Reliability Analysis of Water Distribution Systems

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Abstract: - The most important consideration in the planning and operation of a water distribution system is to satisfy consumer demands. The water supply-demand planning can be done more efficient if one can check the reliability of a water distribution system in advance. If we define the system reliability as “the probability that the flow can reach all the demand-points in the network”, among the reliability methods identified, the minimum cut-set method, defined in this paper, appears to be efficient, and is easily programmed on a computer. The performance of the method is demonstrated by applying it to one example of a water distribution network.

Key-Words: - Reliability analysis, water distribution system, water supply-demand planning, the minimum cut-set method.

1 Introduction

In general, reliability is defined as the probability that a system performs its mission within specified limits for a given period of time in a specified environment. The reliability of a water distribution system is defined by Kaufmann et al. (1977), [6], as the probability that the system will perform its specified tasks under specified conditions and during a specified time. Goulter (1986), [5], and Cullinane et al. (1992), [3], defined the reliability of water distribution system as the ability of the system to meet the demands that are placed on it. The demands are specified in terms of the flow to be supplied and the range of pressures at which these flow rates must be provided.

The most important consideration in the planning and operation of a water distribution system is to satisfy consumer demands. Thus, it is imperative to provide all users with good quality water in adequate amounts at a reasonable pressure at all times, in order to ensure a reliable water distribution system.

The range of combinations of ways in which a failure can occur in a water distribution system constitutes one, and perhaps the major, source of many theoretical and practical difficulties which have been encountered in establishing suitable (comprehensive and computationally tractable) measures of reliability which can be used in the practical design and operation of water distribution systems. If we define the system reliability as “the probability that the flow can reach all the demand-points in the network”, among the reliability methods identified, the minimum cut-set method, defined in this paper, appears to be efficient, and is easily programmed on a computer.

2 Reliability of a system

Reliability of a water distribution system can be defined using the approach of Goulter (1986), [5]: the ability of a water distribution system to meet the demands that are placed on it where such demands are specified in terms of:

- The flows to be supplied (total volume and flow rate);
- The range of pressures at which those flows must be provided.

Hydraulic availability is defined as the ability of the water distribution system to provide service with an acceptable level of interruption in spite of abnormal conditions (Cullinane et al., 1992), [3]. Availability is evaluated in terms of developing the required minimum pressure. Pressures between 137.9 kN/m² and 551.6 kN/m² (Shinstine et al., 2002, [9]) are considered to be desirable pressures under normal daily demands. Goulter and Coals (1986), [5], proposed the use of a discrete
relationship between availability and pressure as shown in Figure 1. The availability during a time period \( t \) can be expressed by the following mathematical relationship:

\[
HA_j = \begin{cases} 
1, & \text{for } P_j \geq PR \\
0, & \text{for } P_j < PR 
\end{cases}
\]  

(1)

Where \( HA_j \) = hydraulic availability of node \( j \); 
\( P_j \) = the pressure at node \( j \); 
\( PR \) = the required minimum pressure.

Cullinane et al. (1992), [3], formulated an approach that describes the availability index as a continuous “fuzzy” function. Using this concept, a significant index value may be assigned to pressure values slightly less than the arbitrary assigned required minimum pressure value, \( PR \). Accordingly, a curve similar to Figure 2 can be developed which resembles the curve of a normal distribution. Thus, the hydraulic availability function can be described mathematically as:

\[
HA_j = P(PR \leq P_j) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{P_j} e^{-\frac{(H-H)}{2\sigma_H^2}} dH
\]

(2)

Where: \( P_j \) = value of nodal pressure; 
\( H \) = mean nodal pressure; 
\( \sigma_H \) = standard deviation of pressure.

2.1 Pipe Failure Probabilities

The probability of failure of pipe \( P_i \) is determined using the Poisson probability distribution:

\[
P_i = 1 - e^{-\beta_1}
\]

(3)

And

\[
\beta_1 = r_i L_i
\]

(4)

Where: \( \beta_1 \) = the expected number of failures per year for pipe \( i \); 
\( r_i \) = the expected number of failures per year per unit length of pipe \( i \); 
\( L_i \) = the length of pipe \( i \).

In order to apply the developed methodology in calculating the complete pipe failure probability, we consider a hypothetical water distribution system as shown in Figure 3.

Mathematically, the probability of occurrence of two independent events \( A \) and \( B \) is given by

\[
P(A \cap B) = P(A) \cdot P(B)
\]

(5)

Assume that a pipe in water distribution system is unable to satisfy the nodal demand. Then, failure is assumed to occur when the flow in the pipe exceeds the capacity of the pipe. According to Hazen-Williams equation, the flow rate in the pipe \( Q_p \), in SI system, is given by (Viessman et al., 1998, [12]):

\[
Q_p = 0.849CHW^0.63AR^{0.849}S^{0.54}
\]

(6)

Where \( CHW \) = the Hazen-Williams coefficient, 
\( A \) = the pipe cross-sectional area (m\(^2\)), 
\( R \) = the hydraulic radius = area/wetted perimeter (m), 
\( S \) = the slope of hydraulic grade line.

If the pipe is considered flowing full, then the cross-sectional area is \( A = \frac{\pi d^2}{4} \) and the wetted perimeter is \( P = \pi d \). Substituting the values of \( A \) and \( R \) in Equation (6), one obtains:

\[
Q_p = 0.27842CHW^{0.63}d^{2.63}S^{0.54}
\]

(7)

The flow rate directed into the pipe will be equal to the pipe distribution factor multiplied by the demand at the junction. Mathematically, it is given by

\[
Q_D = D_p QJ_i
\]

(8)

Where

\( D_p \) = distribution factor of the pipe, 
\( QJ_i \) = water demand at junction \( i \).

Therefore, the performance function, \( Z \), of the pipe can be defined as

\[
Z = Q_p - Q_D
\]

(9)

Consider pipe \( P-1 \) and junction \( j-2 \), as shown in Figure 3. In order to calculate the failure probability \( P(A) \) of the pipe to fulfill the demand, the input
parameters of Equations (7) and (8) are considered as random variables. The probability distributions are assumed for the input variables and their means and coefficient of variations are calculated as shown in Table 1. Assuming a normal distribution for Z, the probability of failure \( P(A) \) is calculated using the following equation:

\[
P(A) = P(Z < 0) = \int_{-\infty}^{0} P_z(Z) dZ =
\]

\[
P\left(z < \frac{X - \mu}{\sigma}\right) = P\left(z < \frac{0 - 0.0182}{0.0502}\right) = \tag{10}
\]

0.3582 = 35.82%

Similarly, the probabilities of other pipes can be calculated. Assuming a normal distribution for Z, the pipe replacement probability \( P(B) \) is calculated using the equation

\[
Z = \frac{\ln(1 + R)F_n}{C_{n+1}} - N(t_0)e^{A(t-t_0)} \tag{11}
\]

Where \( R = \) the discount rate, \( F_n = \) the repair cost at time \( t_n \), \( C_{n+1} = \) the repair cost of the \( (n+1)^{th} \) break. \( t = \) the time in years, \( t_0 = \) the base year for the analysis, \( A = \) the growth rate coefficient (1/year) \( N(t) = \) the number of breaks

\[
P(B) = P(Z < 0) = \int_{-\infty}^{0} P_z(Z) dZ =
\]

\[
P\left(z < \frac{X - \mu}{\sigma}\right) = P\left(z < \frac{0 - 2.3639}{2.533}\right) = \tag{12}
\]

0.17537 = 17.53%

Therefore, the complete failure probability, \( P_{\text{com}} \), is given by

\[
P_{\text{com}} = P(A) \cdot P(B) = 0.3582 \cdot 0.17537 = 0.0628 \tag{13}
\]

Table 1 Statistics of input variables (Demand failure probability), Pipe P-1

<table>
<thead>
<tr>
<th>Input variable</th>
<th>Mean</th>
<th>CV</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{t_{fW}} )</td>
<td>100</td>
<td>0.1</td>
<td>Normal</td>
</tr>
<tr>
<td>( D_p )</td>
<td>2.0348</td>
<td>0.0692</td>
<td>Normal</td>
</tr>
<tr>
<td>( QJ )</td>
<td>0.16712475</td>
<td>0.0133</td>
<td>Normal</td>
</tr>
</tbody>
</table>

2.2 Nodal and System Reliability

The minimum cut-set approach is adopted to calculate the nodal and system reliability, \( R_{\text{node}} \) and \( R_s \). According to Su et al. (1987), [10], and Loganathan et al. (2002), [7], the minimum cut set can be defined as “a set of system components (e.g., pipes) which, when failed, causes the failure of the system”. However, when any component of the set has not failed, it does not cause a system failure, [1-2].

Assuming that a pipe break can be isolated from the rest of the system, the minimum cut sets are determined by closing a pipe or combination of pipes in the water distribution system and using a hydraulic simulation model to determine the values of pressure head at each demand node of the system. In this study, EPANET was used (Rossman, 2000, [8]). By comparing these pressure heads with the minimum pressure head requirements, the reliability model can determine whether or not this pipe or combination of pipes is a minimum cut set of the system or an individual demand node. A minimum cut set for a node is one that causes reduced hydraulic availability at that node, while a minimum cut set for the system is a cut set that reduces the hydraulic availability for any node in the system. To calculate the number of combinations for pipe closure for the cutest determination, it is observed that the failure of two or three pipes is purely a “random” phenomenon. Therefore, in order to determine the pipe combinations for the cutest determination, subsets of pipe combinations should be determined by applying a random approach. For instance, if there are \( K \) numbers of pipes in the water distribution system, then out of those \( K \) pipes, \( T \) subsets should be randomly generated and each subset could have only one pipe or a combination of two or three pipes. A flow chart of the procedure is shown in Figure 4.

According to Shinistne et al. (2002), [9], for \( n \) components (pipes) in the \( i^{th} \) minimum cut set of a water distribution system, the failure probability of the \( i^{th} \) minimum cut set (\( MC_i \)) is

\[
P(MC_i) = \prod_{j=1}^{n} P_i \cdot P_2 \cdot \ldots \cdot P_n \tag{14}
\]

Using the step function for hydraulic availability and assuming that the occurrence of the failure of the components within a minimum cut set is statistically independent, for a water distribution network with four minimum cut sets (\( MC_i \)) with the system reliability, \( R_s \), the failure probability of the system \( P_s \) is then defined ([4], [11]) as

\[
P_s = P(MC_1) + P(MC_2) + P(MC_3) + P(MC_4) = \sum_{i=1}^{4} P(MC_i) \tag{15}
\]

In general form

\[
P_s = \sum_{i=1}^{M} P(MC_i) \tag{16}
\]

The system reliability, \( R_s \), is expressed as
Where $M$ = the number of minimum cut sets in the system.
In order to calculate the nodal and system reliability of the water distribution system, nodal demands and Chezy’s roughness coefficients for pipes are considered as random values.

\[ R_s = 1 - P_s = 1 - \sum_{i=1}^{M} P(MC_i) \]  \hspace{1cm} (17)

3 Discussion and result analysis

The pipe failure combinations required for the cutest calculations are determined by assuming randomness in the simultaneous failure of two or three pipes. Then, steady state hydraulic analysis is performed using the hydraulic simulation software EPANET, and nodal pressures are calculated for different combinations of pipe closures. The nodal and system reliabilities are calculated using the minimum cut-set method. The flow chart of the methodology is shown in Figure 4 and the calculated nodal and system reliabilities are summarized in Table 2.

From the results of this study obtained by applying the developed methodology, the system reliability turned out to be 95.6% and the nodal reliability of all the nodes come out in the range from 97.0% to 100%. This means that the probability that modeled water distribution system will have a required minimum pressure of 35 psi at all the junctions is 95.6 %, and the probability that each junction will have a required minimum pressure of 35 psi varies from 97.0% to 100% depending upon the individual junction as summarized in Table 2.

### Table 2 Nodal and system reliability

<table>
<thead>
<tr>
<th>Node ID</th>
<th>Nodal Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.99971</td>
</tr>
<tr>
<td>3</td>
<td>0.99923</td>
</tr>
<tr>
<td>4</td>
<td>0.99817</td>
</tr>
<tr>
<td>5</td>
<td>0.97075</td>
</tr>
<tr>
<td>6</td>
<td>0.98278</td>
</tr>
<tr>
<td>7</td>
<td>0.99972</td>
</tr>
<tr>
<td>8</td>
<td>0.99991</td>
</tr>
<tr>
<td>9</td>
<td>0.99971</td>
</tr>
<tr>
<td>10</td>
<td>0.99971</td>
</tr>
</tbody>
</table>

| System Reliability | 0.956 |

Fig. 3 Test network (sector network Pitesti)
4 Conclusion
Since the adopted minimum cut set approach for calculating the nodal and system reliability requires the mean and standard deviations of the nodal pressures for hydraulic availability calculations, the mean and standard deviation of the nodal pressures significantly affect the nodal and system reliability. Therefore, higher values of mean and standard deviation of nodal pressures will result in reduced nodal and system reliability. Also, the developed method is more feasible for large water distribution networks. In large networks, it is also possible to consider many random combinations of pipe closures. Therefore, it is recommended to use the developed methodology in large water distribution networks.

References:


