Analysis of Single Phase Moisture and Heat Model of Food Drying

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Abstract: Single phase heat and mass models have been widely used to describe the movement of water and heat during drying. In order to reduce the number of parameters and minimise computer time on the problem, reported and non-reported non-dimension numbers have been used. However, very limited research has been done to analyse the sensitivity of these numbers. In this paper, non-dimension analysis of the mathematical model applicable to drying food is discussed. The model combines the principles of mass balance with heat balance for the movement of moisture and heat. Numerical solutions are presented corresponding to one-dimensional food tissues with constant diffusivity. The main finding of the model is the ability of equations with constant properties to represent the movement of moisture and temperature inside the fruit and, consistent with experiment findings, results providing physical contents are assigned appropriate numerical values.

Key–Words: heat, mass, sensitivity, diffusivity.

1 Introduction

Single phase heat and mass models have been widely used to describe the movement of water and heat during drying. The most common type of single phase modelling in food drying involves one-dimensional heat and mass transfer with empirical values of effective diffusivity. This method has been used by [1, 2, 3, 4] to illustrate drying phenomena. A finite-difference approach [3] or a finite element approach [1, 4] to compute the behaviour of the distribution of heat and mass transfer is widely adopted to solve the resulting equations. Difference between models depends mainly upon assumptions about the surface, such as the use of an equilibrium boundary condition at the surface [5] or convective boundary condition [1].

Dimensionless analysis of the simultaneous heat and mass transfer in food drying has been carried out by Pavon-Melendez et al. [6] and Ruiz Lopez et al. [7]. Pavon-Melendez et al. [6] analysed the transport of moisture and temperature with constant material properties. Non-dimension numbers were obtained and analyses of these non-dimension numbers were used to estimate the mechanisms that control heat and mass transfer during drying but there is no sensitivity of the numbers that were analysed.

Ruiz Lopez et al. [7] conducted a similar analysis with variable properties of material during drying. The results obtained by simulation of heat and mass transfer with variable material properties by experiment were compared with those obtained by analytical solution with equilibrium boundary condition for moisture. However, comparison of the results shows that not all numerical simulations had the same trends in the experiment result but an analytical solution with constant diffusivity reproduces the moisture evolution in an acceptable manner. This demonstrated that variable properties make the system computationally intensive, whereas constant diffusivity can replicate the experiment result.

Barati and Esfahani [8] developed one-dimensional heat and mass transfer in an infinite slab during drying by introducing an analytical solution technique to describe the temperature and moisture evolutions. The result was compared with a set of experiment data found in the open literature. The use of different time scales for moisture and temperature ($F_{o_m} = D t / L^2$, $F_{o_t} = \alpha t / L^2$) mean that the moisture and temperature equations cannot be solved simultaneously; this will make the system of equations independent of each other. To make the system of equations dependent on each other, Barati and Esfahani [9], solved the temperature and moisture equation by changing the Fourier number for moisture and temperature at each time step. This is not good practice in terms of model structure not only because of mathematical ‘correctness’ with regard to the domain, but also the real situation it represents must be the same.
The aim of this work was to analyse the non-dimensional numbers produced in simultaneous heat and mass transfer and conduct sensitivity tests on these numbers. Two different situations were studied, the first of which was an isothermal condition with negligible latent heat, which gives uncoupled boundary conditions to the mass equation at the surface. In the second model, latent heat effect was considered. This is especially important at the fruit-air interface, where evaporation occurs.

2 Physical Problem and Mathematical Formulation

The physical problem involves a single slice of food of thickness $2L$, initially at a uniform temperature $T_0$ and uniform moisture content $M_0$. In the case of an infinite slab of finite thickness $L$, the moisture content $M(t)$ across the slab and unsteady temperature $T(t)$ are expressed by the well known system of partial differential equations (PDEs) for moisture and energy transport [4]. During drying, heat is transferred mainly by convection from air to the product surface and by conduction from the surface towards the product centre. Meanwhile, moisture diffuses outwards towards the surface and is vapourized in the air. Such a coupled mechanism provides the basis for a simultaneous moisture and heat transfer model as,

$$\rho_s \frac{\partial M}{\partial t} = \frac{\partial}{\partial x} \left( D \rho_s \frac{\partial M}{\partial x} \right); \quad 0 < x < L, \quad (1)$$

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \alpha \frac{\partial T}{\partial x} \right); \quad 0 < x < L. \quad (2)$$

Initial and boundary conditions are

at $t = 0$: $M = M_0$, $T = T_0$, $L = L_0$, \quad (3)

at $x = 0$: $\frac{\partial M}{\partial x} = 0$ and $\frac{\partial T}{\partial x} = 0$, \quad (4)

at $x = L$: $h_m(C_{\text{sur}} - C_{\text{air}}) = -D \rho_s \frac{\partial M}{\partial x}$, \quad (5)

$$h(T_{\text{air}} - T_{\text{sur}}) = k \frac{\partial T}{\partial x} - H_v D \rho_s \frac{\partial M}{\partial x}. \quad (6)$$

In equations (1)-(6) $M$ is local moisture content (dry base), $T$ is local temperature, $D$ represents the effective diffusion coefficient, $\rho_s$ is the density of dry solid, $C_p$ heat capacity, $k$ thermal conductivity, $x$ is thickness and $t$ the time. $H_v$ is the heat of vaporization, $T_{\text{air}}$ is air temperature and $T_{\text{sur}}$ is the surface temperature of the fruit, $h$ is the heat transfer coefficient, $h_m$ is the mass transfer coefficient, and $\alpha$ is thermal diffusivity. $C_{\text{air}}$ is the concentration of moisture in the air and $C_{\text{sur}}$ is the concentration in the form of liquid water film at the food surface (example see [10]).

The fruit sample is a hygroscopic porous media, yet it is assumed to be a fictitious continuum. Weak internal evaporation and transport of vapour within the dehydrated fruits towards the external food surface have not been considered. In the first instance, we consider models with no shrinkage ($x = L_0$) associated with isothermal and non-isothermal diffusion and constant moisture diffusivity ($D = D_0$). Non dimensional scaled variables are used: a representative diffusion timescale scaled moisture content, and scaled temperature are defined by

$$\tau = \frac{D_0 t}{L_0^2}, \quad \bar{M} = \frac{M}{M_0}, \quad \bar{x} = \frac{x}{L_0}, \quad \bar{T} = \frac{T - T_0}{T_{\text{air}} - T_0}. \quad (7)$$

The governing equations for diffusion and heat equations as

$$\frac{\partial \bar{M}}{\partial \bar{\tau}} = \frac{\partial^2 \bar{M}}{\partial \bar{x}^2}, \quad \text{and} \quad \frac{\partial \bar{T}}{\partial \bar{\tau}} = Le \frac{\partial^2 \bar{T}}{\partial \bar{x}^2}. \quad (8)$$

Initial conditions associated with constant conditions are $\bar{M} = 1$, and $\bar{T} = 0$, at $\bar{\tau} = 0$.

Taking symmetry boundary conditions in the mid-plane of the drying slice gives

$$\left( \frac{\partial \bar{T}}{\partial \bar{x}} \right) = 0, \quad \text{and} \quad \left( \frac{\partial \bar{M}}{\partial \bar{x}} \right) = 0. \quad (9)$$

At the surface, moisture and temperature boundary conditions of the drying body in contact with drying air become

$$\frac{\partial \bar{M}}{\partial \bar{x}} = -Bi_m C_{\text{air}} \left( \phi(T_{\text{sur}}) f(M) - 1 \right), \quad (10)$$

$$\frac{\partial \bar{T}}{\partial \bar{x}} = Bi \left( 1 - \bar{T}_{\text{sur}} \right) + \lambda \frac{\partial \bar{M}}{\partial \bar{x}}. \quad (11)$$

In the above, several dimensionless controlling parameters are defined by

$$Bi_m = \frac{h_m L_0}{D_0 M_0}, \quad Bi = \frac{h L_0}{k}, \quad Le = \frac{\alpha}{D_0}, \quad \text{and} \quad \lambda = \frac{H_v D_0 M_0 \rho_s}{k(T_{\text{air}} - T_0)}. \quad (11)$$

On the above, $D_0$ represents the constant diffusion coefficient, $T_0$ is initial temperature, $M_0$ is initial moisture.

The relationship between $\alpha$, moisture content $M$ and the saturated pressure can be found in [2] and used by [11]. In non-dimensional form $\phi(T_{\text{sur}})$ and $f(M)$ are given as

$$\phi(T_{\text{sur}}) = \frac{\bar{T}_{\text{sur}}^2}{\bar{M}} + \bar{B} \bar{T}_{\text{sur}} + \bar{C}, \quad (12)$$
### Table 1: Input Parameters used in the simulations of drying of Mango.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi</td>
<td>0.3</td>
</tr>
<tr>
<td>Bi_m</td>
<td>20</td>
</tr>
<tr>
<td>M_0</td>
<td>0.8 w.b</td>
</tr>
<tr>
<td>λ</td>
<td>0.5</td>
</tr>
<tr>
<td>Le</td>
<td>5</td>
</tr>
<tr>
<td>T_air</td>
<td>60°C</td>
</tr>
<tr>
<td>H_c</td>
<td>2345 KJ/kg</td>
</tr>
<tr>
<td>D_0</td>
<td>8x10^{-11} m^2/s</td>
</tr>
<tr>
<td>h</td>
<td>20-250 W/m^2K</td>
</tr>
<tr>
<td>ρ_a</td>
<td>1080 kg/m^3</td>
</tr>
<tr>
<td>h_m</td>
<td>2x10^{-6} m/s</td>
</tr>
<tr>
<td>C_p</td>
<td>2344 KJ/kgK</td>
</tr>
<tr>
<td>α</td>
<td>4x10^{-9} m^2/s</td>
</tr>
<tr>
<td>k</td>
<td>0.475 W/mK</td>
</tr>
</tbody>
</table>

\[
f(M) = \frac{\sigma}{M_0^{\frac{1}{\beta}} + M^{\frac{1}{\beta}}}.
\] (13)

A, B, C, D and E are the value of constant was determined experimentally [7]. In equation (13), \( \sigma = 2.1667/C_{air} \)

### 3 Solution Procedure

#### 3.1 Numerical Solution

The COMSOL Multiphysics program is used to simulate the dehydration process in the drying system, which corresponds to the numerical solution of these model equations. The above system of non-linear partial differential equations, together with the described set of initial and boundary conditions, has been solved by Finite Element Method implementation by COMSOL Multiphysics 3.4. The analysis of transport phenomenon presented here is based on the time evolution of variables, i.e. the dimensionless moisture content and temperature at each exposed surface. For drying simulations, the parameters used in the equations for the generic drying condition for mango are listed in Table I.

#### 3.2 Model verification

Drying experiments were conducted on slices of mango. Before drying, mango were hand peeled and cut into rectangle-shaped slices with thickness of 5 mm, width 5 mm and length 10 mm. The initial temperature and moisture content of the product were 30°C and 80%wb (4.0 db) respectively. Mango were dried using conventional hot air drying at air temperature 60°C, 20% RH and air velocities of 0.5 m/s. Food moisture evolution during drying was calculated by regularly measuring the sample weight loss with a balance every 15 min during the first 75 min. Then, because of the drying rate decrease, measures were taken every one hour until the end of drying.

### 4 Results and Discussions

#### 4.1 Experimental validation

Fig. 1 displays the comparison between the experiment results obtained with dying air temperature of 60°C, RH=20% and air velocities=0.5 m/s and the model prediction using \( Bi=0.3, \) \( Bi_m =20 \) and \( Le=5 \) and \( \lambda =0.5. \) It was found that there was a close correlation between experimental and predicted values (at 5% level). This Fig. illustrates that the model is able to replicate the trend of experiment moisture evolution of a mango slice during drying. The relative MSE is less than 11%. From this Fig., acceptable agreement is observed.

![Figure 1: Moisture comparison between model prediction and experiment results for the first 75 minutes.](image-url)

#### 4.2 Isothermal solution

In view of non-dimension, the timescale of interest is the time taken for moisture to be diffused to the surface along the thickness of the fruits. The diffusion coefficient of moisture \( D_0 \) is given for the fruits around \( 10^{-10} \) m/s and for fruits with a thickness of 5-10 mm (0.005-0.01 m). If the drying time to obtain equilibrium is taken as 15000-28000 seconds [40] for the drying of a mango slice, this gives a time scale approximation of 0.36-1.12. We also take the same time scale for temperature, but equation (8) shows that heat transfer by conduction is \( Le \) times the mass transfer by diffusion. From the values in Table 1, the value for \( Le \) is around 5-100.

In this section, the formulation presented above is simplified by assuming that temperature increases rapidly compared to changes in moisture. For cases in which \( \lambda \) is small, the governing equation for temperature is uncoupled with the governing equation for moisture. The boundary condition for temperature in
equation (11) becomes
\[ \frac{\partial T}{\partial \xi} = Bi(1 - T), \quad \text{at} \quad \xi = 1. \] (14)

Solutions for the temperature equation can be readily obtained analytically as it is analogous to classic diffusion with surface evaporation (see Crank [12] and Carslaw and Jaeger [13]). The solution can be written as an infinite series as
\[ T = \frac{T - T_0}{T_{air} - T_0} = 1 - \sum_{n=1}^{\infty} \frac{2 Bi \times \cos(\beta_n \xi) \exp(-\beta_n^2 \tau)}{\beta_n^2 + B_i^2 + Bi \cos(\beta_n)} \]
(15)
Where the \( \beta_n \)'s are the positive roots of \( \beta \tan \beta = Bi \), the steady state temperature corresponds to \( \frac{\partial T}{\partial \tau} \rightarrow 0 \), and \( T \rightarrow 1 \). This is equivalent to Crank [12]. The time scale for temperature changes is by \( O(\tau^{1/2}) \) and hence \( \tau \gg 1 \) then rapidly compared to change in moisture. At this limit, the boundary condition for the moisture at the surface (equation (10) can be approximated by \( \phi(T) = \phi(T = 1) = \phi_1 \) as
\[ \frac{\partial M}{\partial \xi} = -Bi_m C_{\text{air}} (\phi_1 f(M) - 1) \quad \text{at} \quad \xi = 1. \] (16)

For drying processes with a small \( Bi \) number, a uniform temperature profile in the food can be assumed in the simulation and a single mass transfer model can thus be used to describe the drying process [2]. In this case, when temperature is uniform and increases rapidly compared to the loss of moisture, identified as an isothermal condition, the diffusion equations can be solved without consideration of a heat equation. The only parameter that gives effects in this case is \( Bi_m \). \( Bi_m \) number represents the surface convection mass transfer with respect to the diffusivity of water.

Fig. 2 shows the behaviour of moisture profile, varying the parameter \( Bi_m \). From Fig. 2, \( Bi_m \) has a great impact, if \( Bi_m=1 \). The results suggest that, at any time during the transient process, it is reasonable to assume a uniform moisture distribution across the food. This is not the case for drying, where the moisture gradient within the foods is significant (consistent with the experiment findings for mango [3]). In experiments by [6], when the \( Bi_m \) number is bigger than 30 and change of air velocity is made, the drying curve of moisture is practically overlapped, which shows that drying is by diffusion control. In the simulation in Fig. 2, we also observed that an increase in \( Bi_m \) causes a much faster decrease in moisture and the moisture gradient between the surface and centre is much bigger.

Fig. 3 shows a plot with different values of \( Bi_m \), consistent with the graph showing given by Newmann (in Crank [12]) showing residual moisture left at the surface with time; the bigger the \( Bi_m \) number, the faster the moisture equilibrium with drying air.

### 4.3 Non-Isothermal solution

Drying is a fundamental process involving simultaneous heat and mass transfer under transient conditions. When the \( Bi \) number is large (>>1), coupled heat and mass transfer should be taken into account in the simulation. The isothermal model is modified to solve the heat equation together with the diffusion equation. For this non-isothermal situation, temperature profiles will develop inside the material during drying and a differential energy balance is used to determine this temperature profile. More generally, latent heat is an important consideration and moisture and temperature equations must be solved simultaneously. In food-air interfaces, some heat is used for water evaporation (see equation (6)). This gives a coupled governing equation for heat and mass transfer at the surface.
Fig. 4 shows the prediction of moisture at the surface and at the centre of the sample. Fig. 4(a) shows the moisture profile through the sample of the fruit with increasing time. Moisture decreased but this was a little slower compared to the isothermal case. Figure 4(b) shows temperature and moisture profiles at the surface. The small value of Bi number, Bi=0.3, gives a slower increase in temperature and this affects the moisture profile, which decreased more slowly than in the isothermal case.

Figure 4: Profile of (a) moisture through the sample, with elapse time \( \tau = 0.3 \) in step of 0.25 (b) moisture and temperature at the surface and compared with the isothermal case (\( Bi_m = 20, Bi = 0.3 \) and \( \lambda = 0.5 \))

### 4.4 Sensitivity Analysis

Based on the thermo-physical values reported in the literature [7], the non-dimension value was around 0.5 for \( \lambda \) and 0.2-1 for \( Bi \). For this sensitivity study, we fixed the values as 0.5 for \( \lambda \) and 0.3 for \( Bi \).

Figure 5: Profile of temperature and moisture at the surface against time for different values of \( Le \) (\( Bi = 0.3, Bi_m = 20 \) and \( \lambda = 0.5 \))

Sensitivity analysis was conducted with a fixed value of \( Bi_m = 20 \) and an increased value of \( Le = 5-100 \). Fig. 5(a) shows that the larger the value of Lewis, the quicker the temperature increase. Based on the literature (Pavon Melendez et al. [6]), if the value of \( Le \) is more than 100, this gives an interior temperature that is approximately equal to the surface temperature. Fig. 5(b) shows the moisture at the surface with an increased value of \( Le \). Moisture evaluation at the surface decreased more quickly for a larger value of \( Le \).

To see the effect value of \( \lambda \) and \( Bi \), the simulation was conducted by fixing the value of \( Le \) and \( Bi_m \) and varying another two parameter properties, \( \lambda \) and \( Bi \). The first simulation was conducted with a fixed value of \( \lambda = 0.5 \) and varying the value of \( Bi = 0.1-50 \). As seen in Fig. 6(a), increasing the value of \( Bi \), the temperature increases rapidly and the moisture also decreases rapidly. The second simulation was conducted with a fixed value of \( Bi = 0.3 \) and increased value of \( \lambda = 0.1-10 \). Fig. 7 shows that, when the value of \( \lambda \) increased, the temperature increased slowly but was relatively flat for \( \lambda = 5 \) and 10. This is not the case for drying, where the temperature usually increases nearly the same as the air temperature. Thus, we can conclude that, with the value of \( Bi = 0.3 \), the approximation of the value of \( \lambda \) will be between 0.1-1. This is consistent with the findings of Pavon Melendez et al. [6] that the value of \( \lambda \) is around 0.5 for fruits. Based on parametric study, the choice of parameters \( Bi_m, Bi, Le \) and \( \lambda \) depends on the type of fruits and the drying air temperature.
5 Conclusions

This paper has adopted a simple one-dimensional model of heat and mass transfer that assumes fruit to be a homogenous structure, as discussed by Wang and Brennan [2]. Using numerical simulation, the behaviour of moisture and temperature subject to underlying air flow was calculated for constant diffusivity. With the assumption that a rapidly increasing temperature will generate a uniform temperature profile inside the fruit, the single mass transfer model can thus be used to describe the drying process. This case of moisture transfer by a diffusion process is known as an isothermal model. The results reveal that, with the use of the diffusion process alone, the movement of moisture can simulate the drying process. Taking the $Bi_{\infty}$ number as 20, we may describe the resistance to diffusion within the food as much more than the resistance to convection across the fluid boundary layer. This is consistent with experimental findings by [1] which indicate that the rate of water loss from the fruits appears to be limited by the diffusion of water through the sample but not the rate of evaporation from the surface. However, for typical drying process, it was found that temperature is an important consideration, taking into account a couple of heat and mass transfers in the simulation. This situation applies to the non-isothermal model. Our analysis indicates that an increase in temperature and a decrease in moisture during drying depend on the value of the parameters at the surface boundary, $Bi_{\infty}$ and $Bi$ and latent heat $\lambda$. In general, by changing these parameter values, all these numbers depend upon the type of fruit under study and validation of experiment results.

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References: