Effective Value Calculation Using Wavelet Transform

VIOREL APETREI, CONSTANTIN FILOTE, CALIN CIUFUDEAN
Electrical Engineering and Computer Science Faculty
Stefan cel Mare University of Suceava
University Street no. 13, 720029 Suceava
ROMANIA
viorela@eed.usv.ro, filote@eed.usv.ro, calin@eed.usv.ro

Abstract: - The process of calculating the effective values of voltage and current root mean square (RMS) using Fourier transform (FT) suffers a high computational effort. Since it provides only an amplitude-frequency spectrum, loses time-related information, and is unable to deal with non-stationary waveforms, standard definitions are reformulated in the time-frequency domain using the wavelet transform (WT). The wavelet transform is a powerful tool because it is able to preserve time and frequency information, decreases the computational time and effort by splitting the frequency spectrum into bands or levels. Furthermore, it is able to represent different degrees of distorted waveforms more precisely than FT. In this case, the spectral leakage can be reduced by appropriate selection of the wavelet family and the mother wavelet. When a voltage or current waveform is decomposed and analysed using wavelet transform, the wavelet coefficients can be used to calculate effective values in a way similar to that in the frequency domain using Fourier series. The results obtained by applying the IEEE Standard definitions and the DWT-based definitions for effective RMS show that the differences related to DWT are very small.

Key-Words: - wavelet transform (WT), power quality, electric power components, root mean square (RMS), multi-resolution analysis (MRA), wavelet family, mother wavelet, data acquisition system

1 Introduction
The interest for power quality has increased in the last years both to utilities and their customers. Steady-state waveform distortion due to harmonics is one of the major power system problems. Harmonics are generated by variable speed drives, three phase rectifiers with diode [11], [12], [13], arc furnaces, and other non-linear devices. Given that harmonics can severely degrade the performance of power system equipment, it is essential to monitor their parameters such as voltage, current, and power [1], [2].

The traditional ways of power measurement has been performed in both the time and frequency domain using the Fourier Transform approach.

Regarding RMS and real power as well as their dependent quantities such as reactive power and power factor, the time domain approach is the most efficient and most precise. This happens because the voltage and current waveforms concomitantly sampled at uniform intervals over one or more cycles are the base for all digital methods.

The frequency domain approach allows the determination of influences of distortion and harmonic. On the other hand, it undergoes the constraint of periodicity and the loss of temporal perspective. In spite of the substantial efficiencies offered by the class of Fast Fourier Transform algorithms, it is the most computationally intensive over any span of frequencies because its spectral results are equally distanced in frequency [3].

The wavelet transform (WT), which was introduced by Ingrid Daubechies represents a powerful tool being able to represent any time domain waveform into a time-frequency domain. It allows preserving all information on time and frequency in the analyzed waveform, whereas in the case of FT, the time information is lost during the transformation. Furthermore, instead of performing the computation at every frequency point as in the FT, and by splitting the frequency spectrum into bands, DWT reduces the computational effort [4]. Its base functions, called mother wavelets are irregular, have several vanishing moments and asymmetries, being different from FT functions (sine functions), which are smooth. The WT is able to characterize diverse degrees of distorted waveforms more precisely than FT.

The results of using wavelet transform for RMS and power measurements indicate that the discrete wavelet-based algorithm can measure RMS and power of a number of harmonics within each
frequency band. Nevertheless, the waveform decomposition results using the DWT provide non-uniform frequency bands.

For instance, the frequency band becomes wider at a higher level of decomposition. Consequently, frequency bands at the higher levels have more harmonic components than those at lower levels. For that reason, the discrete wavelet-based algorithm cannot measure the RMS and power of individual harmonic components [5].

In this paper, the RMS value of the voltage and current waveform is obtained using the DWT. The waveform is captured with a data acquisition system which contains voltage and current transducers using the Hall effect.

2 The IEEE Standard 1459–2000 power component definitions

The power component definitions established by the IEEE Standard 1459–2000 [6] for single-phase system with nonsinusoidal situations are presented in this section. Consider the following nonsinusoidal voltage and current:

\[ v_i = \sqrt{2} V_i \sin(\omega t - \alpha_i), \quad i_i = \sqrt{2} I_i \sin(\omega t - \beta_i) \]  

\[ v_H = \sqrt{2} \sum_{h=1} V_h \sin(h \omega t - \alpha_h) \]  

\[ i_H = \sqrt{2} \sum_{h=1} I_h \sin(h \omega t - \beta_h), \]  

where \( v_i, i_i \) represent the power system frequency components (\( f_i = 50 \) or 60 Hz), while \( v_H, i_H \) represent the harmonic components. \( \alpha_i, \beta_i \) represent the fundamental voltage and current phase angle, respectively, and \( \alpha_h, \beta_h \) represent the harmonic voltage and current phase angle, respectively [7].

2.1 RMS Calculations with FFT

The RMS value of the nonsinusoidal voltage is defined as

\[ V = \sqrt{\frac{1}{T} \int_0^T v^2 dt} = V_i^2 + V_H^2 = \sum_{h=1} V_h^2, \]  

where \( T \) is the period. The RMS value of the nonsinusoidal current is defined as

\[ I = \sqrt{\frac{1}{T} \int_0^T i^2 dt} = I_i^2 + I_H^2 = \sum_{h=1} I_h^2. \]

The IEEE Standard 1459–2000 highly recommends definitions based on the frequency domain approach using the Fourier transform (FT). Frequency domain approach experience high computational burden and loss of time information. To overcome these limitations, time-frequency-based approaches are built up to describe power quantities and a number of quality factors for single-phase systems by the use of the wavelet transform due to its capability to save frequency and time information, along with reducing the computational effort as well by splitting the frequency spectrum into bands as an alternative to performing the calculations at each frequency point.

3 Wavelet Transform (WT)

Wavelet analysis is the process of transforming the waveforms defined in the time domain into a time-frequency domain by using wavelet functions called mother wavelet. The process is performed by calculating the wavelet coefficients at every scale (frequency) and position (time), as a result obtaining the continuous wavelet transform (CWT) [5]

\[ C(\text{scale}, \text{position}) = \int_{-\infty}^{\infty} s(t) \psi(\text{scale}, \text{position}, t) dt \]  

where \( C \) corresponds to the continuous wavelet coefficients signifying how close the original waveform \( S \) is to the wavelet function \( \psi \) at this particular scale and position. The wavelet functions at every scale and positions are called daughter wavelets.

Calculating the wavelet coefficients at every scale and position entails a high computational effort. To surmount this shortcoming; the frequency spectrum is divided into uniform bands resulting in the WPT. In WPT, the computational burden increase correlates with the increase in number of decomposition levels. For this reason, when calculating the wavelet coefficients, the frequency spectrum is divided into a limited number of selected scale subsets using powers of two dyadic scales, thus obtaining the DWT.

In DWT, the original waveform \( S \) is first decomposed into approximation “A” and detail “D”. Then, only approximations are successively decomposed, with no further decomposition for the details (Fig.1), hence obtaining the multi-resolution analysis (MRA).

When only approximations are successively decomposed, and no other decompositions of the
Recent Researches in Electric Power and Energy Systems

3.1 Multi-Resolution Analysis (MRA)

The practical usefulness of DWT comes from its Multi-Resolution Analysis (MRA) ability [9], [10], [14], [15] and efficient Perfect Reconstruction (PR) filterbank structures.

Multiresolution analysis (or Multiscale analysis) consists of a sequence of embedded subspaces \( V_0 \subset V_1 \subset V_0 \subset V_-1 \subset V_-2 \ldots \) of \( L^2(\mathbb{R}) \) as shown in Fig.3.

The MRA follows the following conditions:

1. \( V_j \subset V_{j+1}, \quad k \in \mathbb{Z} \)
2. \( V_\infty = \{0\} \) and \( V_\infty = L^2 \)
3. \( f(t) \in V_j \iff f(2^j t) \in V_{j+1} \)
4. \( V_2 = V_0 + W_0 + W_1 \)
5. \( L^2 = \ldots + W_2 + W_{-1} + W_0 + W_1 + W_2 + \ldots = V_0 + W_1 + W_2 + \ldots \ldots \)
6. \( W_\infty + \ldots + W_{-2} + W_{-1} = V_0 \) (8)

A scaling function \( \phi(t) \) (Father wavelet) is introduced such that for each fixed \( j \), the family

\[
\psi_{j,k} = 2^{-j/2} \phi(2^{-j/2} t - k), \quad (j,k \in \mathbb{Z})
\]

(9)
is an orthonormal basis of the subspace \( W_j \).

If \( W_j \) is orthonormal component of \( V_j (W_j \perp V_j) \) in subspace \( V_{j+1} \), then there exists a function \( \psi(t) \) (Mother wavelet) such that for each fixed \( j \) the family

\[
\psi_{j,k} = 2^{-j/2} \phi(2^{-j/2} t - k), \quad (j,k \in \mathbb{Z})
\]

(10)
is an orthonormal basis of the subspace \( W_j \).

Because of the nested subspaces and MRA condition (3), the scaling function satisfies the following 2-scale (dilation or refinement) equation,

\[
\phi(t) = \sqrt{2} \sum_{n=-\infty}^{\infty} h_0[n] \phi(2t - n), \quad (n \in \mathbb{Z})
\]

(11)
where it satisfies the admissibility condition

\[
\sum_{n} h_0[n] = \sqrt{2}.
\]

The wavelet function satisfies similar equation,

\[
\psi(t) = \sqrt{2} \sum_{n=-\infty}^{\infty} h_1[n] \phi(2t - n), \quad (n \in \mathbb{Z})
\]

(12)
with the conditions \( \sum_{n} h_1[n] = 0 \) and

\[
h_1[n] = (-1)^n h_0[-n + 1],
\]

where \( h_0[n] \) and \( h_1[n] \) can be viewed as the coefficients of lowpass and highpass filters. For a function \( f(t) \), the wavelet coefficients \( \{\psi_{j,k}(t), f(t)\} \) describe the information loss when going from projection of \( f(t) \) onto the space \( V_{j+1} \), to the projection onto the lower resolution space \( V_j \).

With MRA, any function \( f(t) \in L^2 \) can be modified by using both scaling function and wavelet function as:

\[
f(t) = \sum_{j,k} \sum_{n=-\infty}^{\infty} C_j (j,k) \phi_{j,k}(t) + \sum_{j} \sum_{n=-\infty}^{\infty} D_j (j,k) \psi_{j,k}(t)
\]

(13)
where \( C_j (j,k) = \langle \phi_{j,k}(t), f(t) \rangle \) are the scaling function coefficients, \( J_0 \) is an arbitrary starting scale for coarsest resolution, and \( J_f \) is an arbitrary finite upper limit for highest resolution with \( J_f > J_0 \).

3.2 Selecting the Wavelet Family and Mother Wavelet

The choice of decomposition levels number, the appropriate choice of the wavelet family along with right mother wavelet can contribute to spectral leakage decrease. The level number depends on the harmonic order displayed by the original waveform. The appropriate wavelet family and suitable mother wavelet can be obtained by evaluating the percentage energy of the wavelet coefficients at each level \( j \)

\[
\% E_j = \frac{E_j}{E} \times 100
\]

(14)
where \( E \) is the energy of the original signal, and \( E_j \) is the energy of the coefficients at each level.
where \( c_j \) are the DWT coefficients at any wavelet decomposition level \( j \) [5]. Consequently, the appropriate wavelet family and mother wavelet assure the minimum energy deviations for all wavelet levels.

The DWT is applied to the phase voltages, line voltages and line currents after being sampled with a sampling frequency \((f_s = 7680 \text{ Hz})\). The approximations and details are obtained with five decomposition levels. The most appropriate wavelet family is Daubechies and the most suitable mother wavelet is ‘db10’ [4]. The pseudofrequency \( F_j \) and \( j \) the pseudo-period of each wavelet level can be calculated using the following:

\[
F_j = \frac{F_s \cdot F_c}{2^j}.
\]

Here, \( F_s \) is the sampling frequency, and \( F_c \) is the central frequency of the chosen wavelet, that is estimated by correlating a periodic signal with the mother wavelet, as presented in Fig. 4.

The pseudofrequency and pseudoperiod for each wavelet level using “db10” mother wavelet are listed in Table 1.

<table>
<thead>
<tr>
<th>Wavelet Levels</th>
<th>Pseudofrequency</th>
<th>Pseudoperiod</th>
</tr>
</thead>
<tbody>
<tr>
<td>d1</td>
<td>2627.4</td>
<td>0.0004</td>
</tr>
<tr>
<td>d2</td>
<td>1313.7</td>
<td>0.0008</td>
</tr>
<tr>
<td>d3</td>
<td>656.8</td>
<td>0.0015</td>
</tr>
<tr>
<td>d4</td>
<td>328.4</td>
<td>0.0030</td>
</tr>
<tr>
<td>d5</td>
<td>164.2</td>
<td>0.0061</td>
</tr>
</tbody>
</table>

### 3.3 RMS Calculations with DWT

The equations of both the RMS values of the voltage and current waveform using the wavelet transform are [3], [5]:

\[
V = \sqrt{\frac{1}{T} \int_0^T v(t)^2 dt} = \sqrt{\frac{1}{T} \sum_{j=0}^\infty \sum_{k} c_{j,k}^2} = \sqrt{V_{app}^2 + \sum_{j>0} V_j^2} \quad (17)
\]

\[
I = \sqrt{\frac{1}{T} \int_0^T i(t)^2 dt} = \sqrt{\frac{1}{T} \sum_{j=0}^\infty \sum_{k} d_{j,k}^2} = \sqrt{I_{app}^2 + \sum_{j>0} I_j^2} \quad (18)
\]

Here, \( V_{app}, I_{app} \) are the RMS values of the voltage and current of the lowest frequency band \( j_0 \), also called approximated voltage (\( V_{app} \)), and approximated current (\( I_{app} \)), respectively. \( \{V_j\} \) and \( \{I_j\} \) are the sets of RMS values of the voltage and current of each frequency band or wavelet-level higher than or equal to the scaling level \( j_0 \), and are called detailed voltage (\( V_{det} \)) and detailed current (\( I_{det} \)), respectively. Also \( c_{j,k} \) and \( d_{j,k} \) are the voltage and current discrete wavelet coefficients at the scaling level \( j_0 \) and sample \( k \), while \( c_{j,k}^:\prime \) and \( d_{j,k}^:\prime \) are the voltage and current discrete wavelet coefficients at any other level \( j \) than the scaling level \( j_0 \) and sample [3].

\[
c_{j,k} = \langle v(t), \phi_{j,k} \rangle, \quad d_{j,k} = \langle v(t), \psi_{j,k} \rangle \quad (19)
\]

\[
c_{j,k}^:\prime = \langle i(t), \phi_{j,k} \rangle, \quad d_{j,k}^:\prime = \langle i(t), \psi_{j,k} \rangle \quad (20)
\]

where \( \phi_{j,k}, \psi_{j,k} \) are the scaling function and wavelet function, respectively, while the symbol \( \langle \rangle \) represents the inner product.

### 4 Measurement of the voltage and current effective values

For the comparative analysis of voltage and current effective values, it was used the data acquisition system shown in Fig.5, which captures the current and voltage waveforms into \( 0 \div 200 \text{ kHz} \) frequency band, \( 10 \div 500 \text{ V} \) voltages range, \( 0 \div 70 \text{ A} \) currents range and contains:
transducers LEM LA-55P, with main characteristics shown in Table 2, which provides the measured signals.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Voltage Transducer LV 25-P</th>
<th>Current Transducer LA 55-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{PN}$</td>
<td>Primary nominal current rms</td>
<td>10 mA</td>
</tr>
<tr>
<td>$I_{PM}$</td>
<td>Primary current, measurement range</td>
<td>0±14 mA</td>
</tr>
<tr>
<td>$R_{M}$</td>
<td>Measuring resistance</td>
<td>30÷350 Ω</td>
</tr>
<tr>
<td>$I_{SN}$</td>
<td>Secondary nominal current rms</td>
<td>25 mA</td>
</tr>
<tr>
<td>$K_{N}$</td>
<td>Conversion ratio</td>
<td>2500 : 1000</td>
</tr>
<tr>
<td>BW</td>
<td>Frequency bandwidth</td>
<td>-</td>
</tr>
<tr>
<td>$V_{C}$</td>
<td>Supply voltage</td>
<td>±12…15 V</td>
</tr>
<tr>
<td>$\varepsilon_{L}$</td>
<td>Linearity error</td>
<td>&lt; 0,2%</td>
</tr>
<tr>
<td>$R_{P}$</td>
<td>Primary coil resistance</td>
<td>250 Ω</td>
</tr>
<tr>
<td>$R_{S}$</td>
<td>Secondary coil resistance</td>
<td>110 Ω</td>
</tr>
</tbody>
</table>

- a plate that amplifies and centers the waveforms obtained from transducer plate on 1,5 V voltage, between 0 ÷ 3 V.
- a switched voltage source which provides ±12 V and +5 V supply voltage.
- a PC computer with Windows XP operation system.
- a MSK28335 kit, which is oriented towards the F28335 features evaluation and offers the possibility to start developing DSC code using the assembler, linker and debugger included in the DMC28x Developer Lite platform.

The voltage and current waveforms captured with an oscilloscope are shown in Fig.6.

Fig.6. The voltage (yellow) and current (green) waveforms

The FFT Math mode of oscilloscope convert a time-domain signal into its frequency components. Fig.7 shows the voltage and current effective value obtained for a 50 Hz frequency and 20 ms period, using the FFT analysis.

Fig.7. The RMS value of the a) voltage and b) current

The MSK28335 DSC board (Fig.8) includes two connectors dedicated for interfacing with external power modules. Each connector provides all the necessary I/O signals to interface with a power module and to drive an electrical motor.

The board includes an RS-232 interface, used for communication with the PC and a CAN transceiver that permits to connect several MSK28335 DSC boards using a CAN-bus network.

The MSK28335 DSC board is equipped with 256-kword external SRAM memory that can be used as data memory, as program memory or both, and an 8-kword serial SPI-connected EEPROM.

For performance evaluation and debugging the MSK28335 DSC board includes a 2-channel 12-bit serial SPI-connected D/A converter with simultaneous update.

The JTAG connector offers compatibility with all programs using the XDS510 or XDS510PP emulator.
The voltage and current waveforms for each phase of a balanced three-phase system are obtained using Capture Units Application of the MSK28335 kit. The application helps to understand how different external signal transitions can be detected by the capture modules. External connections to the capture input pins are required to generate capture events. The waveforms represented in Fig. 9 were imported in Matlab for frequency domain analysis using the Fast Fourier Transform and time-frequency domain analysis using the Discrete Wavelet Transform.

The problem of spectral leakage is reduced as shown from the results indicating the effect of the suitable selection of the wavelet family and the mother wavelet on the differences.

## 5 Conclusion

The use of WT to describe voltage and current RMS offers a time-frequency representation for these components, thus saving time and frequency information. Furthermore, the computational effort is decreased when using DWT, because the calculations are made into frequency bands. Their number decreases with the decrease in number of decomposition level, which is very important, when the phase number increases.

The advantage of using the DWT for computing the voltage and current effective values is to characterize these power components in a time-frequency spectrum, thus preserving time and frequency information.

### References:


