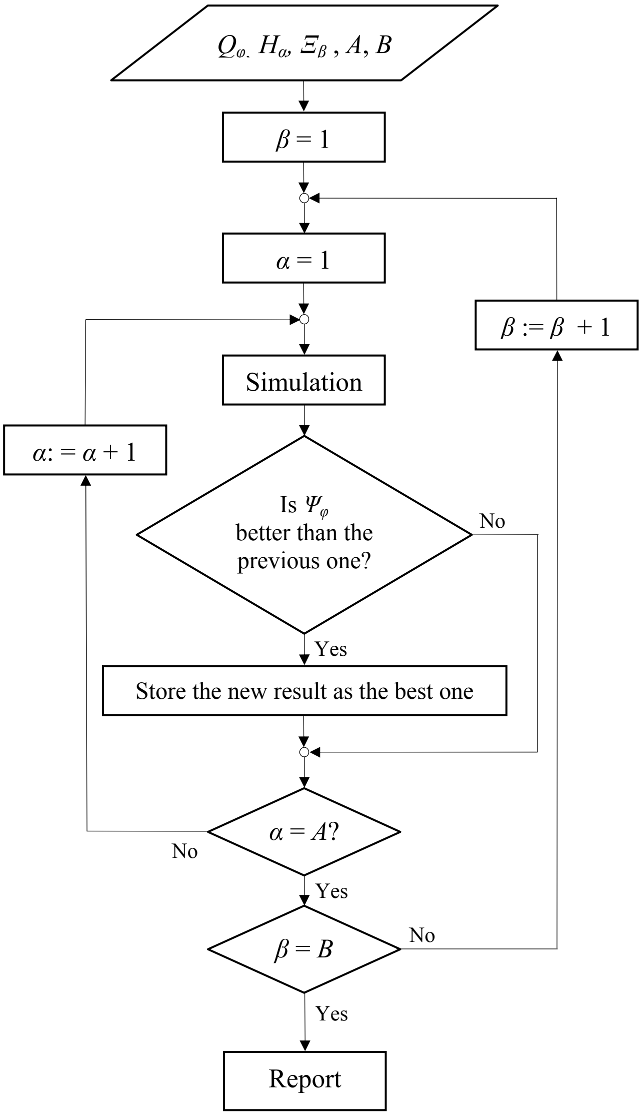

Fig. 1: The control diagram

7 Conclusions

The system presented in the paper hereby shows the general model of the proposed manufacturing structure. It is necessary to make general assumptions which form the base for modelling the synthetic environment representing a highly complex logistic formation. The general model leads to building a specific structure consisting of production stands which are equipped with production tools. To simplify the case we assume that each tool gets worn up throughout the manufacturing process and requires to be replaced immediately. The use of different heuristic algorithms may deliver a satisfactory solution. If not, it is possible to implement a combination of algorithms or even draw elements of the order vector for realization. However, this is possible only by means of the simulation method as a big number of simulation experiments should be carried out. The particular advantage of the use of simulated data consists in obtaining a set of data which will deliver the satisfactory solution. The experimental approach, with its emphasis on simulation-based solution seeking, seems to be the only way of finding an acceptable procedure. The proper insight into the nature of the specific problem, the approximations and assumptions, and other relevant modelling and simulation issues may provide the desired simulator. Simulation, and only simulation, takes into account the combined effect of variability, uncertainty, and complex interdependencies between processes. As there are more criteria, it also seems reasonable to verify different two- or more criterion models. Moreover, different criteria are used to evaluate the production process. They can be connected with input or output streams of objects. Appropriate bounds must always associate each criterion. Another aspect worth analyzing is realizing clients' orders continuously, no matter when they appear. This approach seems reasonable as it would lead to avoiding unnecessary delays consisting in waiting for potential customers who would fill the order vector. From the planning point of view it would mean bringing the whole production system to a standstill when the level of orders does not allow for resuming production.

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