Normalized power spectrum analysis based on Linear Prediction Code (LPC) using time integral procedures

Kazuo MURAKAWA†, Hidenori ITO†, Masao MASUGI††, Hitoshi KIJIMA†††

† Technical Assistance and support center

NTT East

1-2-5 Kamata-honcho, Ohta-ku, Tokyo, 140-0053

JAPAN

murakawa@east.ntt.co.jp

†† Electric and Electronic Department

Ritsumeikan University,

1-1-1 Ichinomitihigashi, Kusatsu-shi, Siga, 525- 8577

JAPAN

masugi@fc.ritsumei.ac.jp

††† Electrical Department

Polytechnic University

2-32-1 Ogawanishi, Kodaira, Tokyo, 187-0035

JAPAN

hkijima@uitec.ac.jp

Abstract: - Recently malfunctions of telecommunication installations caused by switching noises of electric equipment or devices have been increasing. The switching noises usually have low frequency components less than 50Hz and also more than 9kHz or 150 kHz. It is useful to detect power spectrum of noises in order to solve EMC problems. The LPC (Linear Prediction Code) method is known as a powerful frequency estimation method. This paper proposes a new normalized power spectrum analysis technique based on LPC. Introducing time integration on a time series of the noise waves to the LPC shows that noise power spectrum (frequencies and levels) can be estimated more precisely than using the conventional LPC method. Estimated deviations of frequency and NPS (normalized power spectrum) between given frequencies and extracted frequencies of quasi signals are less than 4%, and the proposed technique can also extract low frequencies with short durations from time series of noises.

Key-Words: - LPC (Linear prediction code), frequency analysis, time integral, simulations and experiments

1 Introduction

Reducing CO_2 emissions by minimizing equipment power consumption is one of the more important issues in the world. There are many ways to reduce CO_2 emissions, such as increasing power circuit efficiency, reducing power consumption itself, and using sleep modes. It is common knowledge that solar, wind and geothermal power systems have low CO_2 emissions. One way to reduce equipment CO_2 emissions is to increase the efficiency of ac/dc convertors. For this purpose switching power circuits are used in both telecommunication installations and power systems. Initially, switching frequencies were in the range of few kHz, but now switching frequencies in the range of 9 to 150 kHz are used in order to increase efficiency and downsize power circuits, particularly frequency transformers. A side effect is that these power circuits become a source of electromagnetic noise with a wide frequency range. This power spectrum noise causes malfunctions of equipment connected to ac power mains [1].

Fig.1 shows a malfunction caused by conducted noises. In Fig.1, ONU(Optical Network Unit) is a optical terminator, PC is a personal computer. PC is connected to ONU through E-ther cable. In Figure, a heater generates a high level common mode noise and conducting common mode noise on ac main transmit and cause a malfunction on PC.

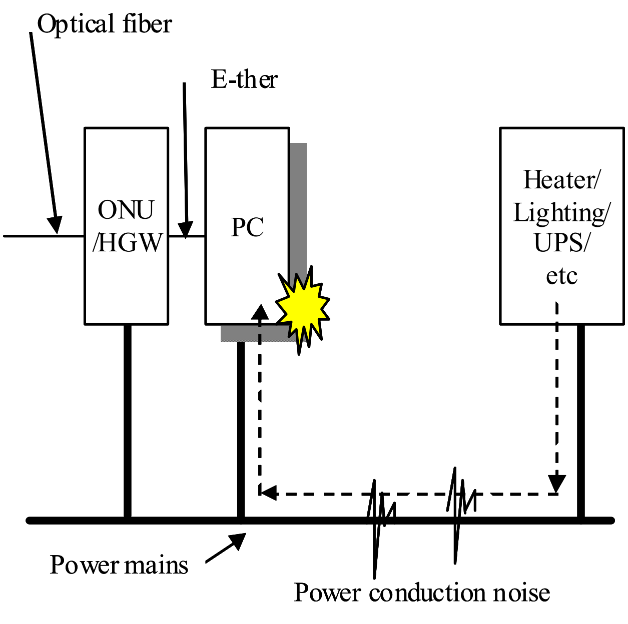

Fig.1 A malfunction caused by conducted noise

Fig.2 shows a measured noise wave shapes induced on both an ac mains and a telecommunication line. An inverter circuit of the heater generates a switching noise as shown in Fig.1. Sharp pulse noises are generated on ac mains and large common mode noises are induced on a telecommunication line as shown in Fig. 2. Fig. 3 shows power spectrums of them by using FFT analysis (Fast Fourier Transform). According to results of Fig.3, the noises have wide frequency ranges. In this case, most of power spectrum is at frequency below 150 kHz as shown in Fig.3.

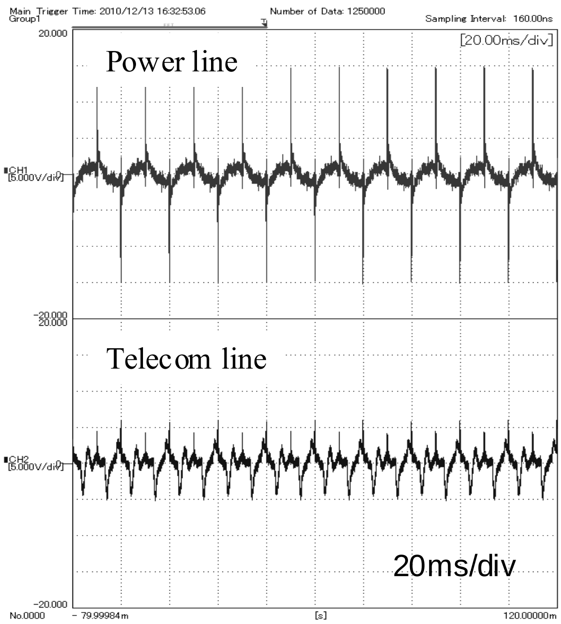

Fig.2 Measured noise wave shapes on a power mains and a telecommunication line

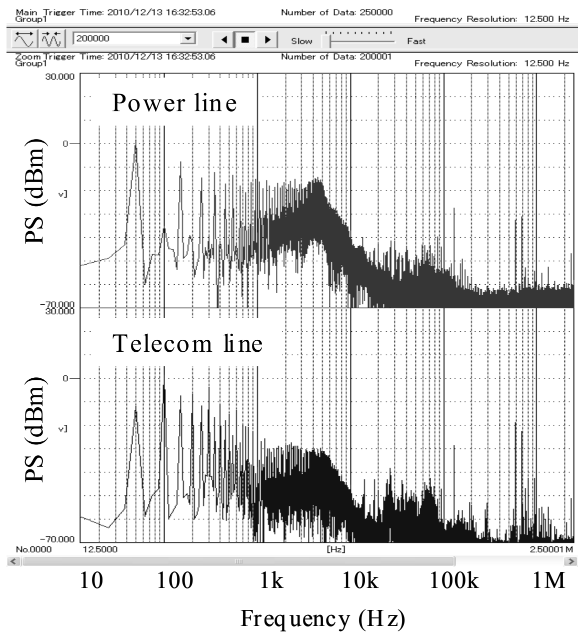

Fig.3 Power spectrums on a power mains and a

telecommunication line
Fig.4 shows obtained noise sources of malfunctions after investigations. There are many noise sources such as heaters, lighting, elevators, electric vehicles and electric fences which can cause malfunction of equipment as shown in Fig.4. Fig.5 shows investigated noise frequencies which cause equipment malfunctions. According to Fig.3, a percentage of a frequency range dc-150kHz is about 60%, 150 kHz-1MHz is about 24%. This means low frequency noise problems still dominant.

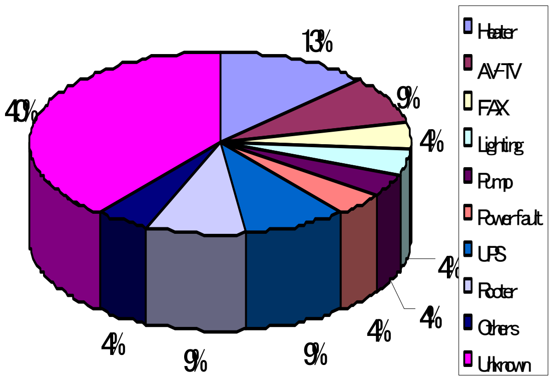

Fig.4 Noise sources that cause equipment malfunction

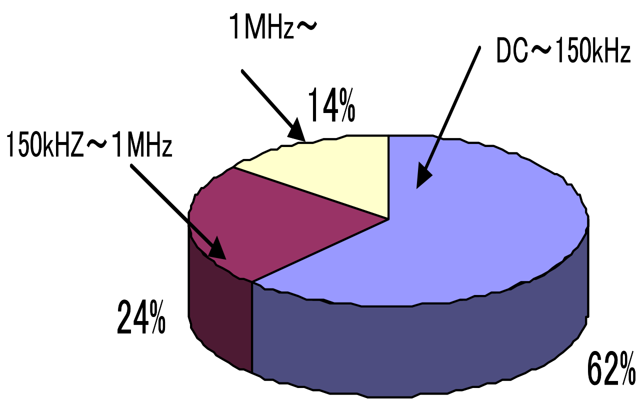

Fig.5 Noise frequencies that cause equipment malfunction

In order to solve these noise problems, the best way is to identify a noise source and to remove it. Since this measure is not always available, it is useful to install appropriate filters on a telecom line or a power line. For selection of appropriate EMC filers, it is required to analyze frequencies of a noise wave shape. One of easy and powerful methods for extraction noise frequencies is a Liner Prediction Code (LPC) method [1]-[3]. The LPC methods have been developed for digital filter designs.

The most importance issues of the digital filter design are to obtain appropriate LPC for the digital filters. Many studied using the LPC methods have been done for noise frequency analysis [4]-[6]. We have also studied the LPC method in order to extract noise frequencies, also studied to improve a frequency extraction accuracy of noise using a time integral procedures [7]-[8]. Calculated spectrums by using LPC methods are assumed to be not real power spectrums but a probability of eigen frequencies or poles of digital filters or signals.

However, LPC methods can be applicable to obtain power spectra of noise according to numerical simulations. In this paper we introduce the conventional LPC method and a proposed LPC method using integration in the time domain for time series of noise wave shapes. Both numerical simulations and experimental calculations show that the proposed LPC technique can usefully be applied to analyze frequency and power spectrum of electromagnetic noise.

2. Power spectrum analysis

This section briefly introduces the conventional LPC method, which extracts frequencies from time series using data from short-duration samples to determine the frequencies of noise waves. The new technique based on LPC is also explained. In this technique, a time series of a noise wave only needs to be transformed into a time series of a noise wave integrated over time. In this section, the formulation of the new technique is introduced in detail.

2.1 The conventional LPC method

The time series of a noise is shown in Fig.6, with the horizontal axis in seconds, and the vertical axis showing voltage $v(t)$. ΔT is the sampling time and $v(n)$ is the n_{th} time series of $v(t)$ where $t = n\Delta T$. We define the estimated time series as follows:

$$v^{(e)}(n) = -\sum_{i=1}^{P} a_i v(n-i) \quad (1)$$

where $a_i (i = 1,2,\cdots,P)$ are unknown estimation coefficients, P is a number of the unknown estimation coefficients, which is determined by numerical simulations. The unknown estimation coefficients $a_i (i = 1,2,\cdots,P)$ are determined by minimizing the following norm I_v as follows:

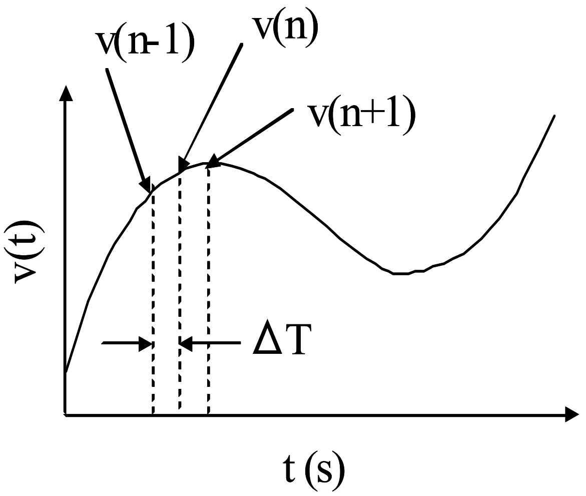

Fig.6 Time series of a noise wave shape

$$I_v = \left\| v - v^{(e)} \right\| \quad (2)$$

where

$$\left\| v - v^{(e)} \right\| = \sum_{n=P+1}^{M} \left| v(n) - v^{(e)}(n) \right|^2 \quad (3)$$

M is an integer which denotes a total amount of a finite segment of the time series $v(n)$. The unknown estimation coefficients $a_i (i = 1,2,\cdots,P)$ are determined by the following derivatives with respect to the complex conjugate of $a_i (i = 1,2,\cdots,P)$ as follows:

$$\frac{\partial I}{\partial a_i^*} = 0 \; (i = 1,2,\cdots,P) \quad (4)$$

Eq. (4) provides a linear equation related to $a_i (i = 1,2,\cdots,P)$, where "*" indicates the complex conjugate. Fourier transformation gives the following equation:

$$V^{(e)}(\omega) = -\sum_{i=1}^{P} a_i \exp(j\omega iT) V(\omega) \quad (5)$$

where $V(\omega)$ and $V^{(e)}(\omega)$ are the power spectrum of $v(t)$ and that of $v^{(e)}(t)$, respectively. The difference between $V(\omega)$ and $V^{(e)}(\omega)$ is given as follows:

$$\Delta V(\omega) = V(\omega) - V^{(e)}(\omega) \quad (6)$$

According to eq. (5) and eq. (6), $V(\omega)$ can be written as follows:

$$V(\omega) = \frac{\Delta V(\omega)}{1 + \sum_{i=1}^{P} a_i \exp(j\omega i \Delta T)} \quad (7)$$

In eq. (7) $\Delta V(\omega)$ is an unknown value, however, by assuming that $\Delta V(\omega)$ is a constant, we can define the estimated power spectrum of $V(\omega)$ as follows:

$$V^{(0)}(\omega) \propto \left| \frac{1}{1 + \sum_{i=1}^{P} a_i \exp(j\omega i \Delta T)} \right| \quad (8)$$

(2)

2.2 The proposed technique

When a noise has frequencies higher than the sampling frequency $(1/\Delta T)$, aliasing [5] will occur. If the sampling duration is shorter than a period of the time series $v(n)$, it is difficult to extract the spectra at lower frequencies. One mitigation is to use a low pass filter to reduce high frequency components and amplify lower frequency components. In this section, we try to use the time integration of time series [9]. We define $s(t)$, the time integration of noise $v(t)$ as follows:

$$s(t) = \int_{-\infty}^{t} v(u)du \quad (9)$$

where the Fourier transform of $s(t)$ can be written as follows:

$$S(\omega) = \frac{V(\omega)}{j\omega} \quad (10)$$

In eq. (10), $S(\omega)$ is the power spectrum of $s(t)$. Using eq. (10), we define the time integral of time series data as follows:

$$s(n) \approx \sum_{m=1}^{n} v(m)\Delta T \quad (11)$$

We also define the estimated time integral of the time series as follows:

$$s^{(e)}(n) = -\sum_{i=1}^{P} b_i s(n-i) \quad (12)$$

where $b_i (i = 1,2,\cdots,P)$ are unknown estimation coefficients. The unknown coefficients $b_i (i = 1,2,\cdots,P)$ can be obtained by minimizing the following norm I_s, defined as:

$$I_s = \left\| s - s^{(e)} \right\| = \sum_{n=P+1}^{M} \left| s(n) - s^{(e)}(n) \right|^2 \quad (13)$$

Taking the derivative of norm I_s with respect to the complex conjugate of $b_i (i = 1,2,\cdots,P)$ gives the linear equation

$$\frac{\partial I}{\partial b_i^*} = 0 (i = 1,2,\cdots,P) \quad (14)$$

which is related to $b_i (i = 1,2,\cdots,P)$, so by solving the linear equation, we can obtain $b_i (i = 1,2,\cdots,P)$. Using the Fourier transform of $S(\omega)$, the following equation is obtained:

$$S^{(e)}(\omega) = -\sum_{i=1}^{P} b_i \exp(j\omega iT) S(\omega) \quad (15)$$

From eq. (15), it follows that:

$$S(\omega) = \frac{\Delta S(\omega)}{1 + \sum_{i=1}^{P} b_i \exp(j\omega i\Delta T)} \quad (16)$$

Using both eq. (10) and eq. (16), we can derive the following equation:

$$V(\omega) = \frac{j\omega\Delta S(\omega)}{1 + \sum_{i=1}^{P} b_i \exp(j\omega i\Delta T)} \quad (17)$$

In eq. (17), $\Delta S(\omega)$ is an unknown value. By assuming that $\Delta S(\omega)$ is a constant, we can write the following equation:

$$V^{(\mathrm{int})}(\omega) \propto \left| \frac{j\omega}{1 + \sum_{i=1}^{P} b_i \exp(j\omega i\Delta T)} \right| \quad (18)$$

In the following sections, the validity of the proposed technique is examined by numerical simulation and by experiment.

3. Normalized power spectrum (NPS) analysis

In this section, we demonstrate the validity of the proposed technique. We first introduce two quasi-noise signals. The given signal is a burst noise as shown in Fig.3, at the power mains frequency.

- **Two-wave model**

This model is used to simulate a situation in which a noise signal has components of the power mains frequency, a burst noise and random noise. The amplitudes of the power frequency and the burst noise are equal. The burst noise is generated for only 5 ms, as described in eq. (20). Sinusoidal wave shapes are used to represent the power mains signal and the burst noise for
this two-wave model.

$$v(t) = \sin(2\pi f_1 t) + RND + W(t)\sin(2\pi f_2 t) \quad (19)$$

where $W(t)$ is a duration defined as follows:

$$W(t) = \begin{cases} 0 \ (5ms \geq t) \\ 1 \ (5ms < t < 5.05ms) \\ 0 \ (5.05ms \leq t) \end{cases} \quad (20)$$

- **Three-wave model**

This model is used in order to simulate another situation, in which the noise signal includes components with the power frequency, a burst noise, random noise and switching noise, with the amplitudes of the power and burst noises equal. The burst noise is generated for only 5 ms, as in the two-wave model. Sinusoidal waves are used to approximate not only the burst noise but also the switching noise in the three-wave model.

$$v(t) = \sin(2\pi f_1 t) + 0.1*\sin(2\pi f_2 t) + RND + W(t)\sin(2\pi f_3 t) \quad (21)$$

In these models, f_1 is the power mains frequency of 50 Hz, f_2 is the burst noise frequency of 50 kHz and f_3 is the switching frequency 5 kHz for the purposes of these simulations. A RND stands for the random noise or white noise, with its amplitude set between -0.5 and +0.5, and the sampling frequency for measuring noise shapes is set to 100 kHz.

3.1 Numerical results of the two-wave model

Fig.6 shows an example of a wave shape for the two-wave model. In Fig.7, the solid line is the noise wave shape calculated by eq.(19) and the broken line shows the wave shape of the integration over time of the two-wave model. The solid line has a 50 Hz sinusoidal wave, a burst wave of 50 kHz, and random noise. The broken line shows a smooth wave shape as shown in Fig.5 because the random noise may be almost removed by the time integral procedure. The duration of the noise wave is only 20 ms (M=2000), *i.e.* a period of 50 Hz, and the sampling frequency is set to 100 kHz $(\Delta T = 10^{-5} s)$. Fig.7 shows calculated normalized power spectra (NPS). The solid line shows the result of this proposed technique, the broken line shows that of the conventional method, and the broken dashed line is that from FFT. In Fig.8, the horizontal axis is frequency in Hz and the vertical axis is the normalized power spectrum of the noise. In this calculation, P (an estimation integer) must be set to 30 after numerical calculations and trials, but then P is 60 in these calculations. Each power spectrum is normalized to its maximum value in Fig.8. Using FFT, the burst noise spectrum can be obtained as shown in Fig.7, however, the power mains signal is not clearly extracted, because the sampling time is extremely short compared with the period of the 50 Hz power signal. The burst noise spectrum can be obtained as well by the conventional method as by FFT, but this does not extract the power mains signal, showing that the LPC method is not suitable for extracting low-frequency signals. In contrast, the proposed technique can extract not only the burst noise, but also the power mains signal. Table 1 shows the extracted frequencies and normalized power spectra (NPS) at given frequencies f_1 (50 Hz) and f_2 (50 kHz). Frequency estimation errors are less than 0.2% at 50 kHz (except for 50 Hz). NPS estimation errors between this method and the FFT method are less than 2.1 dB, as listed in Table 1. This shows that the proposed technique can evaluate the normalized power spectra of noise.

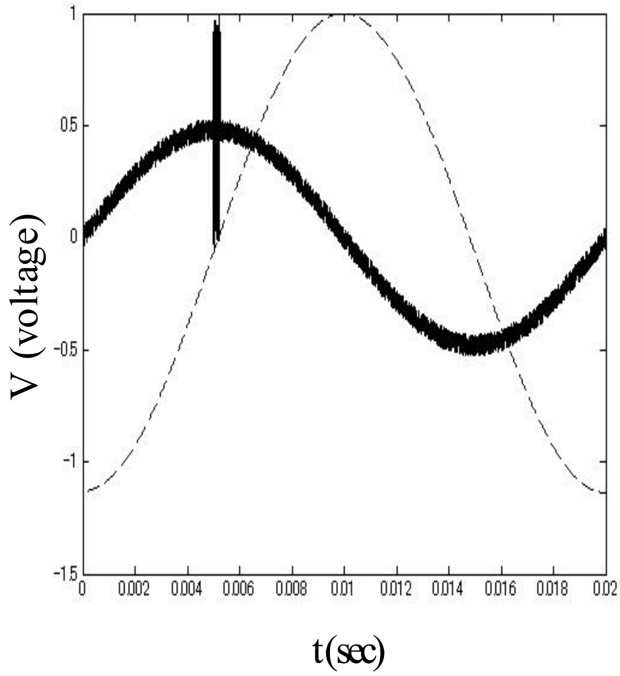

Fig.7 A wave shape for the two-wave model

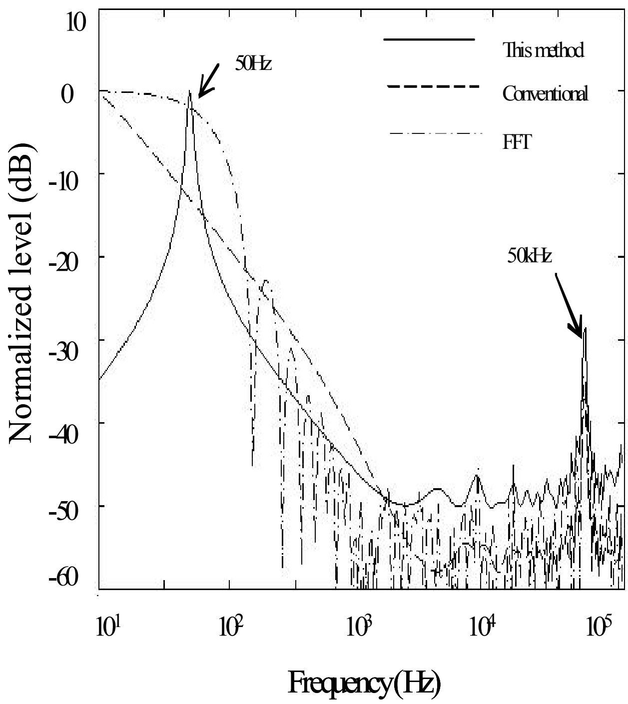

Fig.8 NPS (the two-wave model)

Table 1. Extracted frequency and NPS using the two-wave model

	The FFT	**This technique**	**Errors**	**Remark**
f_1(Hz)	-	50.01	-	-
f_2(Hz)	50.2 k	50.1k	-0.2%	-
NPS_1(dB)	-2.1	0	2.1 dB	@ f_1
NPS_2(dB)	-26.2	-25.1	+1.1 dB	@ f_2

3.2 Numerical results of the three-wave model

Fig.9 shows an example of a wave shape for the three-wave model. In Fig.9, the solid line is the noise wave shape calculated by eq.(21) and the broken line shows the wave shape of the integration over time of the three-wave model. The solid line displays a 50 Hz sinusoidal wave, a burst wave at 50 kHz, random noise, and switching noise simulated by a 5 kHz sinusoidal wave. The broken line shows smooth wave shapes as in Fig.7. Calculation conditions are: the duration of the noise wave is 20 ms (M=2000), the sampling frequency is set to 10 0 kHz $(\Delta T = 10^{-5} s)$, and P is set to be 60.

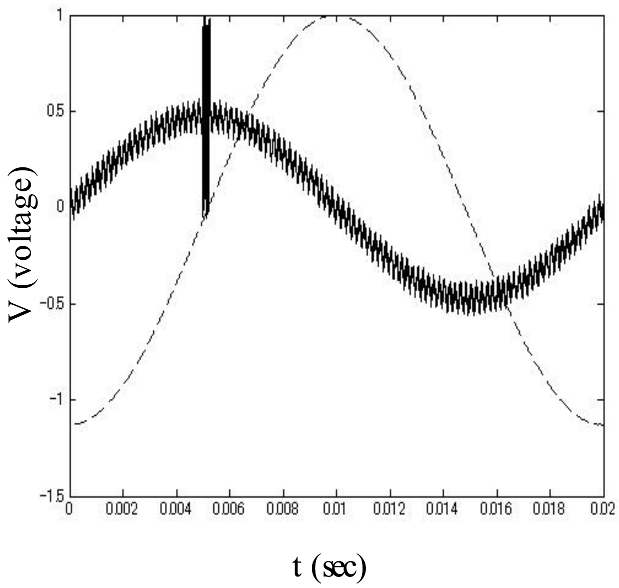

Fig.9 A wave shape for the three-wave model

Fig.10 shows the calculated normalized power spectra (NPS). The solid line shows the result of this proposed technique, the broken line shows that of the conventional method, and the broken dashed line is the FFT result. In Fig.9, the horizontal axis shows frequency in Hz and the vertical axis the normalized power spectrum of the noise. Each power spectrum is normalized to the maximum value observed, as in Fig.10.

FFT can obtain the burst noise spectrum as shown in Fig.10, but the power mains frequency is not clearly apparent in Fig.10. The conventional method can also obtain the burst noise spectrum, as does FFT, but also fails to reveal the power mains frequency. In contrast, the proposed technique can extract both the burst noise and the power mains spectrum. Estimation errors between given and extracted frequencies for 50 Hz, 5 kHz, 50 kHz are less than 0.1% with the proposed method. Errors for the other two methods are less than 0.15% for 5 kHz and 50 kHz, but extraction of the 50 Hz signal would require a longer duration sample of the noise wave for such a low frequency. With the conventional method it appears difficult to extract low frequency noise, as shown in Fig. 9. Deviations of the power peak values at 5 kHz and 50 kHz between FFT and the proposed technique are less than 3 dB and 4 dB, respectively. Therefore, the proposed technique can be used to evaluate noise power spectra.

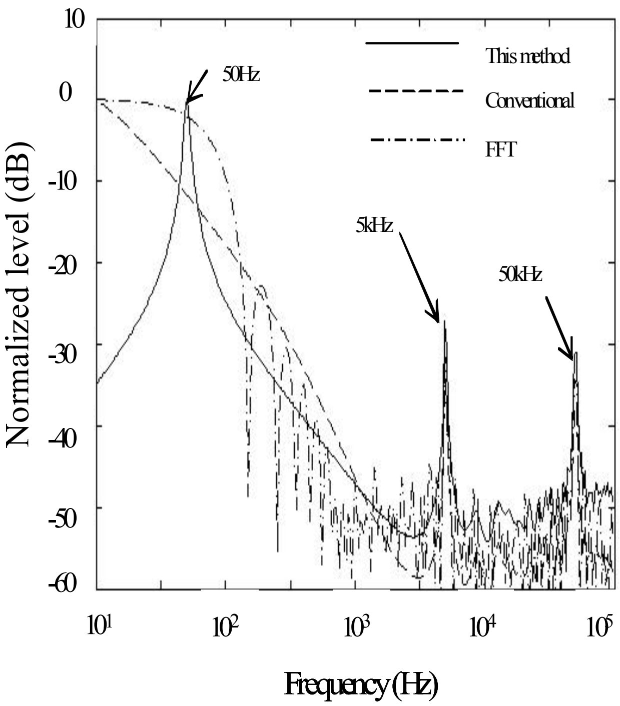

Fig.10 NPS (the three-wave model)

Table 2 shows the extracted frequencies and normalized power spectra (NPS) for the three-wave model at given frequencies f_1 (50 Hz), f_2 (50 kHz) and f_3 (5 kHz). Frequency estimation deviations are less than 0.15% at 50 kHz and 5 kHz, but not for 50 Hz. NPS estimation errors between this method and FFT are les s than 2.2 dB, as shown in T able 2. Therefore, t he proposed technique can evaluate normalized noise power spectra with precision.

Table 2 Calculated frequency and NPS for three-wave model

	The FFT	This technique	Deviation	Remark
f_1(Hz) -		50.01	-	-
f_2(Hz) 50	.2 k	50.1k	-0.2%	-
f_3(kHz) 5.	04k	5.01k	-0.4%	-
NPS_1(dB)	-2.2	0	+2.2 dB	@ f_1
NPS_2(dB)	-26.3	-25.2	+1.1 dB	@ f_2
NPS_3(dB)	-32.3	-31.1	+1.2 dB	@ f_3

3.3 Experimental result with a real wave

To exam ine the validity of the proposed technique, a real measured noise is used. Fig.1 1 shows an exa mple of a real measured noise wave shape(in the common-mode voltage of a power line) in a customer premises. The noise is generated when a thy ristor circuit in a heater operates. There is a very steep burst noise as shown in Fi g.11, and it can also be seen that the noise has a low frequenc y component a round 5 0 Hz. In Fig. 10, t he solid line shows the measured noise and the br oken line is t he time integral of the noise. From the broken line, the period of the noise is about 20 m s. The sam pling frequency is 2 MHz $(\Delta T = 0.5*10^{-6})$, the noise sample duration for freque ncy anal ysis is 33 m s in this case.

Fig.12 shows the calc ulated norm alized power spectrum (NPS) obtained b y FFT, the conventional method, and the proposed method. In Fig.12, th e solid line, th e broken lin e, and t he br oken dashed line show the calculate d result from FF T, the conventional method, and the propos ed technique, respectively. Fig.11 shows that the nois e has 50 Hz, 301 kHz, 603 kHz and 905 kH z frequency components, and also 50 Hz. Frequencies of 50 Hz, 301 kHz, 60 6 kHz and 908 kHz are assu med to be noise com ponents generated b y switching power circuits.

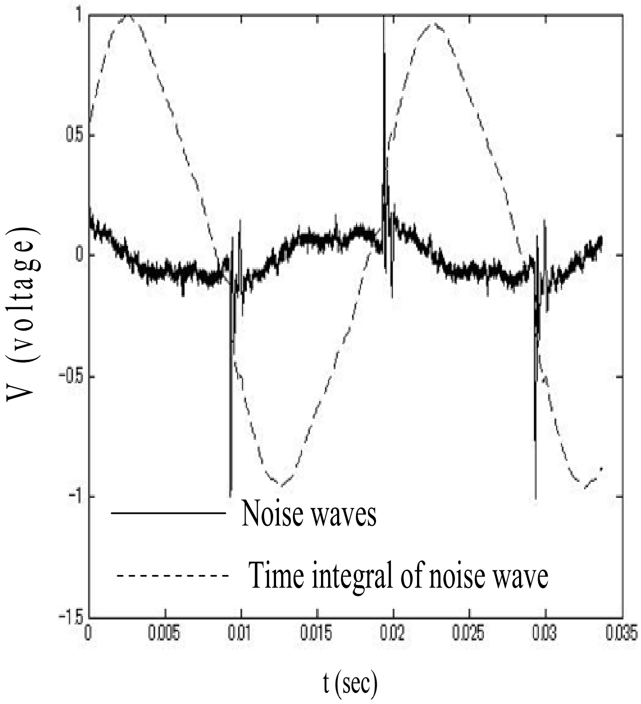

Fig.11 A real measured noise wave shape at a customer premises (common-mode voltage of a power line)

The burst noise is not sinusoidal, but a damped oscillation as shown in Fig.12. The rise ti me of the damped oscillation is about 0. $33*10^{-6}$s. The characteristic frequency i s about 3 01 kHz, so the generated frequencies lie around 300 kHz and its harmonics, as shown in F ig.12. T able 3 shows the extracted frequencies and NPS for a real noise wave. There are many NSP peaks in Fig.11, representative noise frequencies (f_1 ,f_2, f_3 and f_4) are about 50 Hz, 301 kHz, 6 06 kHz and 908 kHz. Frequency estimation dif ferences between FF T and the proposed technique are less than 0.3 % (except for 50 Hz). NPS esti mation dif ferences are between 4 dB and 10 dB (except for 50 Hz).

According t o Fig.1 1, t he dam ped oscillation peak level is about 1.1, and the peak level of the power mains frequency of 50 Hz is about 0.1 as shown in Fig.10, with the estimated difference between FFT and this technique at 50 Hz being -20.1dB (=$20\log_{10}(1.1/0.1)$). This shows that FFT is likely to overestimate the NPS level at 50 Hz, or extract a wrong frequency in this case. If this conclusion is correct, the estimated difference between FFT and this technique at 50 Hz is 5 dB as shown inFig.12. Numerical simulations show that the proposed technique can extract dominant power spectrum figures with high accuracy using a short-duration time series.

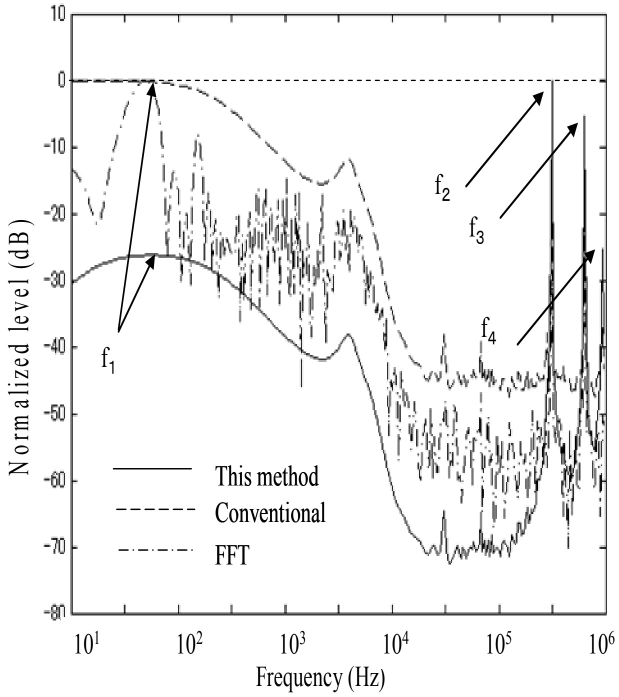

Fig.12 NPS of a real noise

Table 3 Extracted frequency and NPS for a real noise wave

	The FFT	This technique	Deviation	Remark
f_1(Hz)	48.1	50.1	+4%	-
f_2(Hz)	302.1k	301.2k	-0.3%	-
f_3(Hz)	604.2k	606.3k	+0.3%	-
f_4(kHz)	907.2k	908.3k	+0.1%	-
NPS_1(dB)	0.1 (-20.1)	-25.1	-25 dB (-5 dB)	@ f_1
NPS_2(dB)	-10	0	+10 dB	@ f_2
NPS_3(dB)	-13	-6	+7 dB	@ f_3
NPS_4(dB)	-30	-26	+4dB	@ f_4

4 Conclusions

This paper shows that there are still serious malfunctions of telecommunication equipment that occur in the field, mainly due to noise from the power circuits of electronic and electrical equipment. This noise causes communication link interruption, throughput reduction and sound quality degradation. Most of the noise is below 150 kHz in this case. To reduce the malfunctioning of telecommunication equipment, it is important to extract noise power spectra (frequencies and levels) to enable selection of the most suitable noise filters to put on power lines or telecommunication lines when it is impractical to simply eliminate the source(s) of the noise.

This paper also proposes a new technique based on the conventional LPC method. The new technique uses a time integral procedure on the noise signal. The procedure is to minimize random noise effects in frequency analysis, therefore, the new technique can extract power spectra, and the frequency extractions produced differ from numerical simulations and actual measurements by less than 4 %. The new technique can produce normalized power spectra of noise without requiring lengthy time series data, using shorter time series than FFT requires in general.

Numerical results presented here show that the proposed technique can be used for normalized power spectrum analysis of noise with dominant frequencies of less than 1 MHz.

References

[1] K.Tominaga, Y .Ogura, K.Murakawa:" Rec ent noise trou bles for teleco mmunication equipment an d future works ", IEICE, B-4-66, Sep. 2011.

[2] J. W. Cooley and J. W . Turkey: "An algorithm for the m achine of complex Fourier series", Math. Comput., p.297, Apr. 1965.

[3] E. O. Bir gham: "The Fast Fourier Transform", Prentice-Hall, 1974

[4] R. W. Hamming: "Digital Filter", Prentice-Hall, 1977.

[5] I. Daubecies : "The wavelet transfor m, ti me frequency lo calization and signal an alysis", IEEE Trans. Inf. Theory, vol.36, no.5, pp.961-965, Sept. 1990.

[6] G. Frenante and D. P . Adorno: "A wavelet analysis of 1/f and white noise in microwave transistors", Micro-electron, Reliability, vol.41, pp.99-104, 2001.

[7] J. D. Markel: "Digital inverse filtering – A new tool formant trajectory estimation", IEEE Trans. Audio Electron., v ol.AU-20, n o.2, pp.129-137, June 1972.

[8] F. Ishi yama and H. Y amane: "Identification of source of acoustic noise c aused b y electromagnetic disturbances using f ormant analysis", IEEE Int. S ymp., EMC, vol. 2, pp.697-700, Aug. 2002.

[9] F. Ishiy ama, K. Muraka wa, H. Yamane: "Formant analy sis of Imm unity test", IEICE, B -4-51, May 2003.