

# Simulation Models for Supporting Hedging in Illiquid Markets

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*Abstract:* Market liquidity is a very important topic for financial risk management. This paper proposed the use of an integrated model (Monte Carlo, System Dynamics and Discrete Event Simulation) in order to support energy management in a steel plant. After the introduction, which presents a literature review, we have analyzed the use of System Dynamics (SD) and Discrete Event Simulation (DES). Finally, we have analyzed a case of a on-line real time tool related to steel production. In this case we used the techniques previously described emphasizing their strengths.

*Key-Words:* liquid markets, illiquidity markets, System Dynamics, Discrete Event Simulation, steel production

## 1 Introduction

Markets liquidity is an issue of very high concern in financial risk management. In a perfect liquid market the option pricing model becomes the well-known linear Black–Scholes problem while nonlinear models are used when illiquid market effects are taken into account. The Black–Scholes pricing allows investors to calculate the “fair” price of a derivative whose value depends on an underlying. One of the major assumptions of Black–Scholes model is that the market in the underlying asset is perfectly elastic, this occurs in perfectly liquid markets, but the case is clearly unrealistic[1]. The presence of price impact of investors’ trading has been widely documented and extensively analyzed in the literature. In the presence of asymmetric information use an equilibrium approach to investigate how informed traders reveal information and affect the market price through trading [1].

In perfectly liquid markets one can find a dynamic trading strategy that reproduces the movement of the price of a given derivative asset by judiciously trading the underlying asset and cash. Any option in a complete and perfectly liquid market can be replicated by following the appropriate trading strategy [7]. The presence of price impact of investors’ trading has been widely documented and extensively analyzed in the literature [2,3,4]. Sircar and Papanicolau (1998) claim: “Increases in market volatility of asset prices have been observed and analyzed in recent years and their cause has generally been attributed to the popularity of portfolio insurance strategies for derivative securities” [4]. In the literature are

presented pricing models that predict increased implied volatilities with minimal assumptions beyond those of the Black-Scholes theory. They are characterized by a nonlinear partial differential equation that reduces to the Black-Scholes equation when the feedback is removed [4].

Some authors consider a model where the asset price process is driven by some exogenous source of randomness (i.e. Brownian motion) and by the trading strategy of a representative agent who is hedging derivatives and the rest of the market that is made by small operators [i.e. 5]. Representative hedger is a large trader and the implementation of his hedging strategy has a feedback effect on the price of the stock: the stock price rises (falls) if the representative hedger buys (sells) additional shares of the stock. Some authors proposed an extensive simulation study over the Frey’s “perturbate” nonlinear Black-Scholes model. The simulation is made in order to better understand the implications of market illiquidity for derivative asset analysis [6]. Frey and Patie use a simulation study for the tracking error for different hedging strategies in an illiquid market [6].

Other authors exposed the quantitative feedback effect of a large trader strategies over the prices defining the excess demand function [7]. They analyze the difficulties encountered if one tries to build an option pricing theory in illiquid markets and argued that in many cases the possibility of market manipulation will make option trading impossible. They use a modeling technique based on solving Partial Differential Equations (PDE) with a process of numerical integration [7]. Frey uses a partial differential equation for perfect replication

trading strategies and option pricing in feedback markets [8,9]. He considers a model where the asset price process is driven by some exogenous source of randomness (in his case a standard Brownian motion) and by the trading strategy of a representative agent who is hedging derivatives. He obtains a formula for the impact of hedging on market volatility and characterizes perfect hedging strategies by a non-linear version of the standard Black-Scholes partial differential equation [5]. Some authors - in order to solve the nonlinear PDE numerically - have implemented an efficient numerical scheme, which is used to study for a number of different payoffs the properties of hedge cost [6].

The paper is structured as follows. In paragraph two are analyzed some simulation techniques: System Dynamics, SD, (as a special case of Time Stepped) and Discrete Event Simulation (DES). The paragraph three contains the analysis of a case.

## 2 Simulation Techniques: Monte Carlo, Time Stepped and Discrete Events

Looking around is possible to find many modeling techniques associated to the concept of "simulation". In financial and trading application Monte Carlo method is wide spread, however other less frequented used paradigms could be very useful in improving modeling. In particular, Time Stepped (System Dynamics) simulation could be very useful to improve the quality and the fidelity of the models [7]. Discrete Event Simulation is very powerful to implement the complex interaction among traders extending the hedging paradigm toward an integrated view.

Before describing the possible application of these simulation paradigms a simple yet concise introduction to both time stepped (System Dynamics) and Discrete Event Simulation will be made.

### 2.1 System Dynamics formalism

System Dynamics is a powerful tool to support strategic Decision Making Processes. It is often used as a tool to model changes in demand across supply chains. By incorporating models of price processes into such supply chain models, it may be possible to construct a system dynamics model that adequately represents the complex interactions of price and quantity [10].

Developed at MIT in the fifties, System Dynamics is a methodology that focuses on the

internal structure of a system, underlying its feedback loops, cause-effect relations and delays. It deals with internal feedback loops and is based only on two building blocks: Stocks and Flows. Figure 1 shows flow and stock.

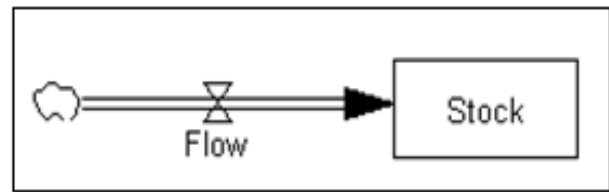


Fig. 1 Flow and Stock

Instead of Discrete Event Simulation, that build bottom-up, System Dynamics is able to capture complexity from a top down approach that is much suitable for data driven applications. Using System Dynamics is possible to see not just events, but also patterns of behavior over time.

System Dynamics is an approach to understanding the behavior of complex systems over time. It gives the opportunity to see not just single events but also pattern of behaviors over time and their development. This model helps in studying and analyzing the system in a quantitative way by expressing complex differential equations into simple diagrams.

Such models are usually built and simulated using several computer software: some of them integrated into ERP (Enterprise Resource Planning). From the mathematical point of view Flows could be considered as variables and Stocks as integrals or Stocks could be regarded as variables while Flows their derivatives.

The following describes a Wiener Process /Brownian Path implemented in classical Matlab-like script.

```
N = 1E6;           % number of timesteps
randn('state', 0); % initialize random nr generator
T = 1;             % final time
dt = T/(N-1);     % time step
t = 0:dt:T;
dW = sqrt(dt)*randn(1,N-1); % Wiener increments
W = [0 cumsum(dW)]; % Brownian path
```

As described above in Matlab can be translated through the SD, as shown in Figure 2.

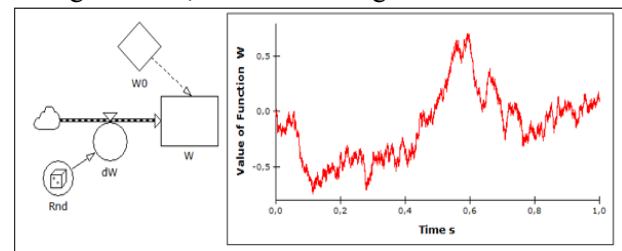


Fig. 2

The right part of the figure above represents flow and stock, the graph shows the typical Brownian motion obtained from the simulation.

Using System Dynamics Stock/Flow modeling paradigm was able to implement the Black-Scholes model [10]. The main advantages of using the SD are described below.

Given the non-linear, behavior driven, and interconnected characteristics of trading in illiquid markets, modeling concepts from SD theory provide appropriate and attractive features.

More the graphical based modeling approach allows the equations and the complexities to be added and integrated step-by-step and thus validated in a more simple way rather than using Matlab or Spreadsheet based scripting.

Models can incorporate qualitative and semi-qualitative data as well as live data coming from trading platforms (i.e. Bloomberg, Reuters, etc.) in order to provide fully integrated decision support systems to trade in illiquid markets.

Finally SD models are much more maintainable than Matlab or Spreadsheet based models since are more easy to understand due to the explicit graphically link among variables and their derivatives.

According to Pézier (2004) [11], the financial models fall into three main categories:

- probabilistic/statistical models describing uncertainties about the future values of market factors (geometric Brownian motion, stochastic volatility models, GARCH models etc.);
- pricing models relating the prices and sensitivities of instruments to underlying market factors (i.e. Black-Scholes option pricing model);
- risk aggregation models evaluating the corresponding uncertainties on the future values of portfolios of financial instruments (value-at-risk-VaR models).

Some authors describe a unique SD model able to capture the above views applied to a 10 year data base to quantify financial risks based on the combination of methods and techniques such as parametric VAR, historical and Monte Carlo simulations, Bayesian inference and game theory [12].

## 2.2 Use of Discrete Event Simulation in Energy Management and Hedging

In Discrete Event Simulation (DES), the operation of a system is represented as a chronological sequence of events. Each event occurs at an instant in time and marks a change of state in the system

(i.e. a transaction from one state to another according to Markov's models). This approach produces extremely fast simulations allowing the possibility to do on-line simulation during trading.

Just to give an idea about this advantage in term of computational time: a one year simulation of GNG logistics among two terminals took more than 4 hrs with Time Stepped simulation while with DES it requires only 8 s.

The price to pay for this heavy reduction of computational overhead is an increased level of complexity in the modeling phase since the entire process has to be described from event to event. At the same time is possible to design a unique platform where physical needs (i.e. commodities for production, energy consumption, etc.) can meet hedging opportunities in a unique optimization framework.

Hedging is an approach born mainly to "secure" commodities from the unexpected market's volatility.

The main aim of this activity is strictly related to a set of "physical" needs such as:

- provide raw materials for a production process;
- supply energy at a certain price;
- guarantee margins for next period contracts.

As "opponent" of speculator the hedger is looking for a way to transfer its risk using market as opportunity. In perfect liquid markets this could be done quite easily while in illiquid one (i.e. energy) this goal could be more difficult to achieve. At the same time market could offer something more than an hedging opportunity: it could make possible to achieve competitive advantages over competitors. In order to achieve this is necessary to create a more sophisticated integrated model where all the key aspects are taken properly into account:

- physical constrains (i.e. production capabilities);
- market situation and opportunities (i.e. spot prices, options, etc);
- business requirements (i.e. due dates, flexibilities, etc).

## 3 Simple case: a on-line real time tool for supporting energy management in a steel plant (continuous casting facility)

We have analyzed a simple case: a on-line real time tool for supporting energy management in a steel plant (continuous casting facility).

Steel production (from ore to casting) is a process where energy consumption took an important part in the overall cost structure (i.e. 40% over the final cost for Electric Arc Furnace plants). Today world crisis has reduced the total production in western countries leading to a dramatic search for cost reduction. The data relating to the steel production, between 2000 and 2009, show for the European Union a pass from 193,387 (thousand tonnes) produced in 2000, to 139,366 produced in 2009, a decrease of about 27.9%. The reduction is even more evident for the North America where, from 2000 to 2009, steel production was reduced by about 39%. In contrast, for example, the China, for which the steel production has increased by about 349% from 2000 to 2009. Figure 3 shows the trend of steel production in the European Union from 2000 to 2009, while Figure 4 shows the trend of the production in China.



Fig. 3 Steel production in European Union (Data from World Steel Association)

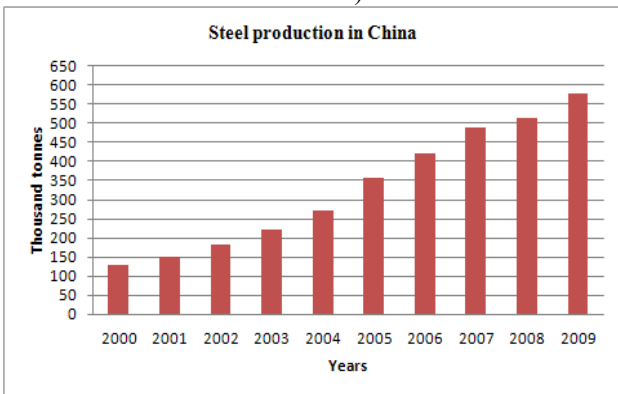


Fig. 4 Steel production in China (Data from World Steel Association)

The industrial companies that operate mainly with high automation level plants are characterized by process chains strongly linked to technical and operational decisions of production. Managers are often forced to work in a highly stressful environment and characterized by high uncertain. Working in an existing plant that is far from

saturation (i.e.  $Ut\% \approx 40-50\%$ ) give the manager the opportunity to move the production in a time where the energy cost is lower [13]. This is especially true when using EAF (Electric Arc Furnace) as source of liquid steel that is an extremely flexible machine.

Generally is possible to have some advantages in such kind of day planning (i.e. peak and off-peak hours). However this is just a glimpse of what could be done by connecting “the plant” with “the market” in real time [14]. Figure 5 shows the general concept of on-line/real time simulation.

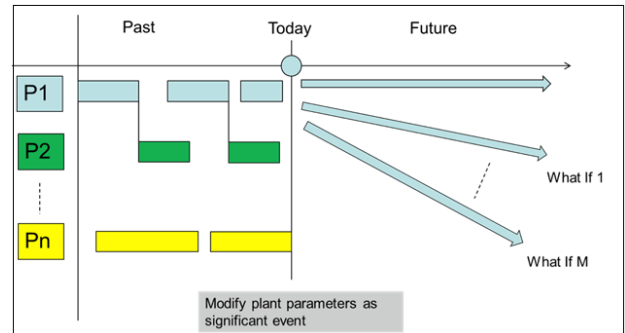


Fig. 5 On-Line/Real Time Simulation Schema

As already pointed out the existing plants produce less but with the same costs. In this regard it is necessary to buy the electricity, the cost of which weighs heavily on the total cost, under the best conditions. Figure 6 shows, on the right side, the steel process considered, while, on the left side, the blue curve represents the power consumption of the system.

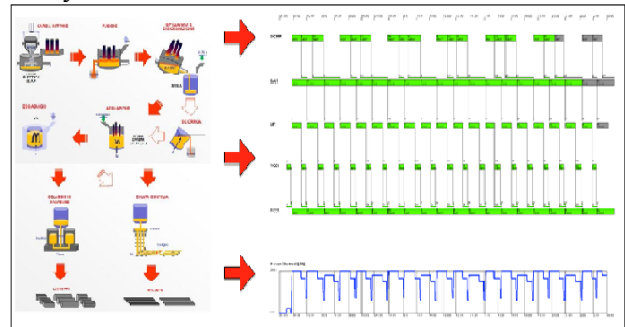


Fig. 6 – The General Architecture

The Discrete Event Simulation was used to obtain the power consumption daily, minute by minute (MW). The energy market can be analyzed using the System Dynamics, described in paragraph 2.1. Using these two techniques and comparing the results obtained it is possible to buy energy under the best conditions and reschedule, thus, the production. The decision is then based on the continuous analysis of the market through on-line simulation over the process schedule and the market opportunities. In particular the control over the experimental

errors is made with reference to several works available in literature using the innovative MSPe methodology [15][16][17][18].

#### 4 Conclusion

The paper successfully proposed the use of an integrated model (Monte Carlo, SD and DES) in order to support energy management in a steel plant.

A literature review and a simple yet integrated case study was presented and discussed.

Trading and hedging in illiquid markets require a more sophisticate approach: models became more complex to handle. While several approaches are available in literature the more promising solution came from the exploitation of several models into a unique platform. Beside traditional Matlab-based and Spreadsheets simulation effective approach require a more sophisticate simulation tool.

System Dynamic graphical formalism allows the implementation of more readable and maintainable models that could grow up into valuable solutions.

Finally Discrete Event Simulation could bridge the gap between “market-technical-only” models and “industrial-physical-process” creating a value added possibility for hedging just managing the “other side”.

#### References:

- [1] R. Company, L. Jódar, J.R. Pintos, A consistent stable numerical scheme for a nonlinear option pricing model in illiquid markets, *Mathematics and Computers in Simulation*, Vol.82, 2012, pp. 1972–1985.
- [2] P. Jorion, Risk management lessons from Long-Term Capital Management, *European Financial Management*, Vol.6, No.3, 2000, pp.277-300.
- [3] W.F. Sharpe, G.J. Alexander, J.V. Bailey, *Investments*, Prentice Hall, New Jersey, 1999.
- [4] K.R. Sircar, G. Papanicolau, General Black-Scholes models accounting for increased market volatility from hedging strategies, *Applied Mathematical Finance*, Vol.5,1998, pp.45-82.
- [5] R. Frey, *Market Illiquidity as a Source of Model Risk in Dynamic Hedging*, Risk Publications, London, 2000.
- [6] R. Frey, P. Patie, *Advances in Finance and Stochastics: Essays in Honour of Dieter Sondermann*, K. Sandmann and P. Schonbucher, Springer, 2002.
- [7] P.J. Schonbucher, P. Wilmott, The feedback effect of hedging in illiquid markets, *SIAM J. APPL. MATH*, Vol. 61, No. 1, 2000, pp. 232–272.
- [8] R. Frey, Asset Price Volatility and option Hedging in Imperfectly Elastic Market, Ph.D. thesis, University of Bonn, Germany, 1995.
- [9] R. Frey, Perfect option hedging for a large trader, *Finance and Stochastics*, Vol. 2, No. 2, 1998, pp. 115-141.
- [10] D. Cooke, *Using System Dynamics Models to Enhance the Visualization of Stochastic Price Processes*, Proceedings of the 22 International Conference, July 25 – 29, 2004, Oxford, England, UK.
- [11] J. Pézier, Risk and risk aversion, *The professional risk managers handbook*, C. Alexander and E. Sheedy, 2004.
- [12] R. M. Chain, M. Castellano, *ERM quantitative risk analysis methods and techniques applied to a small commercial bank*, Proceedings of the 30th International Conference of the System Dynamics Society, July 22-26, 2012, St. Gallen, Switzerland.
- [13] R. Revetria, A. Testa, R. Mosca, A. Bertolotto, *A Flexible Modeling Approach for Supporting Rapid Business Simulations*, Proceedings of 10th International Conference on Software Methodologies, Tools and Techniques (SoMeT\_11), September 28-30, 2011, Saint Petersburg, Russia.
- [14] R. Mosca, R. Revetria, *On-Line and real time simulation framework for supporting production optimization in steel manufacturing industry*, Proceedings of the 2011 Winter Simulation Conference, December 11-14, 2011, Phoenix, AZ, USA.
- [15] Mosca R., Giribone P. (1982), “Optimal lenght in O.R. simulation experiments of large scale production system”, Proceedings of IASTED "Modelling, Identification and Control", Davos (CH), 1982, pp. 78-82.
- [16] R Mosca, AG Bruzzone, L Cassettari, M Mosca (2009), “Risk analysis for industrial plants projects: an innovative approach based on simulation techniques with experimental error control”, *Proceedings of the European Modelling and Simulation Symposium (EMSS'09)*
- [17] Mosca R., Mosca M., Cassettari L., Giribone P.G., (2010) “The Stochastic analysis of investments in industrial plants by simulation models with control of experimental error: theory and application to a real business case” ,

*Applied Mathematical Sciences, Vol. 4, 2010,*  
*no. 76, 3823 - 3840*

- [18] Cassettari L., Mosca R., Revetria R.,  
(2012) “Monte Carlo Simulation Models  
Evolving in Replicated Runs: A Methodology  
to Choose the Optimal Experimental Sample  
Size”, *Mathematical Problems In Engineering*,  
*Article Number: 463873 DOI:*  
*10.1155/2012/463873*