Analysis and Synthesis of Band-Pass and Notch Memristor Filters

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Abstract: The main purpose of this paper is to present the possibility of application of the Williams's memristor in passive band-pass and notch filter circuits. At frequencies higher than 20 *Hz* the Williams's memristor have behaviour similar to this of a linear resistor and the distortions of the signals passed through the memristor circuit are negligible. The resistance of the memristor element can be tuned by external pulse voltage source. Changing the passive RC filters' characteristics and cut-off frequencies might be achieved with substituting of the resistors with memristors. In this paper two circuits of band-pass and notch filters with memristors are synthesized and investigated in MATLAB environment. The basic conclusion is that the characteristics of the memristors used in the filters could be linearized in the working frequency range. The memristors internal states and the filters frequency characteristics respectively can be adjusted by external voltage or current pulses with determined duration and amplitude.

Key-Words: memristor, computer simulation, band-pass filter, notch filter, characteristics, cut-off frequency

1 Introduction

The memristor as a fourth nonlinear circuit element was predicted in 1971 by Professor Leon Chua [1]. The first physical prototype of the memristor was invented in 2008 by Stanley Williams from the Hewlett-Packard scientific laboratories [2, 3]. This memristor consists of two sub-layers of titanium dioxide, sandwiched between two platinum electrodes. The basic unique property of this nonlinear element is the memorizing the full amount of charge which has passed through it [4, 5]. Many research results and simulations from the memristor investigation have been made in the last few years [6, 7, 8]. The main properties and the principle of operation of Williams's memristor have been described. Some physical relationships between the basic electrical quantities of the memristor are presented. It is established that for frequencies higher than 20 *Hz* the Williams's memristor is similar to a linear resistor [2, 4]. By applying to the memristor of voltage pulses with determined duration, shape and magnitude, the boundary between the doped and undoped regions moves so that the equivalent resistance of the element changes. This phenomenon could be used in some tuneable circuits like electronic oscillators, amplifiers and filters.

In Section 2 a band- pass filter is synthesized and investigated using a cascade connection of a low-pass and a high-pass memristor filters. A notch memristor filter is synthesized and analyzed in Section 3 as a parallel connection of a low-pass and a high-pass filter circuits. Final conclusions and remarks about filters investigated and their characteristics are presented in Section 4.

2 Synthesis and analysis of a passive band-pass memristor filter

The circuit of a memristor passive band-pass filter is presented in Fig. 1. It is obtained by cascade connection of the low-pass filter and the high-pass filter groups described in [9].

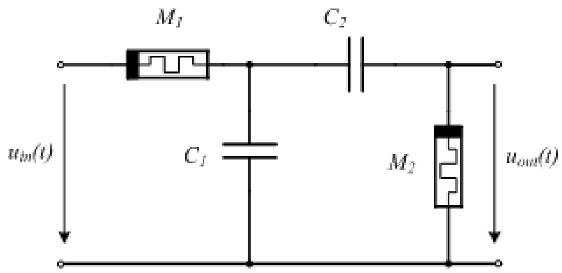

Fig. 1. Band-pass passive memristor filter

The low-pass filter group contains the elements M_1 and C_1 and the high-pass filter circuit is constructed by the elements M_2 and C_2.

The cut-off frequency of each filter group is presented with Eq. (1) [10]:

$$f_{cut-off\ N} = \frac{1}{2\pi R_N C_N} \quad (1)$$

To obtain band-pass filter the cut-off frequency of the low-pass filter must be higher than the cut-off frequency of the high-pass filter group [10].

The resistance of the memristor element R_{eq} is given with Eq. (2) [2]:

$$R_{eq}(t) = M(t) = R_{ON}\frac{w(t)}{D} + R_{OFF}\left(1 - \frac{w(t)}{D}\right) \quad (2)$$

where R_{ON} = 100 Ω is the resistance of the memristor in closed state, R_{OFF} = 16 $k\Omega$ is the resistance of the element in fully open state. The factor *w(t)/D* is the normalized width of the element in the initial moment and it is denoted with *a* and with *x(t)* [2]:

$$a = x(t) = \frac{w(t)}{D} = \frac{q(t_0)}{Q_D} \quad (3)$$

The normalized width is equal also to the normalized initial charge $q(t_0)$ with respect to the maximal amount of charge Q_D that the memristor might memorize. In the present investigations we examine the circuit at three different values of the inner memristor state: a_1 = 0,1; a_2 = 0,2; a_3 = 0,5. The capacitance of the capacitor C_1 has a value of 750 *pF* and C_2 has a value of 1 μF. The transfer function of the circuit analyzed $T_{U1}(s)$ is obtained as a multiplication of the transfer functions of a low-pass and of a high-pass filter [9, 10, 11]:

$$T_{U1}(s) = \frac{U_{out}(s)}{U_{in}(s)} = \frac{1}{R_1C_1s+1}\cdot\frac{R_2C_2s}{R_2C_2s+1} \quad (4)$$

The pole-zero map of the band-pass filter investigated is presented in Fig. 2. It is obtained in MATLAB environment by using Eq. (4).

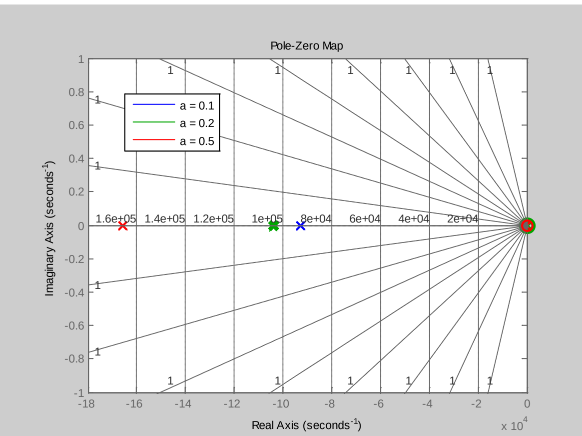

Fig. 2. Pole-zero map of the band-pass memristor filter

There is one zero in the coordinate outset of the diagram and three pairs of poles are available for the three values of the initial normalized charge respectively.

The amplitude-frequency characteristics of the band-pass filter are presented in Fig. 3 in logarithmic scale. It is obvious that with increasing the initial normalized charge *x(t)* the pass band of the filter moves to the higher frequencies.

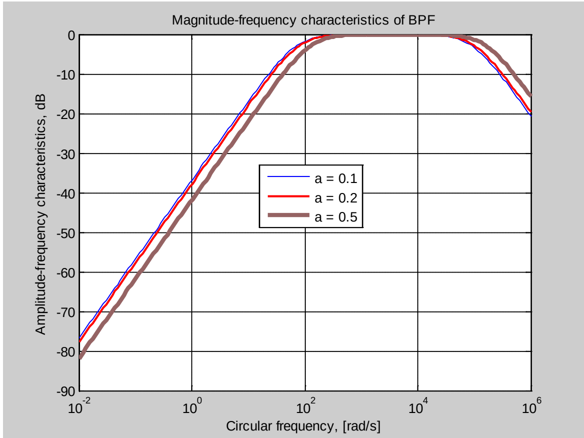

Fig. 3. Magnitude-frequency characteristics of the band-pass passive memristor filter at different initial charge

The phase-frequency characteristics of the circuit are given in Fig. 4. It is clear that with increasing the initial normalized charge *x(t)* the phase of the output voltage of the filter increases too.

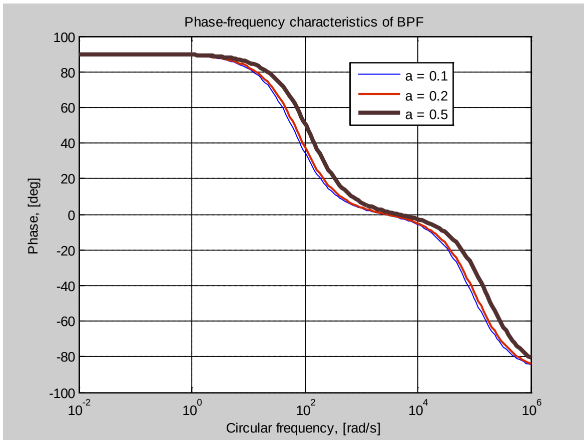

Fig. 4. Phase-frequency characteristics of the band-pass passive memristor filter at different initial charge

3 Synthesis and analysis of a notch memristor filter

Theoretically a notch filter may be constructed by parallel connection of the passive low-pass and high-pass filters described in [9]. But it is specified by simulation in PSPICE environment that the

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magnitude-frequency characteristic of the circuit obtained is similar to the characteristic of a low-pass filter. This phenomenon is due to the action of the capacitor of the low-pass filter which is connected in parallel to the output of the circuit.
To obtain a notch filter it is necessary to use a memristor connected in series with the capacitor of the low-pass filter. The circuit of the notch-filter obtained is given in Fig. 5.

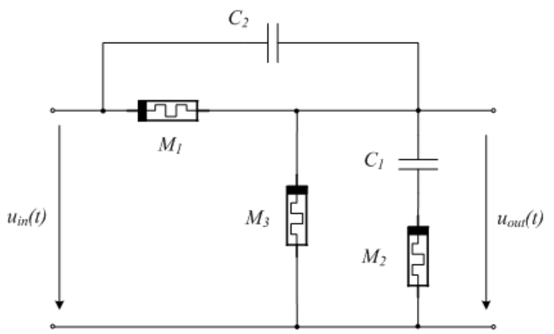

Fig. 5. Notch passive memristor filter

The transfer function of the circuit is obtained by using of the Ohm's law and the Kirchoff's laws and is presented with Eq. (5) [10, 11]:

$$T_{U2}(s) = \frac{U_{out}(s)}{U_{in}(s)} = \frac{N(s)}{D(s)} \qquad (5)$$

where the numerator *N(s)* is:

$$N(s) = (R_2 R_3 C_1 s + R_3)(-R_1 C_2 s + 1) \qquad (6)$$

and the denominator is:

$$D(s) = -R_1 R_2 R_3 C_1 C_2 s^2 + (R_1 R_2 C_1 + \\ + R_1 R_3 C_1 + R_2 R_3 C_1 - R_1 R_3 C_2)s + (R_1 + R_3) \qquad (7)$$

The capacitors used in the circuit have the following values:

$$C_1 = 1\,[\mu F],\ C_2 = 68\,[nF]$$

For initial analysis we choose the states of the memristors with determined initial charges so that their resistances have the following values:

$$R_1(M_1) = 330\ [\Omega],\ R_2(M_2) = 270\ [\Omega]$$
$$R_3(M_3) = 4.7\ [k\Omega]$$

After finishing the computer simulation we use a tolerance of the memristor resistances + 10% for a new analysis. The final simulation is made with using of tolerance of memristor resistances – 10%.

The pole-zero map of the notch filter investigated is presented in Fig. 6. It is obvious that for positive tolerance of the memristor resistances the poles and the zeros of the transfer function move to the left on the diagram. For negative value of the resistance tolerance of the memristors the poles and the zeros of the transfer function are moved to the right.

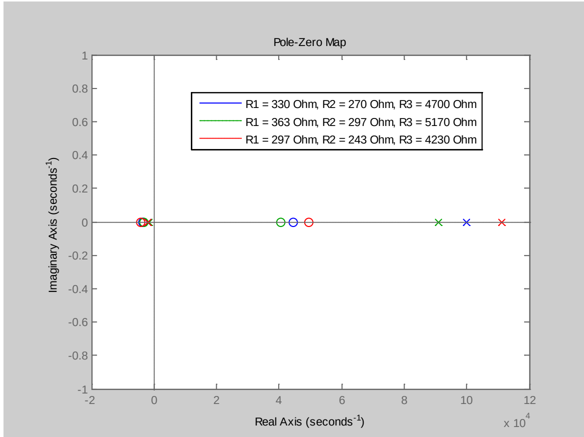

Fig. 6. Pole-zero map of the notch memristor filter for different memristor resistances

The amplitude-frequency characteristics of the circuit analyzed are given in Fig. 7.

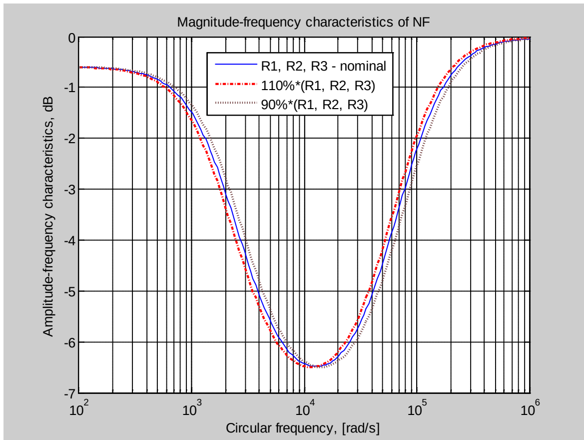

Fig. 7. Magnitude-frequency characteristics of the notch passive memristor filter for different initial resistances

It is obvious that when the resistances of the memristors have a positive tolerance then the magnitude-frequency characteristic moves to the lower frequencies. For the negative tolerance of the resistances the amplitude-frequency characteristic are moved to the higher frequencies.
The phase-frequency characteristics of the notch filter are presented in Fig. 8. It is obvious that these characteristics move to the higher frequencies when the tolerance of the memristor resistances is negative. When the memristors have positive tolerances of their resistances then the phase-

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frequency characteristics are placed in the lower frequency range.

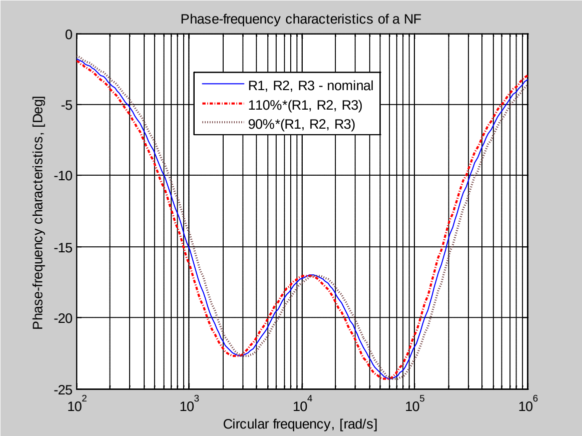

Fig. 8. Phase-frequency characteristics of the notch memristor filter for different memristor resistances

The resistances of the memristors used in the notch filter could be changed independently by autonomous external voltage sources. Then we can adjust the width of the notch band.

4 Conclusion

From the results presented above it is clear that the changes of the initial normalized charges of the memristors cause changing the amplitude-frequency characteristics and the phase-frequency characteristics of the filters investigated. The changes of the memristors parameters in the circuit of the band-pass filter cause the pass-band of the filter to move up or down. In this circuit it is possible to change and tune the initial charges of the memristors independently so we can adjust the boundaries of the pass-band of the band-pass filter. Changes of the resistances of the memristors in the notch filter cause the notch band to move up or down. With adjusting the memristors resistances the position of the poles and zeros in the pole-zero maps changes which causes changes the characteristics of the circuits. The boundaries of the notch band of the notch filter may be changed independently by separately adjusting the memristors.

The resistances of the memristors could be changed easily by applying to the memristors voltage or current pulses from external sources. The circuits of the passive memristor filters investigated could be used in audio electronic adjustable devices (like low-frequency amplifiers), MEMS [12, 13], etc..

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