Reduced Order Modeling for Supersonic Cavity Flows

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Abstract: In this study, Proper Orthogonal Decomposition (POD) is used to obtain reduced order models for different supersonic cavity flow configurations. The preliminary study is performed for an open cavity with a length to depth ratio (L/D) of 5.07 and a Mach number of 1.5. The POD method is applied to x-velocity data obtained as a result of CFD simulations of the flow. The reduced order modeling method is applied to different cavity configurations that have different L/D ratios; 1, 3, 7.6 and 10. With POD results, effects of L/D ratios to the flow are presented. As the flow becomes more complex, the necessary number of modes to represent the system also increases. For each case, results of the developed reduced order models which represent the main flow characteristics with less data show good agreement with CFD results.

Key-Words: Supersonic Cavity Flow, Compressible Flow, Proper Orthogonal Decomposition, Reduced order modeling

1 Introduction

In aeronautics applications, to increase the performance of the air vehicles and provide some advantages such as increase in the lift or drag reduction, cavity configurations are used to represent interior storages. In contrast to these provided advantages, due to the undesired interactions that occur in the cavity flow region as pressure fluctuations and resonant acoustics modes, the performance of air vehicles might decrease [1].

There is a need of exact understanding of the supersonic cavity flow mechanism to suppress its dangerous flow interactions. Many researchers investigated different cavity flow mechanisms. Zhang and Edwards [2] used k-w turbulence model to perform cavity flow simulations for Mach numbers of 1.5 and 2.5 in different cavity configurations with L/D ratios of 1,3 and 5. Their results showed that cavity flow is more uniform in cavity with L/D ratio of 1, above the L/D ratio of 3, the flow becomes more complicated and oscillation frequencies increase. Unalmis et al. [3] investigated the effect of L/D ratio on cavity flow experimentally and according to their results, higher L/D ratios mean higher acoustics oscillations. Ayli et al. [4] performed CFD simulations on supersonic cavity flow with different L/D ratios. The cavity flow mechanism with L/D ratio of 5.07 is explained in detail. A comparison of the effects of different L/D ratios on the flow structure is presented.

The complex cavity flow mechanism can be represented in a simple way by using mathematical methods. Proper Orthogonal Decomposition method is used in literature to get reduced order models for system. The purpose of POD is to obtain reduced order models for the systems by reducing the degree of the data sets [5]. POD is defined by Karhunen and Loéve. The combination of Karhunen-Loéve decomposition, principal component analysis and singular value decomposition is mainly used by researchers due to the compatibility of methods with each other [6, 7, 8]. This process is important for further control approaches.

Several researchers studied the application of reduced order modeling to cavity flow. In the study of Rowley et al.[9]., open cavity configurations with different L/D ratios were studied. They used POD to get reduced order models of the systems for flow control applications. Nagarajan et al. [10] studied POD based modeling of an open cavity with L/D ratio of 2 and Mach number of 0.6. Their purpose was to obtain a model for the control of cavity acoustics. In the study of Bortz et al. [11], simulations were performed for an open cavity with a Mach number of 0.85 and L/D ratio of 4.5 to control cavity acoustics. Colonius [12], Caraballo et al. [13] and Kasnakoglu [14] are other researchers who studied POD based reduced order model of cavity flow.

The objective of this study is to apply POD to cavity flows in order to obtain reduced order models of the systems and to compare the characteristics of cavity flow for different L/D ratios based on reduced order modeling. The study includes cavity configurations with L/D ratios of 1, 3, 5.07, 7.6 and 10. The number of modes necessary to represent the cavity flows is investigated for each cavity configuration and the results of each L/D ratio are compared to each other.

2 Methodology

The problem includes a cavity configuration which is given in Fig. 1.

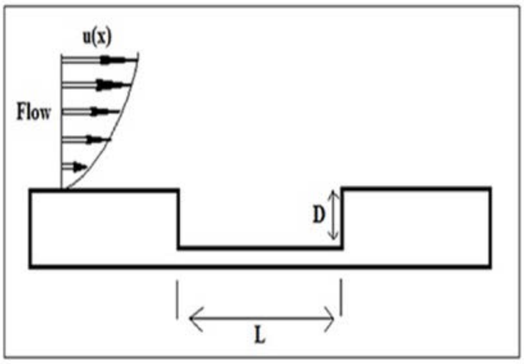

Figure1. Schematic view of the flow field

The length and depth values of configurations studied are given in Table 1. POD is applied to the x-velocity results obtained as a result of computational fluid dynamics simulations.

Table 1. L/D Ratios of Cavities Tested

	L (m)	**D(m)**
L/D=1	0.0238	0.0238
L/D=3	0.0714	0.0238
L/D=5.07	0.12065	0.0238
L/D=7.6	0.180975	0.0238
L/D=10	0.2413	0.0238

In the computational study, k-ω turbulence model is utilized. The computations are second order accurate in time and space. Two-dimensional compressible, time-dependent Navier-Stokes equations are solved.

After a CFD simulation is performed for M time steps, snapshots obtained include x-velocity data for M number of time steps. x-velocity data in each snapshot are collected in $U_i(x)$ matrix. As a result, the following equation is obtained;

$$U_i(\vec{x})=U_1(x),U_2(x),U_3(x),....U_M(x) \quad (1)$$

To eliminate requirements of scaling in further steps as defined in the studies of Newman [15] and Deane et al. [16], average of all data n is calculated and subtracted from each data matrix.

$$V_i(\vec{x})=U_i(\vec{x})-\frac{1}{M}\sum_{i=1}^{M}U_i(\vec{x}) \qquad i=1,2,...,M \quad (2)$$

To find basic functions which represent the dominant structures in system, the function given below is used [15];

$$\phi(\vec{x})=\sum_{i=1}^{M}\alpha_{ik}V_i(\vec{x}) \qquad k=1,2,...,S \text{ (mode number)} \quad (3)$$

By using The Method of Snapshots developed by Sirovich [17], an MxM dimensional covariance matrix is obtained as [18, 19];

$$C\phi_i=\lambda_i\phi_i \qquad i=1,2,3,...,M \quad (4.a)$$

$$(C)_{ij}=\frac{1}{M}\int_{\Omega}V_i(\vec{x})V_j(\vec{x})dx \qquad i,j=1,2...,M \quad (4.b)$$

This covariance matrix can be solved mathematically. By using singular value decomposition, eigenvalues and eigenvectors are obtained [7, 20];

$$C=R\sum P^T \quad (5)$$

R contains the eigen vectors. After obtaining the eigenvalues, they are sorted starting from the largest as $\lambda_1 > \lambda_2 > \lambda_3 > \ldots \lambda_M$. By examination of energy information, the number of modes to represent the system can be obtained [21].

For the reconstruction of the reduced order model, the following equation is used [22];

$$U=\bar{U}+\sum_{k=1}^{S}\alpha_k\phi_k \quad (6)$$

U is the original data set, $\bar{U}$ is the matrix for the mean values , α_k are time coefficients, $ø_k$ are basis functions and S is total number of modes.

3 Results

3.1 POD results for the cavity configuration with L/D ratio of 5.07

As a result of POD application to the supersonic cavity flow with L/D ratio of 5.07 energy distribution and energy contents of modes are obtained and presented in Table 2. (in Appendix). This energy distribution is obtained by using energy contents of each eigenvalue obtained as a result of POD.

The first mode includes 70.65% of total energy of the flow after which, the energy content values decrease rapidly. 99.18 % of the total energy of the

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system can be represented using 12 modes. There are many small structures which affect the main characteristics of the flow due to turbulent nature of the flow. It is clearly seen in Table 2 (in Appendix) that, there are many small turbulent structures which contain small energy values that affect the main flow. For future studies for flow control, the small structures are not important since the main idea is to control larger structures from which the smaller structures develop. In Fig. 2 (in Appendix) modes are given. As it seen, there are some serrated structures due to the small turbulent structures.

The dominant changes of characteristics of flow are presented with the time coefficient history of modes which is given in Fig. 3 (in Appendix). The motion of dominants structures in time can be seen clearly. The amplitudes and energy contents of modes are directly proportional to each other. As one mode has higher energy content, it also has a higher mode amplitude. For Mode 1 and Mode 2, the structures show a sinusoidal periodic oscillation. Mode 3 and Mode 4 include periodical but unpredictable oscillations due to the effects of small turbulent structures.

3.2 The Effects of L/D based on POD results

POD is applied to the CFD results of several cavity configurations with different L/D ratios. The necessary number of modes to represent the flow without loss of information for all cavity configurations is presented in Table 2. Systems can be represented with number of modes corresponding to 94-97 % of the total system energy.

With the increase in L/D ratio, the number of modes increases. Only in the cavity with L/D ratio of 10, POD results show divergence from the general trend due to the being in transition region. The cavity with L/D ratio of 1 is a deep cavity and the cavity flow interactions are in low level [4, 23], the representation of this cavity configuration using the least number of modes shows that the flow is more uniform than others. Between the range of L/D ratio of 3 and 10, the flows become more complicated with the increase in L/D ratio and the number of POD modes to define the systems increases with increasing L/D.

All modes include an energy value. The amount of these energy values is related with how the modes include characteristics of the system at hand. Energy content of each mode is given in Table 2. All cases include multiple modes except for the L/D ratio of 1. Including multiple modes means that the flow is affected by small structures more. This conclusion shows good agreement with the CFD results of the same cavity configurations by Ayli et al [4] and Ayli [23].

4 Conclusion

The CFD simulations of several supersonic cavity configurations are performed and Proper Orthogonal Decomposition method is applied to results of the CFD simulations. The physical interpretation of the POD results show good agreement with CFD results of the study of Ayli et al. [4,23]. With the increase in L/D ratio, regularity of periodic structures decreases and the flow becomes more complex [4, 23]. The number of POD modes to represent these systems also increases as L/D ratio increases. POD results of the cavity with L/D ratio of 10 are not consistent with the general trend. This L/D ratio is in the range of transition region between open cavity and closed cavity; therefore the behavior of the flow is not clear as others [4, 23].

For control phenomena, the control of the larger structures is the main objective. Therefore, in this study, it is shown that supersonic cavity configurations can be represented with POD modes that include %95-97 of the total energy of the flow. For flow control purposes, these large scale structures represented by first several modes can be targeted and small scale fluctuations do not exist anymore since they develop from the larger ones.

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Appendix

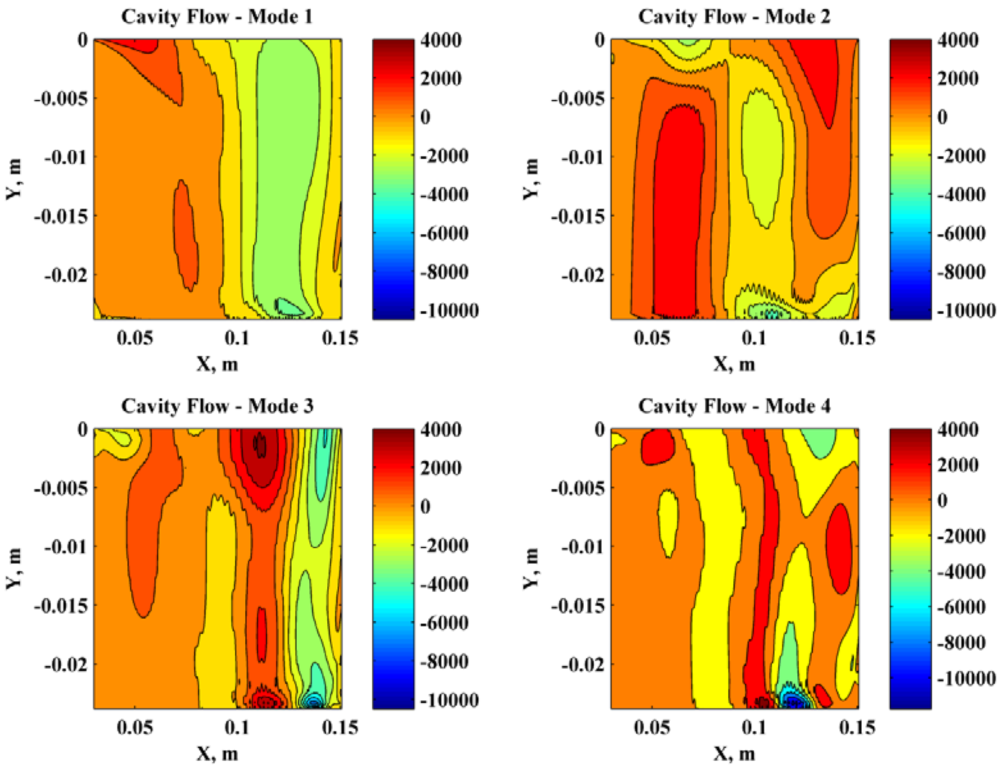

Figure 2. POD Modes of Cavity Flow with L/D Ratio of 5.07

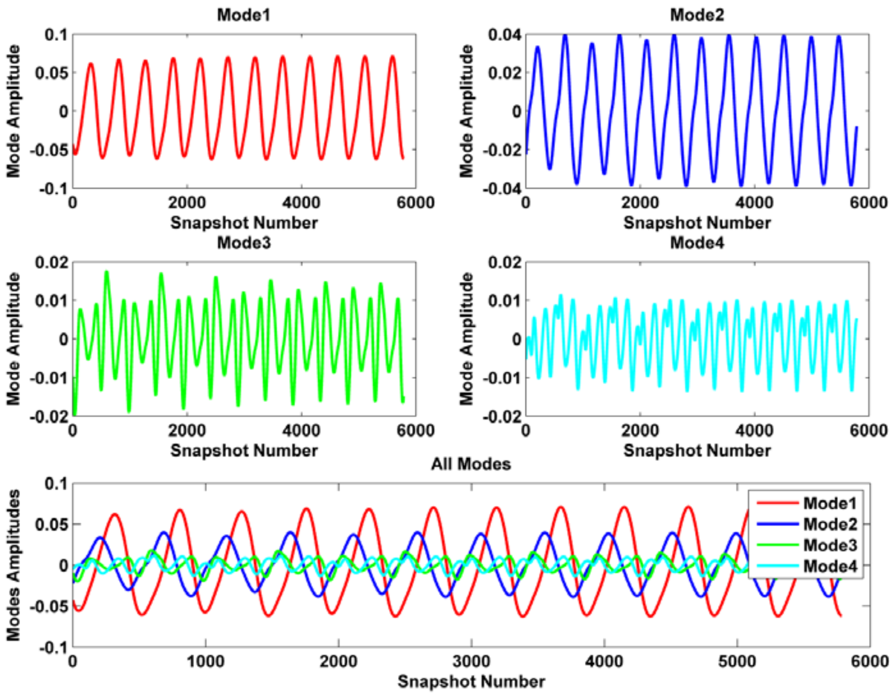

Figure 3. Time Coefficient History of Cavity Flow with L/D Ratio of 5.07

Table 2. Energy Content of each Mode for All Cavity Configurations

Energy Content of each Mode for All Cavity Configurations, %										
Mode Number	**L/D=1**		**L/D=3**		**L/D=5.07**		**L/D=7.6**		**L/D=10**	
1	94.33	94.33	67.02	67.02	70.65	70.65	60.87	60.87	46.38	46.38
2	-	2.88	20.60	20.60	21.19	21.19	18.15	18.15	31.06	31.06
3	-	1.64	4.86	4.86	2.63	2.63	7.12	7.12	12.15	12.15
4	-	0.56	3.10	3.10	1.54	1.54	5.17	5.17	4.2	4.2
5	-	-	-	0.090	-	0.96	2.16	2.16	3.02	3.02
6	-	-	-	0.078	-	0.61	1.86	1.86	-	1.67
7	-	-	-	0.067	-	0.58	-	1.14	-	0.69
8	-	-	-	0.048	-	0.32	-	0.71	-	-
9	-	-	-	0.030	-	0.23	-	0.55	-	-
10	-	-	-	0.0026	-	0.18	-	0.40	-	-
11	-	-	-	0.0020	-	0.15	-	0.36	-	-
12	-	-	-	-	-	0.13	-	0.30	-	-
13	-	-	-	-	-	-	-	0.23	-	-
14	-	-	-	-	-	-	-	0.17	-	-
Total Energy Content,%	94.33	99.41	95.58	99.17	96.01	99.18	95.33	99.19	96.82	99.18

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