Availability of City Buses in the Municipal Transport System

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Abstract: This article presents the way of defining availability of a single technical object (mean of transport), based on the built semi-Markov model of operation and maintenance process realized in the transport system. A unique type of transport system is the municipal bus transport system. In this article, the complete consideration was presented based on the chosen real system of transport means operation and maintenance – municipal bus transport system in a chosen urban complex. In systems of this type, the direct implementation of passenger transport remains the responsibility of elementary subsystems of the driver – bus type. It is on the availability of these subsystems that the possibility of appropriate implementation of transport task depends. By identifying the system under investigation and the multi-state operation and maintenance process being carried out within it an event-based and mathematical model was built, assuming that the model of that process was a homogeneous semi-Markov process X(t). In order to determine availability of city buses on the basis of semi-Markov model of operation and maintenance process, the operation and maintenance states have been divided into availability and unavailability states of an object (city bus) to start execution of the task assigned to it. Afterwards limiting probabilities of staying in the process states X(t) and availability of city buses were determined for the data obtained from the investigations of the real operation and maintenance process.

Key-Words: municipal transport system, city buses, availability

1 Introduction

Technological object (element or system) availability is the object’s feature which is characteristic from the point of view of the possibility of timely obtaining or maintaining the state of efficiency (facilitating the realization of goals) [10, 11].

A unique type of transport system is the municipal bus transport system with the main goal of passenger transport. The size of the transport goals is determined by the frequency of rides and a complex number of persons on board, in an assigned period of time, on a defined route. The operation and maintenance process of the city buses may be divided in general into the operational use process and the serviceability assurance process. In the operation process the basic task of transport system is obtained, at the same time generating profits due to the performance of the task. The activities performed inside the serviceability assurance process are aimed at restoring the task serviceability of the operated and maintained city buses. The realization of the process described is to bring back serviceability of the required number of the city buses within the specified period of time. In systems of this type, the direct implementation of passenger transport remains the responsibility of the utilization subsystem comprised of elementary subsystems of the operator – transport means (driver – bus) type. It is on the availability of these subsystems that the possibility of appropriate implementation of transport goals depends. Availability of transport means remains on an appropriate level as a result of the repair processes implemented in utilization subsystem by technological support units and in the efficiency implementation in repair posts at the bus depot.

The above results in the availability of transport system being dependent on the possibility of correct control of the process of operation realized both in the utilization and efficiency implementation subsystems. Due to a random nature of the factors affecting the course and effectiveness of the operation and maintenance process being realized within a complex system, the stochastic process, with extensively applied Markov and semi-Markov processes are most frequently applied for mathematical modelling of the operation and maintenance process [1, 2, 4, 5, 6, 7, 8]. Implementing semi-Markov processes makes it possible to create and analyze the mathematical model of the operation and maintenance process in the case of the variables characteristic of the defined process states being distributions other than exponential.
This article presents the way of defining availability of a single technological object – means of transport, based on the built semi-Markov model of operational process realized in the tested municipal transport system. The urban bus transport operation and maintenance system forming one of the subsystem of an urban transportation system within a selected conurbation was chosen as the investigation object.

2 Model of the Operation and Maintenance Process of City Buses

On the basis of identification of the real transport system under investigation and the multi-state operation and maintenance process being executed in it, an event-based and mathematical model was built for the analysed process, assuming that a homogeneous Markov process \( X(t) \) is the model of this process.

The model of operation and maintenance process was created on the basis of the analysis of state space as well as operational events pertaining to city buses used in the analyzed authentic transport system. Due to the identification of the multi-state operation and maintenance process of city buses, crucial operation states of the process as well as possible transfers between the defined states were designated. Based on this, a graph was created, depicting the changes of operation and maintenance process states, shown in figure 1.

![Directed graph representing the operation and maintenance process of city buses](image)

**Fig.1.** Directed graph representing the operation and maintenance process of city buses, where:
1 - stopover at the bus depot, 2 - obtain the transport task, 3 - damage to the bus at the bus depot, 4 - carrying out of the transport task, 5 - damage to the bus on route, 6 - repair by technical support unit without losing a ride, 7 - repair by technical support unit with losing a ride, 8 - emergency exit, 9 - repair at the efficiency implementation subsystem posts

Using the semi-Markov processes in mathematical modeling of the operation process, the following assumptions were put forward:
- the random process \( X(t) \) being the mathematical model of the operation and maintenance process is a homogenous process,
- the modelled operation and maintenance process has a finite number of states \( i = 1,2,...,9 \),
- if technological object (city bus) at moment \( t \) is in state \( i \), then \( X(t) = i \), where \( i = 1,2,...,9 \),
- at moment \( t = 0 \), the process finds in state \( i = 1 \).

The homogenous semi-Markov process is unequivocally defined when initial distribution (1) and its kernel \( Q(t) \) are given [3, 5, 6]:

\[
p_i(0) = \begin{cases} 1 & \text{when } i = 1 \\ 0 & \text{when } i \neq 1 \end{cases}, \quad i = 1,2,...,9, \quad (1)
\]

\[
Q_i(t) = P[X(t \geq t_i) = j, t_{n+1} - t_n \leq t | X(t_n) = i, i_j] \quad i, j = 1,2,...,9 \quad (3)
\]

means that the state of semi-Markov process and the period of its duration depends solely on the previous state, and does not depend on earlier states and periods of their duration, where \( \tau_1 < \tau_2 < ... < \tau_n \) are arbitrary moments in time, and

\[
Q_i(t) = p_{ij} \cdot F_{ij}(t), \quad (4)
\]

where:

\[
p_{ij} = \lim_{t \to \infty} p_{ij}(t) = \lim_{t \to \infty} \left[ P[X(t) = j | X(0) = i] \right] \quad (5)
\]

is a conditional probability of transfer from state \( i \) to state \( j \), and

\[
F_{ij}(t) = P[t \geq \tau_n - t_n \leq t | X(t_n) = i, X(t_{n+1}) = j] \quad i, j = 1,2,...,9 \quad (6)
\]
is a distribution function of random variable $T_{ij}$ signifying period of duration of state $i$, under the condition that the next state will be state $j$.

3 Limit Probability of Staying in States of Semi-Markov Process

Limit probability $P^*_i$ of staying in states of semi-Markov process were assigned on the basis of limit theorem [3, 6]:

If hidden Markov chain in semi-Markov process with finite state $S$ set and continuous type kernel contains one class of positive returning states such that for each state $i \in S$, $f_j = 1$ and positive expected values $E(T_i), i \in S$ are finite, limit probabilities:

$$P^*_i = \lim_{t \to \infty} P(t) = \frac{P_i \cdot E(T_i)}{\sum_{i \in S} P_i \cdot E(T_i)} ,$$  \hspace{1cm} (7)

where probabilities $P_i, i \in S$ constitute a stationary distribution of a hidden Markov chain, which fulfils the simultaneous linear equations

$$\sum_{j \in S} \sum_{i \in S} P^*_i P_{ij} = P^*_j, \quad j \in S, \quad \sum_{i \in S} P^*_i = 1 .$$  \hspace{1cm} (8)

In order to assign the values of non-conditional duration periods for process states, values of limit probabilities of Markov chain hidden in the process and values of limit probabilities of semi-Markov process, based on the directed graph shown in figure 1, the following were created matrices $P$ of the states change probabilities and matrix $T$ of conditional periods of duration of the states in process $X(t)$:

$$P = \begin{bmatrix} 0 & P_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{23} & P_{24} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{39} \\ P_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{59} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{99} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{99} \\
\end{bmatrix} ,$$  \hspace{1cm} (9)

$$T = \begin{bmatrix} 0 & T_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & T_{23} & T_{24} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_{39} \\ T_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_{49} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_{56} & T_{57} \\ 0 & 0 & 0 & 0 & 0 & 0 & T_{57} & T_{58} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & T_{58} & T_{79} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & T_{79} & T_{89} & 0 \\ T_{81} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix} ,$$  \hspace{1cm} (10)

where $T_{ij}$ – the period of duration of technological object in $i$-th state, under the condition that the next state will be $j$-th state.

Based on source data obtained from operation research performed in authentic municipal bus transport system, the values of the elements of matrix $P$ (9) as well as matrix $T$ (10) were estimated. Then matrices $P$ and $T$ are thus defined:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.056 & 0.944 & 0 & 0 & 0 & 0 & 0 \\ 0.291 & 0 & 0 & 0 & 0 & 0.089 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.833 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix} ,$$

$$T = \begin{bmatrix} 0 & 5.788 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.199 & 0.145 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.197 \\ 0 & 0 & 0 & 3.154 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.049 & 0.297 & 0.444 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.092 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.290 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.169 \\ 3.153 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix} ,$$

where the values of conditional duration periods of process $T_{ij}$ states were shown in [h].

Then, with the use of the Mathematica software, for the data shown in matrices $P$ and $T$, values of limit distribution of semi-Markov process. Results were shown in table 1.

<table>
<thead>
<tr>
<th>Table 1. Values of probabilities $P^*_i$ of limit distribution of the semi-Markov process</th>
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<tbody>
<tr>
<td>$P^*_1 = 0.3377$</td>
</tr>
<tr>
<td>$P^*_4 = 0.5204$</td>
</tr>
<tr>
<td>$P^*_7 = 0.0019$</td>
</tr>
</tbody>
</table>
4 Availability of Transport Means

In order to determine the availability of transport means based on the semi-Markov operation process model, the operational states of the technical object should be divided into availability states $S_i$ and unavailability state $S_U$ of the technical object to realize the assignment. Technical object availability states are states during which the technical object is efficient and/or supplied or will be repaired in a period of time shorter than the time reserve which is to serve the purpose. Unavailability states are states in which the technical object is inefficient and/or unsupplied.

Availability of transport means determined based on the semi-Markov operation process model is defined as the sum of limit probabilities $P_i^*$ of being in states belonging to the set of availability states $[3, 5, 9]$

$$A_{TM} = \sum_i P_i^*, \, \text{dla} \, S_i \in S_A, \, i = 1, 2, ..., 9. \quad (11)$$

In the presented model the following technical object availability states were defined: 1 - stopover at the bus depot, 2 - obtain the transport task, 4 - carrying out of the transport task, 6 - repair by technical support unit without losing a ride. Then, for the values given in the table 1, the value of availability of transport means is:

$$A_{TM} = P_1^* + P_2^* + P_4^* + P_6^*$$

$$A_{TM} = 0.3377 + 0.0086 + 0.5204 + 0.0003$$

$$A_{TM} = 0.8670$$

5 Summary

Availability of transport means $A_{TM}$, used in the municipal bus transport system, determined on the basis of semi-Markov operation process model depends directly on the values of limit probabilities $P_i^*$ of being at the states of the analyzed process, and indirectly on the values of the time values $\bar{t}_y$ and number $n_i$ of entries to the states of process $X(t)$.

Value of availability of transport means $A_{TM}$ depend on many factors:
- reliability of the transport means in use,
- serviceability and repair efficiency of transport means,
- effectiveness of repair processes carried out at logistics subsystem posts and by technical support units,
- efficiency of the subsystem fuel intake posts,
- equipping the logistics subsystem posts and the technical support units with tools and devices ensuring high efficiency of the performed repairs.

Value of availability of transport means obtained on the basis of operational data is not very high. However, one should take into consideration the fact that high availability of the utilization subsystem comprised of $N$ number of transport means may be obtained as a result of using appropriate structure which integrates technical objects and numbers of reserve objects. The model of defining availability of utilization subsystem comprised of $N$ transport means will be prepared at future stages of the work conducted.

References: