Abstract: Experimental and numerical study of a thermally developing laminar pipe flow of a heterogeneous nanofluid was carried out. Effect of nanoparticle distribution on flow and heat transfer characteristics was investigated. With Iron-Oxide nanoparticle volume concentration of 0.009 in water, up to 15% heat transfer enhancement was predicted for the heterogeneous compared to the homogeneous nanofluid. Pressure drop was shown to decrease by 5%. Experimental results show a heat transfer enhancement up to 80% with the heterogeneous compared to homogeneous nanofluid.

Key-Words: Heat transfer, heterogeneous, laminar, nanofluid, Nusselt number, pipe

1 Introduction

Suspended particles of high thermal conductivity can enhance heat transfer in fluid suspensions. Researchers have long been working to improve the thermal conductivity of liquids with suspended solid particles [1]. Previously, using micro-scale particles led to pipe erosion and pump damage, as well as sedimentation and fouling. However, recent advances in materials technology has made it possible to produce nanoparticle suspensions that can overcome such limitations.

Fluids suspended by nanoparticles are termed nanofluids [2]. They are made by suspending the nanoparticles in liquids such as water, ethylene glycol, oil, radiator fluids, etc. The suspended nanoparticles can change the properties of the base fluid, leading to great potential for heat transfer enhancement. Reviews of the state-of-the-art research on nanofluids are given, for example, in [3, 4]. Trisaksri and Wongwises [3] presented a critical review of heat transfer characteristics of nanofluids. They summarized developments in research on the heat transfer characteristics of nanofluids for the purpose of suggesting some possible reasons why the suspended nanoparticles can enhance the heat transfer of conventional fluids and to provide a guide line or perspective for future research. Keblinski et al [5] presented a critical analysis of the experimental data in terms of the potential mechanisms and showed that by accounting for linear particle aggregation, the well established effective medium theories for composite materials are capable of explaining the vast majority of the reported data without resorting to novel mechanisms. In principle, therefore, one can model homogeneous nanofluids using effective medium approximation.

With the homogeneous mixture approach, the fluid phase and particles are assumed in thermodynamic equilibrium. This approach is relatively simple because it does not require the complexity of solving additional transport equations. As such, it has been applied in several theoretical studies of convective heat transfer in nanofluids, [6, 7, 8, 9, 10, 11]. Effect of base fluids on flow and heat transfer characteristics of nanofluids was investigated by Koo and Kleinstreuer [9]. Using two different base fluids, namely ethylene glycol and water, they have carried out a numerical study on nanofluids in micro channels, and have reported that base fluids with higher Prandtl number result in higher heat transfer enhancement.

Akbarinia [10] has carried out a numerical simulation to investigate the effect of nanofluid consisting of water and alumina on laminar flow and heat transfer characteristics in curved tubes. Using single phase with effective properties, they have reported that concentration of the nanoparticles does not have significant effect on the secondary flow, while the axial velocity, Nusselt number, and skin...
friction factor have been affected. Rea et al. [11] have investigated flow and heat transfer characteristics of alumina-water nanofluid in heated tubes. They reported that the experimental results are in good agreement with traditional model predictions for laminar flow, and that the heat transfer coefficient in the entrance and fully developed regions is estimated to increase by 17% and 27%, respectively, for alumina nanofluid at 6% by volume.

A common ground in previous nanofluid studies is that all nanofluids used were made to be homogeneous. While uniform suspension of colloids has been shown to enhance heat transfer, the enhancement comes at a significant cost in pumping power consumption due to significant increases in nanofluid density and viscosity with increasing colloid concentration. High concentration colloids near the centerline of pipes contribute to pressure drop and negligibly to heat transfer enhancement. Therefore, a novel approach by which having particles stratified in regions of high thermal gradients can lead to more heat transfer enhancement for a given overall power consumption, compared to homogeneous nanofluids.

2 Problem Formulation
Assuming the flow is steady, incompressible, and axisymmetric with no viscous heating, and the nanoparticles and base fluid forming a continuous nanofluid, the governing equations become

Continuity

\[
\frac{\partial}{\partial x} (\rho u_x) + \frac{\partial}{\partial r} (\rho u_r) + \frac{\rho u_r}{r} = 0
\]  

(1)

Axial Momentum

\[
\frac{1}{r} \frac{\partial}{\partial x} (r \rho u_x u_x) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u_r u_r) =
\]

\[
-\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_r}{\partial r} \right) \right]
\]  

(2)

Radial Momentum

\[
\frac{1}{r} \frac{\partial}{\partial x} (r \rho u_x u_r) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u_r u_r) =
\]

\[
-\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_r}{\partial x} \right) \right]
\]  

(3)

Energy

\[
\rho c \left[ u_x \frac{\partial T}{\partial x} + u_r \frac{\partial T}{\partial r} \right] = \frac{\partial}{\partial x} \left[ k \frac{\partial T}{\partial x} \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ kr \frac{\partial T}{\partial r} \right]
\]  

(4)

2.1 Boundary conditions
Uniform velocity inlet, no-slip condition, uniform nanoparticle volume fraction and heat flux at the wall, and prescribed outlet pressure.

2.2 Nanoparticle Distribution Function
In order to stratify the nanoparticles within the boundary layer, the following nanoparticle distribution function is used

\[
v(r) = v_w \frac{1 - e^{ar}}{1 - e^{aR}}
\]  

(5)

Equation (5) is used to distribute the nanoparticles across the pipe, where \(v_w\) is the nanoparticle wall boundary volume concentration and “\(a\)” is adjusted to fix overall nanoparticle volume concentration (by volume) to 0.009. In this study, the nanoparticle wall boundary volume concentration was set to 0.2. Distribution of the nanoparticles in this case is shown in Figure 4.

3 Nanofluid Properties

<table>
<thead>
<tr>
<th>Table 1 Property values</th>
<th>Conductivity W/(m K)</th>
<th>Density kg/m³</th>
<th>Specific heat J/(kg K)</th>
<th>Viscosity kg/(m s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base fluid</td>
<td>0.6</td>
<td>1000</td>
<td>4182</td>
<td>0.0013</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>13.5</td>
<td>5180</td>
<td>670</td>
<td></td>
</tr>
</tbody>
</table>

The base fluid is a homogeneous mixture of water and 0.009 (by volume) Iron-Oxide. Following the
effective medium theory [5] the Weiner model [12] is used to evaluate the thermal conductivity of the nanofluid as follows

\[ k = k_b + \frac{3.0 v k_b (k_c - k_b)}{2.0 k_b + k_c - v(k_c - k_b)} \]  \hspace{1cm} (6)

The density is evaluated using volume-weighted average of the liquid and particle densities.

\[ \rho = \rho_c + (1-v)\rho_b \] \hspace{1cm} (7)

Assuming thermal equilibrium, the specific heat is given by

\[ c = [\rho_c c_c + (1-v)\rho_b c_b]/\rho \] \hspace{1cm} (8)

Equation (8) has been validated with the experimental work of Zhou and Ni [13], and has shown to provide good agreement with measurements. Eq. (9) is used for viscosity [14].

\[ \mu = \mu_b (1+2.5v) \] \hspace{1cm} (9)

And viscosity of the base fluid is given as a function of temperature as follows

\[ \mu_b = -(1.83 \times 10^{-9})T^3 + (4.75 \times 10^{-7})T^2 \]
\[ - (4.33 \times 10^{-5})T + 0.00173 \] \hspace{1cm} (10)

4 Experimental Setup

A schematic of the experimental apparatus is shown in Figure 1. The loop was constructed at MIT Reactor Thermal Hydraulics Laboratory with stainless steel tubing, and the fluid was pumped throughout the system by a miniature gear pump. The volumetric flow rate was measured with a flow turbine meter, which has an accuracy of 0.5% of the reading in the range of study. The flow meter was positioned just after the pump discharge. The vertical test section was a stainless steel tube with an inner diameter (ID) of 5.5 mm, outer diameter (OD) of 6.4 mm, and a length of 1.0 meter. The test section had twelve sheathed and electrically insulated T-type thermocouples clamped along the test section along the equal-distance axial locations 10 cm apart, and two similar T-type thermocouples inserted into the flow channel before and after the test section to measure the bulk fluid temperatures. The test section used in the experiment was resistively heated by a DC power supply. After being heated the fluid was cooled using a recirculating chiller that provided flow to a coil placed in the accumulator. A data acquisition system was used to record the output of instruments. Additional loop components included a needle valve to control the flow rate throughout the loop and a drain valve. NdFeB, grade 42 block permanent magnets were used to generate magnetic fields along the height of the test section. These magnets had a length of 10.16 cm, a width of 1.27 cm and a thickness of 0.635 cm. The surface field of each magnet is about 3000 Gauss. In the data presented below, only two magnets were used which were attached to the walls opposite to each other. Accordingly, and unlike the numerical simulation, the magnetic field was not axisymmetric.

![Fig. 1 Loop setup for temperature measurements](image-url)
conducted with de-ionized water for experiment validation. Iron-Oxide nanofluid was tested in the laminar flow loop at a volumetric loading of 0.009%. The heterogeneous nanofluid was formed at the test section using two opposite slab-magnets attached to the outer wall. Further details of the set up are found in (R. Azizian et al, "Effect of Magnetic Field on Laminar Convective Heat Transfer of Magnetite Nanofluids." to be submitted to Journal of Heat Transfer.

5 Numerical Procedure
The axisymmetric Navier-Stokes and energy equations were solved using ANSYS 12.1. A structured mesh was built using Gambit 2.0. The mesh consisted of 10,000 cells. The mesh was non-uniform in order to optimize distribution in the boundary layer and entrance region where relatively higher gradients are expected. A snap shot of the mesh in the entrance region is shown in Fig. 2.

The simulation was carried out using SIMPLE algorithm [15] and second-order differencing schemes. The linearized equations were solved using Gauss-Seidel method, in conjunction with an algebraic Multigrid scheme [16]. A uniform heat flux boundary condition was applied at the wall.

6 Uncertainty Analysis
There are mainly two sources of uncertainty in CFD, namely modeling and numerical [17]. Modeling uncertainty can be evaluated through experimental or analytical validation while numerical uncertainty can be approximated through grid independence. Numerical uncertainty has two main sources, namely truncation and round-off errors. Higher order schemes have less truncation error, and as was outlined earlier, the discretization schemes invoked were second-order. In explicit schemes, round-off error increases with increasing iterations, and is reduced by increasing significant digits (machine precision). However, having used Gauss-Seidel iterative procedure in a steady-state simulation renders the calculation insensitive to round-off error.

Fig. 2 Snap shot of the mesh in the entrance region

Fig. 3 Nusselt number distribution along the pipe
A comparison between the numerical model and the analytical solution [18] is depicted in Fig. 3. The Nusselt number has been normalized by the fully developed value of 4.36. The numerical prediction with the higher resolution is in good agreement with the analytical solution. The grid independence is also verified. Therefore, both the numerical and modeling errors are considered negligible.

7 Results and Discussion
The normalized velocity and nanoparticle distributions across the pipe are shown in figure 4 at x/d = 100. The heterogeneous nanofluid has resulted in a slight decrease in the axial velocity around the pipe.
center of the pipe. The nanoparticle distribution profile is reflecting the boundary condition of 0.2% Iron-Oxide by volume at the wall boundary, and decreasing to zero around the center.

![Nusselt number distribution along the pipe](image1.png)

**Fig. 5** Nusselt number distribution along the pipe

Nusselt number distribution along the pipe is depicted in Fig. 5 for both the heterogeneous and homogeneous nanofluids. The experimental results show heat transfer enhancement up to 80%, while the simulation predicts only 15%. Several reasons can be behind the discrepancy. For one, Einstein’s model used to determine viscosity of the high-concentration nanofluid near the wall may have resulted in an under-estimation of the actual viscosity. This would lead to lower fluxes at the wall. Also, a uniform distribution of the nanoparticles was used along the pipe, when in fact the actual distribution would take place gradually and take more of a bell-shape distribution as is reflected in the experimental profile. Finally, an axisymmetric distribution would lead to less density of the nanoparticles than the actual distribution in the experiment.

The normalized friction coefficient of the heterogeneous nanofluid along the pipe is depicted in figure. 6. A sudden decrease in wall friction is followed by an approximately equal increase. These opposing trends resulted in a net decrease of 5% in pressure drop along the pipe. The decrease in the friction factor is attributed to a decrease in strain rate caused by the increase in viscosity at the wall.

![Normalized friction factor distribution along the pipe](image2.png)

**Fig. 6** Normalized friction factor distribution along the pipe

### 8 Conclusion

Experimental and numerical study was carried out on a thermally developing laminar pipe flow of a heterogeneous nanofluid. Effect of nanoparticle distribution on flow and heat transfer characteristics was investigated. With Iron-Oxide nanoparticle volume concentration of 0.009 in water, up to 15% heat transfer enhancement was predicted for the heterogeneous compared to the homogeneous nanofluid. Experimental results showed a heat transfer enhancement up to 80% with the heterogeneous nanofluid. A 5% decrease in pressure drop was predicted. This study shows that heterogeneous nanofluids can be effective in augmenting heat transfer in pipes without increasing the pressure drop compared to homogenous nanofluids.

### 8 Acknowledgements

The numerical simulations of this study were conducted at King Saud University, Riyadh, Saudi Arabia. The experiments were performed at Massachusetts Institute of Technology, Cambridge, MA. Special thanks are due to Prof. Lin-wen Hu of the MIT Research Reactor Lab for designing the experimental setup and supervising conduction of the experiments.

### References:


