Study of the Fuel Consumption for Station-Keeping Maneuvers for GEO satellites based on the Integral of the Perturbing Forces over Time

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Abstract: - This paper provides a study of the estimate of the amount of fuel required for station-keeping maneuvers to maintain a satellite in its nominal position. The method used is based on an integral of the perturbing forces over the time. The result of this integral is the velocity change that the satellite needs to perform all the time to compensate the deviations caused by the perturbing forces. The perturbing forces considered in the present paper are the Luni-Solar perturbation and the J_2 effect of the gravitational field. The solutions presented here are obtained by numerical methods.

Key-Words: - Astrodynamics, Orbital Maneuver, Station-keeping, Orbital Perturbation, Third-Body Perturbation, J_2 Perturbation, Artificial Satellites.

1 Introduction
Orbital maneuvers have been studied for many years and still today this topic is relevant. The study of orbital dynamics can be related to several aspects of its performance. The optimization of some parameters and the satisfaction of the constraints are essential to guarantee the efficiency of the mission. The optimization of the fuel consumption in orbital maneuvers is an essential study since there is no advantage in refueling the satellite in space, so the amount of fuel in the satellite can determine its lifetime. In the literature there is an extensive study about optimal maneuvers, some of them can be found in [1] to [21].

The present paper is concerned with an estimation of the fuel consumption for station keeping maneuver needs to keep the satellite on its correct orbit. This amount can be estimated by the integral of the perturbing forces over the time. The perturbing forces studied here are the third-body perturbation of the Sun and the Moon and the J_2 term of the geopotential. A satellite in a geosynchronous orbit is used for the numerical simulations.

2 Problem Formulation
The integral over the time measures the magnitude of the perturbation for one orbital period of the satellite. This integral is called PI (“Perturbation Integral”) and it is given by:

\[ PI = \int_0^T |\nabla U| dt \quad (1) \]

where \( U \) is the potential of the perturbations and \( \nabla U \) is the gradient of the potential. Equation (1) can be written in terms of the eccentric anomaly \( E \) as follows [16]:

\[ PI = \frac{1}{n} \int_0^{2\pi} |\nabla U| \left( 1 - e \cos(E) \right) dE \quad (2) \]

The PI value in Equation (2) provides the amount of velocity change that the satellite receives from the perturbation forces.

The value of PI is an important quantity because it can point out the most economical orbit to plan a mission.

3 Disturbing Body
The third body perturbations considered here are due to the Sun and the Moon. The orbits of the Sun and the Moon are assumed to be circular. The Sun mass is \( 1.98892 \times 10^{30} \) kg, while the Moon’s mass is \( 7.349 \times 10^{22} \) kg. The inclination of the Sun’s orbit is 23.5 degrees, while the inclination for the Moon’s orbit varies from 18 to 28 degrees. Although the Moon’s orbit varies, the difference on the Moon’s inclination barely affects the PI value. At extreme cases, the difference of the PI value can reach 2%. In this way, the inclination for the Moon’s orbit was considered to be 18 degrees in this paper.

There are some important notations of the keplerian elements used in this paper, as follows:
where $G$ is the gravitational constant, the “$S$” sub-index is related to the Sun, while the “$M$” sub-index is related to the Moon. The $x, y, z$ are the coordinates of position and $m$ is the mass of the body. Remember also that the reference system is the inertial frame with the $x$ axis pointed to the Vernal point and the Earth at the center of the reference system. The distances $r_{EM}$, $r_{ES}$, $r_{M}$ are the distances between the Moon and the Earth, between the Sun and the Earth, between the spacecraft and the Moon and between the spacecraft and the Sun, respectively.

The perturbing forces due the third-body perturbation will be omitted here due to the lack of space. A wide and explicit explanation about this topic can be found in [16].

### 3.1 The Perturbation Integral of the Third-body

The initial position of the Sun and the Moon will affect the PI value. In order to evaluate the PI value without considering their initial positions, it is considered all the initial positions of the perturbing bodies with the help of extras integrals which considers the average meaning of the PI value evaluated with respect to the true anomaly of the perturbing body. The integral with the third-body perturbation becomes:

\[
PI = \frac{1}{2\pi n} \int_{0}^{2\pi} \int_{0}^{2\pi} |\nabla U| \left(1 - e \cos(E)\right) dE \, df_{osun}
\]

where $f_{osun}$ and $f_{omoon}$ are the initial true anomalies of the Sun and Moon, respectively. If only one perturbing body is considered, for example the Sun, the integral will become [16]:

\[
PI = \frac{1}{2\pi n} \int_{0}^{2\pi} \int_{0}^{2\pi} |\nabla U| \left(1 - e \cos(E)\right) dE \, df_{osun}
\]

### 4 $J_2$ Perturbation

The Earth has not a symmetrical distribution of mass. The Earth is oblate and this effect can be described by the $J_2$ coefficient. The $J_2$ coefficient is part of the spherical harmonics which contribute to describe the asymmetry of the Earth. This paper will consider the $J_2$ coefficient for the perturbation of the gravitational model of the Earth. The value of this coefficient is: $J_2 = 0.00108263$ [17].

As given by [17], the acceleration due to the Earth’s gravity field, if considered the three coefficients, is given by Equation (6):

\[
\nabla U = g_r \mathbf{i}_r + g_\theta \mathbf{i}_\theta
\]

where

\[
g_r = 3J_2 \left(\frac{R_e}{r}\right)^2 P_2 (\cos \phi)
\]

\[
g_\theta = 3GM \left(\frac{R_e}{r}\right)^2 \cos \phi \sin \phi
\]

where the vector unit $\mathbf{i}_r$ is the radial direction and $\mathbf{i}_\theta$ the southward direction, and $R_e$ is the equatorial radius of the planet, $\theta$ is the co-latitude and $P_2$ is the second degree of the Legendre polynomial. Equation (6) provides the gradient of the potential needed to calculate the PI value as given by Equation (2).

### 4.1 The Perturbation Integral of the $J_2$ coefficient

As the integral of the third-body perturbation, the PI value depends on the initial position of the satellite if it’s considered the $J_2$ perturbation and the rotation of the Earth along the orbit. In order to take the average of the PI value with all the possible initial positions, the integral in Equation (2) is then calculated for different values of the initial eccentric anomaly and then the average is taken by considering all the PI values obtained.

### 5 Results

This section presents the results of the numerical simulations of the integral for two kinds of orbits for different orbital parameters. The orbits proposed are the Geostationary and a MEO (“Medium Earth Orbit”) one.
5.1 The Geostationary orbit

The orbital parameters for the geostationary orbit considered in this paper are: \( a = 42,164 \text{ km} \), \( e = 0 \), \( 1 = 0^\circ \), \( \Omega = 0 \), \( \varpi = 0^\circ \). From Figs. 1 to 5 it's possible to analyze how the PI value changes for different orbital parameters.

In this case the average for different values of the PI with different initial positions related to the initial latitude was not considered, since the geostationary orbit keeps the same latitude if the station keeping maneuver is applied to correct the shifts due to the perturbations. In this way, another initial parameter is necessary, the initial time and date to position correctly the satellite. The date and time are 01/01/2010 at 5:30 A.M.

In Fig. 1 it's possible to note that there's a minimum value for the PI, at approximately 4.5X10^7 m. Before this semi-major axis value, the J_2 perturbation has the biggest influence on the PI value, and after this semi-major axis, the influence of the Sun and the Moon becomes prevailing. The J_2 perturbation decreases as the semi-major axis increases, since the satellite departs from the Earth gravitational perturbing force.

In Fig. 2, for the semi-major axis interval considered, the Moon's perturbation has a larger influence when compared to the Sun. Also, the Sun's and Moon's perturbations increase as the semi-major axis increases, because the satellite approaches to the perturbing bodies.

Fig. 3 shows that, as the eccentricity increases, both the J_2 and Luni-Solar perturbations increase, since the apogee becomes bigger, increasing the third body perturbation, and the perigee becomes smaller, increasing the J_2 perturbation.
Fig. 4 shows how the PI value varies for different inclinations. Specially for inclinations smaller than 0.25 rad and bigger than 2.85 rad, the Luni-Solar perturbations decreases the J$_2$ perturbation if they are all considered for the PI value when compared to the J$_2$ perturbation curve only. The maximum value for the PI with all perturbations included is when $I = \pi/2$.

Fig. 5 evaluates how the Luni-Solar perturbation affects the PI value as the inclination varies. In this figure, it's possible to note that for the Moon’s perturbation, the PI value has its minimum if the satellite lies in a perpendicular plane to the Moon’s orbit. And for the higher values of PI, the satellite has a coplanar orbit with the Moon. This happens because satellite’s coplanar orbits are closer to the Moon than the perpendicular orbits, in average mean with the time.

5.2 The MEO orbit

The orbital parameters for the geostationary orbit considered in this paper are: $a = 15,000$ km, $e = 0$, $I = 0^\circ$, $\Omega = 0$, $\omega = 0^\circ$. From Figs. 6 to 12 it’s possible to analyze how the PI value changes for different orbital parameters.

In this case the average for different values of the PI with different initial positions related to the initial latitude was considered. In Fig. 6 the semi-major axis varies from 2000 to 35000 km. The J$_2$ perturbation is critical, especially from 2000 until 5000 km, where the PI value is extremely high for the station-keeping maneuvers.

In Fig. 7 the semi-major axis varies from 10000 to 35000 km. From Fig. 7 it’s possible to see that the J$_2$ perturbation has the largest influence on the PI and the Luni-Solar perturbation is minimal compared to that.
Fig. 8 shows the Sun’s and Moon’s perturbations for the same semi-major axis length shown in Fig. 7. It’s possible to note that, as the semi-major axis increases, the Luni-Solar perturbation increases as well. This occurs because as the semi-major axis increases, the Sun and the Moon becomes closer, increasing the third-body perturbation.

In Fig. 9, all the perturbations included in this paper are considered for the change of the eccentricity for this orbit. Once again, the J₂ perturbation has the largest influence as the eccentricity varies and the PI value increases as the eccentricity increases as well.

In Fig. 10 it was considered only the Luni-Solar perturbation as the eccentricity varies, and once again, as the eccentricity increases, the PI perturbation increases as well.

In Fig. 11 it’s possible to see that the minimum PI values for the J₂ are close to the minimum value with all perturbations considered, since the J₂ perturbation has the largest influence on the PI value. The minimum values of PI for all perturbations occur when the inclinations are 0.55 and 2.6 rad and the maximum value occurs when \( I = \pi/2 \).

Fig. 12 shows clearly how the Sun and the Moon influences the PI value. As mentioned in the geostationary orbit, the MEO orbit has the same analysis that was made for the Luni-Solar perturbation. The third-body has the largest influence if the satellite orbit is coplanar and the minimum value of the PI if the orbit is perpendicular.

6 Conclusion
This paper is concerned with an estimate of the costs of station-keeping maneuvers for different orbits. This approach used the integration the perturbing forces over the time. The forces
considered were the J₂ perturbation and the third-body perturbation due to the Sun and the Moon.

The solution of the integral, the PI value, shows an estimate of the cost for the orbit maintenance with the total amount of velocity the satellite needs to receive to keep constant the nominal orbit. By varying the orbital elements, the potential low cost maintenance orbit for the station-keeping maneuver can be found. The PI value can point out orbits which has a low cost for station-keeping maneuvers and that suffers less deviations from the perturbations considered.

This paper presented some cases where the J₂ had the largest influence on the PI value, and other cases where the third-body perturbation were prominent. The change on the orbital elements made for both of the orbits generated a map of the PI values for each case. There were found minimum PI values at some figures as the semi-major axis or the inclination were varying. From increasing the eccentricity, it was possible to note that the PI value always increases. The J₂ perturbation has a strong influence on the station-keeping maneuver and on the perturbing forces for the cases proposed, but the Sun’s and the Moon’s perturbation plays an important role in some cases as well.

References: