Numerical Analysing the Parabolic Catenary

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Abstract: This paper deals with calculation of a parabolic catenary. The horizontal reaction is determined by calculation. Four numerical methods were chosen and computational complexity of those methods was compared. These are the direct iteration, regula falsi, bisection and tangent method. The Matlab software was used to develop the computational algorithm task. The studied problem was a parabolic catenary with an additional condition – the total length.

Key-Words: parabolic catenary, algorithm development, numerical methods, analysis, horizontal reaction

1 Introduction
This paper analyses the numerical methods used to determine the parabolic catenary in Fig. 1. Catenaries are commonly used analysis methods for cable structures discussed in [1, 2]. Several approaches based on numerical methods with special catenary element are available [3, 4] when analysing the catenary or cable structures. Nonlinear analysis of cable structures is shown in [5, 6, 7]. The solution can be made on the basis of discrete analysis [8].

This paper uses four methods for determining of the horizontal reaction of a parabolic catenary. Computational complexity is compared for those methods. The methods are direct iteration, regula falsi, bisection and tangential methods. The Matlab software [9] was used computations. The studied problem was a parabolic catenary with an additional condition – the total length. The development of an algorithm was based on methods described in [10]. When analysing the steel structures, it is recommended in some cases to use the probabilistic approach [11].

2 Calculating the parabolic catenary
The cable which is suspended in two joints and loaded with a continuous load applied onto the horizontal projection is referred to as a parabolic catenary [12] [13]. Because of a very low bending stiffness, the load-carrying cable is considered in calculations to be an element which does not bear bending moments. The only internal force which arises in the structure is tensile force. All geometric and static quantities are expressed by means of horizontal reaction. Of importance for description of the structure is determination of the horizontal force. The horizontal force cannot be described using conditions of balance only. It is necessary to choose an additional condition.

In this case, the additional condition is the specified length of the cable. When calculating the horizontal force, the equation for the cable length (1) is taken as a basis. The horizontal force \( H \) was determined using iteration methods.

3 A model case of the parabolic catenary
The distance between the suspended points for this structure is \( l = 30 \) m and the difference in height is \( h = 1 \) m. The continuous load applied onto the cable projection is \( q = 0.8 \) kN/m.

\[ L = \frac{H}{2q} \left[ l \sqrt{l + x^2} + \ln \left( l + \sqrt{l + x^2} \right) + x \sqrt{l + x^2} + \ln \left( x + \sqrt{l + x^2} \right) \right] \] (1)

\[ \lambda_a = \frac{q l}{H}, \quad \lambda_b = \frac{q (l - x_d)}{H} \quad \text{and} \quad x_d = \frac{l^2 + H h}{2q} \] (2)
The additional condition – the length of the cable – is $L = 33$ m.

4 Iteration methods
All iteration methods are based on the equations (1) and (2). Another condition for those methods is selection of specific criteria, for instance, the value of the first approximation or the termination condition. In order to compare the solutions, same values for identical criteria were maintained. The use of the numerical methods was based on [14] and [15].

Fig. 2 Parabolic catenary – results
Direct iteration

Fig. 3 Parabolic catenary – results
Regula falsi
4.1 Direct iteration
For a graphical representation of this method see Fig. 2. The initial equation (1) was modified and one side shows the horizontal force only – that side of the equation represents the linear function (in blue), while the other side of the equation comprises other input parameters.
This method requires that the unknown variables have to be on the both sides of the equation. Then, the zero approximation and iteration cycles result in the final value. This is the intersection of the red and blue curves. The zero approximation needed for iteration is 1 kN. Two termination conditions were specified. The first termination condition is the exact number of iteration steps being 100. The second termination condition is the deviation between two subsequent calculated values being 0.001 kN. With this method, divergence was an issue. Because the curve which represented the modified equation (1) was convex, it was moving towards infinity in each subsequent iteration step. Therefore, the algorithm was modified in order to use an inverse function (in green). It was not necessary to determine the entire inversion function. It is, however, more efficient for the calculation to add a double of the difference between the original function and I and III quadrant axis to the original function. This resulted in convergence. For the required deviation the horizontal force was 14.8853 kN. For the required number of steps, 100, the horizontal deviation was 14.8911 kN.

4.2 Regula falsi
This method is sometimes referred to as the false position method or the chord method. For general background see Fig. 3. Once the equation (1) is adjusted to be homogeneous, this method gives intersection of a curve with a horizontal axis. The initial condition is the interval in which the required value is located. In every subsequent iteration step, a chord line is created between the outer points. Then, the value of the outer point in the interval is replaced with the value obtained by intersection of the chord and horizontal axis. The first three iteration steps are represented by the chords of the curve. Because of the shape of the curve under investigation, this method iterates very slowly. In order to accelerate convergence of this method, it would help making the input interval narrower so that the outer point could be as close as possible to the required value. The input interval comprises the required result and is limited by the lower boundary 1 kN and by the upper limit 40 kN. The condition which will stop the iteration is the deviation between the two subsequent calculated values. In order to keep the input conditions, this deviation is again 0.001 kN. The horizontal force calculated using this method is 14.8937 kN.

4.3 Bisection method
The bisection method or the interval dividing method is similar to the regula falsi because of its input criteria. The interval under investigation is limited again by 1 kN and 40 kN.
The termination condition is again the difference between two subsequent iteration values: 0.001 kN. The equation for the cable length (1) was modified and is homogeneous now. In the next iteration step, the previous interval with the required value is reduced. The value of the outer point of the interval changes after the functional value of the function under investigation is compared in the half which precedes the interval. If the difference against the next value is within the specified termination deviation, the calculation will be interrupted. The bisection method is described in Fig. 4. The first five iteration steps are described there as horizontal lines. It is evident that the interval with the required value (14.8910 kN) becomes smaller more quickly. Unlike the regula falsi method, the bisection method does not depend much on the shape of the curve under investigation and iterates considerably faster.

4.4 Newton method

This iteration method is shown in Fig. 5. The initial and termination conditions are identical with those used in the direct iteration method. In the zero approximation point, the tangent to the curve under investigation is found. Then, the intersection with the horizontal axis is found. The next tangent is constructed in the functional value of that point. The tangent represents the next iteration step. In the chart, the first four iteration steps are visible. The approach the final value, 14.8905 kN, relatively quickly. In order to develop an algorithm for this method it is necessary that derivations should be calculated in each iteration point [5]. The model was calculated using the three-point forward formula (3) with the 0.01 differentiation. If other methods were used, the time needed for the modelling by means of the Newton method did not extend. The number of iteration steps were not be influenced too.

\[
f'(x) = \frac{-3f(x) + 4f(x+\text{dif}) - f(x+2\text{dif})}{2\text{dif}} \tag{3}
\]

5 Comparison of the iteration methods

When using the methods described above, the resulting H was 14.89 kN for the given input values. Using this value, other geometric and force parameters can be determined. Fig. 6 shows deflection of the cable for the specified values of the structure. Fig. 7 shows how the calculated horizontal forces depend on the number of steps of each method. The zero step represents the initial approximation values. In terms of necessary steps, the regula falsi method is the most demanding – it requires 276 steps. The reason for such a high value is the shape of the function under investigation and the initial values. The result is also proved by the chart which shows the iteration steps used in the regula falsi method. See Fig. 3. The least number of iterations (7 steps) was needed by the Newton method. Tab. 1 shows
Fig. 6 Parabolic catenary – cable deflection

Fig. 7. Parabolic catenary – comparison of the methods
the number of iteration methods in each method as well as the time needed for the calculation. Except for the Newton method, the time correlates with the number of steps. The reason for more time needed in the Newton method is a rather long operation in one step, the reason being calculation of derivations in each point. The shortest time needed for calculation of the horizontal reaction was for the bisection method. Unlike the Newton method, the bisection method does not have enough input conditions. It is necessary to specify the interval where the required value is located. For this reason, it is recommended to use the tangent method. In this method, it is sufficient to determine the zero approximation and the difference. The direct iteration needed also a shorter time than the Newton method. But the direct iteration faces a similar problem as the bisection method.

6 Conclusion
The Matlab software [9] was used to model by means of iteration methods the horizontal reaction of a planar parabolic catenary.

<table>
<thead>
<tr>
<th>Method</th>
<th>H [kN]</th>
<th>Calc. steps</th>
<th>Time (s)</th>
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<tr>
<td>Iteration (step)</td>
<td>14.8911</td>
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<td>0.1393</td>
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<tr>
<td>Iteration (deviation)</td>
<td>14.8853</td>
<td>44</td>
<td>0.0602</td>
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<tr>
<td>Regula falsi</td>
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<tr>
<td>Bisection</td>
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<td>0.0349</td>
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<tr>
<td>Newton method</td>
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<td>7</td>
<td>0.0777</td>
</tr>
</tbody>
</table>

Table 1. Comparison of the methods – results

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References:


