Sequences of Swing-by Maneuvers Using the Planet Saturn

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Abstracts: This paper search for successive swing-by maneuvers that use the planet Saturn as the body for the closest approach. The patched-conics method is used to compute the trajectories. It is also assumed the presence of only two massive bodies (Sun and Saturn) that are in circular and planar orbits. It is then searched for geometries of the swing-by maneuvers that cause a series of passages by the planet. Those orbits have to be resonant with the motion of Saturn. It is necessary to verify if the orbits are physically possible, having in mind that the periapsis of the orbits around the Sun needs to be above its surface, as well as the closest approach with Saturn needs to be above the surface of the planet.

Key Words: Astrodynamics, Close Approach Maneuvers, Space Trajectories, Successive Swing-Bys.

Introduction

The literature shows several aspects of the Swing-By maneuver. It is a maneuver used to change the energy of a spacecraft by making a passage close to a massive body. The description of this type of maneuver can be seen in several publications, like [1]-[7]. Applications of this maneuver are also widely available, like in studies of transfer orbits to/from the Lagrangian points [8]-[10]; in the description of real missions that used this concept, like in [11]-[17]. Also variations of this problem is studied, like using the combination of impulsive maneuvers with the close approach [18], the presence of an atmosphere of the planet during the passage [19], elliptical orbits for the main bodies [20], the simultaneous passage of a group of particles instead of a single one [21]-[24], the combination with multiobjective optimization [25] or in the scattering of comets by a planet [26].

In the present research the goal is to find series of close passages by Saturn to allow a spacecraft to study the space in that region of the Solar System by changing its orbit without allowing an escape to occur. The dynamics is assumed to be given by the “patched-conics”. The orbital characteristics of the orbits involved are studied (velocity, energy, orbital elements and angular momentum) after each passage to show the evolution of the trajectories. Fig. 1 shows a sketch of the maneuver.

Mathematical model

The system is assumed to be formed by the Sun ($M_1$), Saturn ($M_2$), that is orbiting the Sun in a circular orbit, and a spacecraft $M_3$ that starts its motion orbiting the Sun and then makes a close approach with Saturn. The spacecraft is assumed to have a negligible mass.

The variables used to identify one trajectory are: $r_\infty$ (minimum distance from the center of Saturn to the spacecraft), $\vec{v}_\infty^-$ and $\vec{v}_\infty^+$ (velocities of the spacecraft
with respect to Saturn before and after the maneuver, \( \vec{v}_2 \) (velocity of Saturn with respect to the Sun), \( \delta \) (half of the deflection angle, that is defined as the curvature made by the close approach) and \( \psi \) (the angle of approach). More details can be found in reference [8].

The distance that the spacecraft needs to pass from Saturn (\( r_{ap} \)) to increase its apoapsis and also going to a resonant orbit with Saturn, to allow a new encounter, can be calculated by [3]:

\[
r_{ap} = \frac{\mu}{v^2_\infty} \left[ \frac{1}{\sin(\delta)} - 1 \right]
\]  

(1)

where

\[
\delta = \text{ArcCos}\left[ \frac{1}{\sqrt{2}} \sqrt{\frac{A^2+4B(C+B+C)}{A^2+B^2}} \right]
A = \frac{\sin(2\pi+\beta)}{2}; \quad B = \cos(2\pi + \beta) \quad \text{and} \quad C = -\frac{\Delta E}{2v^2_\infty}
\]  

(2)

In order to verify the orbits found here, the Tisserand’s criterion is used. It means that the orbits before and after the passage have to follow the equation [2]:

\[
\frac{1}{a_1} + 2\sqrt{a_1(1-e_1^2)} \cos i_1 \approx +2\sqrt{a_0(1-e_0^2)} \cos i_0
\]  

(3)

When the orbital elements \( a_1,e_1 \) and \( i_1 \) are the ones before the passage and the orbital elements and \( a_0,e_0 \) and \( i_0 \) are the ones after the passage.

**Numerical Study**

The spacecraft starts in a given orbit around the Sun which is specified by its apoapsis and periapsis distances. Table 1 shows resonant orbits for the spacecraft, given the number of periods of Saturn before the following approach, the number of orbits of the spacecraft in this same period of time, the period of the orbit of the spacecraft (in days), the semi-major axis (km) of the orbit, and the order of the resonance. Then, it is possible to organize the orbits to put them in order of crescent values of the energy. Next, it is possible to find the values of \( r_{ap} \) for every passage. The assumptions used here are:

1) The close approach occurs at the point A (Fig. 1);
2) No perturbations affect the spacecraft;
3) The two-body (Sun-spacecraft) energy is constant after and before the passage by Saturn;
4) The angular momentum (C) and the energy (E) are measured before and after the maneuvers.

It is then necessary to remove orbits that have periapsis below the surface of the Sun. Table 2 shows the useful orbits. It shows the number of the maneuver, the period (days), the distance of the closest approach (in units of radius of Saturn), semi-major axis (km), eccentricity, energy (km²/s²), periapsis distance (km), apoapsis distance (km), half of the deflection angle (degree), angle of approach (degree), order of the resonance and the time elapsed since the start of the maneuvers (days). The initial orbit for the spacecraft is assumed to have a periapsis of 155,000,000 km, which is near the orbit of the Earth around the Sun, and apoapsis of 1,858,220,000 km, that is a little bit higher than the orbit of Saturn around the Sun. Then, it is built Table 2, where the resonant orbits are organized in crescent values of the energy. There is also no problem of having values for the periapsis that is inside the Sun. Some of the orbits (the first 6) have periapsis below the initial value, so they also intercept the orbit of the Earth around the Sun and there is a potential risk of collision, which is neglected in the present study. It is clear that there are decreasing values for the distance of the closest approach to compensate the increase of the velocity of approach. The sequence is limited to 17 revolutions before a situation where a value below the surface of Saturn is found. This situation is also a characteristic of the sequence of orbits shown and initial conditions where there is an escape orbit can be found for Saturn.
Table 1: Resonant orbits for the spacecraft passing by Saturn.

<table>
<thead>
<tr>
<th>Number of revolutions of the Moon between two successive close approaches</th>
<th>Number of revolutions of the spacecraft between two successive close approaches</th>
<th>Period of the spacecraft (years)</th>
<th>Semi-major axis of the spacecraft (km)</th>
<th>Order of the resonance</th>
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<td>1</td>
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<td>7.751 x 10^6</td>
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</table>

Table 2 - Sequence of orbits performing close approaches with Saturn.

<table>
<thead>
<tr>
<th>Man.</th>
<th>orbital period (day)</th>
<th>r_{\text{se}} (Radius of Saturn)</th>
<th>a (10^6 Km)</th>
<th>e</th>
<th>Energy</th>
<th>R_{\text{in}} (10^6 Km)</th>
<th>R_{\text{out}} (10^6 Km)</th>
<th>(\delta) (Deg)</th>
<th>(\psi) (Deg)</th>
<th>resonance</th>
<th>time(days)</th>
<th>Tisserand's Criterion</th>
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</table>
Fig. 2 - Energy of the spacecraft as a function of time.

Fig. 3 - Semi-major axis of the spacecraft as a function of time.

Fig. 4 - Eccentricity of the spacecraft as a function of time.

Fig. 5 - \( r_{ap} \) distance of the spacecraft as a function of time.

Fig. 6 - Apogee distance of the spacecraft as a function of time.

Fig. 7 - Perigee distance of the spacecraft as a function of time.
Those results show the evolution of the resonant orbits encountering Saturn. The energy, the angular momentum, the semi-major (axis) and the apoapsis distance increase after each close approach as forced by the objective of the sequence. The energy goes from -89.56 km²/s² after the first passage until -18.48 km²/s² after the last one, in crescent steps. This variation in energy causes the semi-major axis to go from 27,793,200 km to 729,803,000 km, which corresponds to a variation of the apoapsis from 1,457,280,000 km (a little above the orbit of Saturn) to 6,466,380,000 km. It is a very large interval, so the spacecraft travels in different regions of the Solar System. The eccentricity has an oscillating sequence, with a first decrease series and then an increasing sequence. The time span for this sequence is 1472 years. The values of $r_{ap}$ decrease from passage to passage, to compensate the increasing values of the velocity of approach.

CONCLUSION

This study was made to show the evolution of the trajectories, as well as the amplitudes of the variations of the velocity, energy and angular momentum of an orbit due to a series of close approaches with Saturn. Analytical equations are used to make the calculation of the distance of the closest approach that generates a specified orbit. Then, a series of resonant orbits with Saturn that has increasing values for the apoapsis to cover a large area of the space around the orbit of Saturn is found. Using these equations it is possible to establish a sequence of close approaches that meets the goals. The results showed that it is possible to find useful sequences of close approaches to study the space near Saturn by using these natural changes of orbits to pass by different positions in the space without the expenses of applying a control to the spacecraft.

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