The use of Analytic Hierarchy Process for the life extension analysis of Air Defense Integrated Systems

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Abstract: The main objective of this paper is the achievement of research based on theoretical studies and experimental data processing in engineering and management solutions for the modernization of ADIS (Air Defense Integrated Systems). The authors have in mind the resource extension, recovery, and modernization processes, including the integration of missile systems with short range of action in a flexible structure. In order to emphasize the manner in which the ECM capability may affect the single shoot kill probability, we used the Analytic Hierarchy Process (AHP), a multi-criteria method in decision-making. The AHP is a very flexible and powerful tool because the scores, and therefore the final ranking, are obtained on the basis of the pairwise relative evaluations of both the criteria and the options provided by the user.

Key Words: ADIS (Air Defense Integrated Systems), Electronic counter measures capability, Analytic Hierarchy Process (AHP), eigen vector (value).

1. Introduction

Special integrated aerospace systems, relating to ADIS, it's a subject of the hour because the evaluation at national level and in NATO disclose the existence of real threat in continuous development, in particular ballistic missiles carrying mass destruction weapons. Development of an integrated defense system based on detection capabilities, making interception and combat is the key point in combating this kind of threat. Upgrading involves significant costs, creating an integrated framework of understanding the necessity of such particularly complex processes being essential in this process.

The Air Defense Integrated Systems architecture contains: Mobile command post, Operating system and data processing, Launchers, Radar research station, Maintenance, The education and training system, Simulators, and Industrial facilities;

The problematic of the study regarding the usage increase processes of special integrated airspace systems is of tremendous relevance. In this area, we intended to achieve a research based on theoretical studies and analysis of experimental data concerning modernization solutions of these systems, chasing differentiated technological progress influence [1-3].

The special airspace system represents an assembly of anti-air defense and anti-missile nets that include groups of algorithms, technical and technorganizational means [4,5,7,8], whose specific mission is to survey the air space, to detect and identify air targets and to guide active propulsion system vehicles, launched from the ground, against enemy targets. These special airspace systems are integrated into larger nets that include the analysis and conduct of operations and are active elements of these nets, as well.
The key performance indicators for ADIS are:
1. Probability of target shooting down with a single shot,
2. Missile speed,
3. Protection against electronic interference and jamming,
4. High level of mobility,
5. Maximum range,
6. Reliability.

Among criteria that are basing the resources extension, revitalization and modernization processes [9,10], we focused on the analytically study of protection against electronic interference and jamming.

The revitalization/modernization costs represent the direct technical costs.

\[ C_C = \frac{C_t + C_a + C_i + C_as}{10^6} \]  

where:
- \( C_t \) - the direct technical costs;
- \( C_a \) - training costs;
- \( C_i \) - infrastructure costs;
- \( C_as \) - auxiliary costs.

Tacking into account that in the revitalization processes the performance parameters do not change, the training costs tends to zero. Contrariwise, modernization involves modification of parameters, assuming non-null training cost. The same situation for \( C_i \) si \( C_as \).

The assessment efficiency program of life extension could be analized by considering the budget allocation per year at a half of the necessary cash flow amount, with an expected growth of 25% si a capital cost factor of 20%, the NPV (Net Present Value) method may be used [6] to check the viability risk of this complex project:

\[ NPV = -I_0 + \sum_{i=1}^{n} \frac{E(FCF_i)}{(1 + WACC)^i} \]  

where:
- WACC - weighted average cost of capital
- \( I_0 \) - the initial investment
- \( i \) - number of years before producing cash flow

The NPV should be positive to consider the project feasible.

In order to highlight the manner in which the anti-jamming capability may affect the probability of hitting the target, we used the Analytic Hierarchy Process (AHP), a multi-criteria method in decision-making, and may aid the decision maker to set priorities and make the best decision [11-14]. By reducing complex decisions to a series of pairwise comparisons, and then synthesizing the results, the AHP helps to capture both subjective and objective aspects of a decision. In addition, the AHP incorporates a useful technique for checking the consistency of the decision maker’s evaluations, thus reducing the bias in the decision making process.

The AHP considers a set of evaluation criteria, and a set of alternative options among which the best decision is to be made. It is important to note that, since some of the criteria could be contrasting, it is not true in general that the best option is the one which optimizes each single criterion, rather the one which achieves the most suitable trade-off among the different criteria [5,8,9].

This method generates a weight for each evaluation criterion according to the decision maker’s pairwise comparisons of the criteria. The higher the weight, the more important the corresponding criterion. Next, for a fixed criterion, the AHP assigns a score to each option according to the decision maker’s pairwise comparisons of the options based on that criterion. The higher the score, the better the performance of the option with respect to the considered criterion. Finally, the criteria weights are combined and the options scores, thus determining a global score for each option, and a consequent ranking. The global score for a given option is a weighted sum of the scores it obtained with respect to all the criteria.

The AHP is a very flexible and powerful tool because the scores, and therefore the final ranking, are obtained on the basis of the pairwise relative evaluations of both the criteria and the options provided by the user. The computations made by the AHP are always guided by the decision maker’s experience, and the AHP can thus be considered as a tool that is able to translate the evaluations (both qualitative and quantitative) made by the decision maker into a multi-criteria ranking. In addition, the AHP is simple because there is no need of building a complex expert system with the decision maker’s knowledge embedded in it.

On the other hand, the AHP may require a large number of evaluations by the user, especially for problems with many criteria and options. Although every single evaluation is very simple, since it only requires the decision maker to express how two options or criteria compare to each other, the load of the evaluation task may become unreasonable. In fact the number of pairwise comparisons grows quadratically with the number of criteria and options. For instance, when comparing 10 alternatives on 4
criteria, $4\cdot 3/2 = 6$ comparisons are requested to build the weight vector, and $4\cdot (10 - 9/2) = 180$ pairwise comparisons are needed to build the score matrix.

However, in order to reduce the decision maker’s workload the AHP can be completely or partially automated by specifying suitable thresholds for automatically deciding some pairwise comparisons. The AHP can be implemented in three simple consecutive steps:

- Computing the vector of criteria weights;
- Computing the matrix of option scores;
- Ranking the options.

Each step will be described in detail in the following. It is assumed that $m$ evaluation criteria are considered, and $n$ options are to be evaluated. A useful technique for checking the reliability of the results will be also introduced.

In order to compute the weights for the different criteria, the AHP starts creating a *pairwise comparison matrix* $A$. The matrix $A$ is a $m \times m$ real matrix, where $m$ is the number of evaluation criteria considered. Each entry $a_{jk}$ of the matrix $A$ represents the importance of the $j$th criterion relative to the $k$th criterion. If $a_{jk} > 1$, then the $j$th criterion is more important than the $k$th criterion, while if $a_{jk} < 1$, then the $j$th criterion is less important than the $k$th criterion. If two criteria have the same importance, then the entry $a_{jk}$ is 1. The entries $a_{jk}$ and $a_{kj}$ satisfy the following constraint:

$$a_{jk} = \frac{1}{a_{kj}}$$

Obviously, $a_{jj} = 1$ for all $j$. The relative importance between two criteria is measured according to a numerical scale from 1 to 9, as shown in Table 1, where it is assumed that the $j$th criterion is equally or more important than the $k$th criterion.

The phrases in the “Interpretation” column of Table 1 are only suggestive, and may be used to translate the decision maker’s qualitative evaluations of the relative importance between two criteria into numbers. It is also possible to assign intermediate values which do not correspond to a precise interpretation. The values in the matrix $A$ are by construction pairwise consistent.

On the other hand, the ratings may in general show slight inconsistencies. However these do not cause serious difficulties for the AHP.

Each of these judgments is assigned a number on a scale, associated with, is very much more important, rather more important, as important, and so on down to very much less important, than attribute. One common scale is:

<table>
<thead>
<tr>
<th>Intensity of importance</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two factors contribute equally to the objective</td>
</tr>
<tr>
<td>3</td>
<td>Somewhat more</td>
<td>Experience and judgement slightly favour one over the other.</td>
</tr>
<tr>
<td>5</td>
<td>More important</td>
<td>Experience and judgement strongly favour one over the other.</td>
</tr>
<tr>
<td>7</td>
<td>Very much more</td>
<td>Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice.</td>
</tr>
<tr>
<td>9</td>
<td>Absolutely more</td>
<td>The evidence favouring one over the other is of the highest possible validity.</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values</td>
<td>When compromise is needed</td>
</tr>
</tbody>
</table>


The main task of application consists in a better understanding of the actual possibilities of upgrading, the factors that are involved in this process, the risks, project costs proposed, under a complex set of restrictions and costs.

Modernization brings the ability to use the system in actual conditions and fully or partially corrects the obsolescence of the equipment. In the same time assures efficient and complex aerospace system efficiency and understanding of project management appropriate to obtain the proposed results in compliance with the performance requirements. In terms of expected results the application offers whole suggestive images that allow project manager to lead efficiently at low cost and high performances.

We applied this method to decide if upgrading a technical system is opportune or not. To do this we had to compare six characteristics. Let’s denote them $C_1$, $C_2$, …, $C_6$.

We first provide an initial table for pairwise comparisons in which the principal diagonal contains entries of 1, as each factor is as important as itself. These (pairwise) comparisons are presented in table 2.
The next step is to build the (comparison) matrix corresponding to the pairwise comparisons. It will be a 6 (lines) by 6 (columns) matrix, and will be denoted by \( A \). The diagonal elements of this matrix are always 1 as we mentioned before, and all we have to do is filling up the upper triangular matrix. In order to do this, we have to respect the following obvious rules (taking into account that we fill the rows of the matrix, i.e. we compare the \( C_i \) characteristic with \( C_{i+p} \), where \( i \) is the row index in the matrix):

1. If the judgment value (be it \( a_{i,i+p} \in [1,9] \)) is on the left side of 1 (see Table 2), it means that \( C_i \) is better than \( C_{i+p} \), so we fill this value in the matrix.

2. If the judgment value (be it \( a_{i,i+p} \in [1,9] \)) is on the right side of 1 (see Table 2), it means that \( C_{i+p} \) is better than \( C_i \), so we fill in the matrix the value \( \frac{1}{a_{i,i+p}} \).

Observing Table 2, the obtained matrix is the following:

\[
A = \begin{pmatrix}
1 & 2 & 9 & \frac{1}{4} & 5 & 9 \\
\frac{1}{2} & 1 & 9 & 4 & 1 & 9 \\
\frac{1}{9} & \frac{1}{9} & 1 & \frac{1}{1} & 1 & \frac{1}{3} \\
4 & \frac{1}{4} & 9 & 1 & 6 & 4 \\
1 & \frac{1}{5} & 5 & \frac{1}{6} & 1 & 3 \\
\frac{1}{9} & \frac{1}{9} & 3 & \frac{1}{4} & \frac{1}{3} & 1 \\
\end{pmatrix}
\]

According to Saaty’s theory, the next step is to compute the priority vector, which is the normalized Eigen vector of the matrix, corresponding to the maximum eigen value.

We have calculated the priority vector by using the Matlab program. The result of this numerical determination:

- Maximum eigen value: \( \lambda_{\text{max}} = 7.4526 \)
- The corresponding eigen vector (normalized), i.e. the priority vector:

\[
W_{\text{p}} = \begin{pmatrix}
0.2520 \\
0.2976 \\
0.0194 \\
0.3030 \\
0.0931 \\
0.0350
\end{pmatrix}
\]

The priority vector may be approximated by using the following method, especially when dealing with small matrices. It is based on the normalization of each column of the matrix.

\[
a_{ij} \rightarrow \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}}, \forall j = 1, n; \quad n = 6
\]

The priority vector will consist in the average value of each line

\[
w_{ij} = \frac{\sum_{j=1}^{n} a_{ij}}{n}, \forall i = 1, n; \quad n = 6
\]

- The normalized matrix:
The priority vector:

\[
\begin{bmatrix}
1 & 2 & 9 & 1 & 4 & 5 & 9 \\
1 & 2 & 1 & 9 & 4 & 1 & 9 \\
1 & 2 & 1 & 1 & 1 & 1 & 1 \\
9 & 9 & 9 & 9 & 5 & 5 & 3 \\
4 & 1 & 4 & 9 & 1 & 6 & 4 \\
1 & 5 & 1 & 5 & 1 & 1 & 3 \\
1 & 1 & 1 & 3 & 1 & 4 & 3 \\
9 & 9 & 9 & 9 & 4 & 3 & 1 \\
\end{bmatrix}
\]

\[
A_{max} = \begin{bmatrix}
13 & 17 & 1 & 1 & 17 & 27 & 0 \\
77 & 38 & 4 & 23 & 46 & 79 & 0 \\
71 & 76 & 4 & 13 & 27 & 79 & 0 \\
53 & 40 & 36 & 92 & 43 & 12 & 79 \\
52 & 1 & 1 & 1 & 1 & 1 & 1 \\
77 & 18 & 4 & 52 & 97 & 79 & 0 \\
3 & 17 & 15 & 1 & 2 & 9 & 0 \\
89 & 76 & 36 & 35 & 27 & 79 & 0 \\
53 & 40 & 12 & 23 & 81 & 79 & 0 \\
\end{bmatrix}
\]

\[
W = \begin{bmatrix}
10 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

\[
\varepsilon = \max \left( \frac{W_i - W_{A_{max}}}{W_{A_{max}}} \right) \times 100 = \max \left( \frac{6.77026 - 9.09091}{9.09091} \right) = 9.09091\%
\]

An error level of approximately 10% is considered to be large enough, so we can conclude that the comparison matrix which is 6 by 6 is not small enough for applying the approximating method.

**4. Discussions. Results interpretation**

Further study is dedicated to analyze the consistency, the comparison matrix consistency in practice. In principle, a comparative judgment is consistent if it respects the transitivity principle. A basic, but very reasonable, assumption is that if attribute A is absolutely more important than attribute B and is rated at 9, then B must be absolutely less important than A and is valued at 1/9. These pairwise comparisons are carried out for all factors to be considered, usually not more than 7, and the matrix is completed. The matrix is of a very particular form which neatly supports the ensuing calculations.

As these qualitative judgments are transformed into quantitative assessments, it follows that one may define first a reciprocal array consistency. Thus a reciprocal matrix \( a_{ij} > 0; a_{ij} = \frac{1}{a_{ji}} \) is said to be consistent if respects the relationship:

\[ a_{jk} \cdot a_{kp} = a_{jp}, \forall j, k, p \]

The following theorem (Saaty) is proved:

A reciprocal matrix of \( n \times n \) type is consistent if and only if its characteristic polynomial has the form:

\[
P(\lambda) = \lambda^n - n \cdot \lambda^{n-1}
\]

Thus the eigen value of this kind of matrix (solution of equation \( P(\lambda) = 0 \)) will be zero (root of \( n-1 \) multiplicity order) and \( n \) (simple root). So, choosing the maximum eigen value and the corresponding eigen vector as priority vector appears a natural action. It also ensues that the consistency of the study can be appreciated through the difference \( \lambda_{max} - n \).

Ideally it must be null. However, it is very unlikely that following the comparisons between analyzed criteria, a consistent (comparison) matrix (as defined above) will be obtained. As a consequence, the following consistency indicators (Saaty) are introduced:

- Consistency index (CI):
  \[
  CI = \frac{\lambda_{max} - n}{n - 1}
  \]

- Random consistency index (RI). This has been achieved by random generation of reciprocal matrix (500 items), filled with the values \( \frac{1}{9}, \frac{1}{8}, \ldots, \frac{1}{1}, \ldots, 8, 9 \) and CI index determination. Their average values, corresponding to arrays of type 3 × 3, ..., 10 × 10, are shown in Table 3.

<table>
<thead>
<tr>
<th>n</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0.58</td>
<td>0.9</td>
<td>1.09</td>
<td>1.24</td>
<td>1.33</td>
<td>1.44</td>
<td>1.57</td>
<td>1.69</td>
</tr>
</tbody>
</table>

- Rate of consistency (CR) is defined as the ratio of the two above mentioned quantities:
  \[
  CR = \frac{CI}{RI}
  \]

It is accepted that an analysis of this kind is consistent if \( CR < 10\% \).

In the discussed case:

\[
CI = 7.4526 - \frac{6}{6-1} \approx 0.29
\]

\[
CR = \frac{0.29}{1.24} \approx 0.23 = 23\%
\]

The value is too large to be accepted, so it requires a revising of judgments (comparisons) we have made. Thus, it results that the increasing order of the importance of the 6 criteria is \( C_9, C_6, C_5, C_4, C_1, C_2, C_3 \). If the judgments were consistent, then this order ought to be found on each line or column of the comparison matrix.
matrix, which does not happen. For example, if considering the fourth line, it results the following order of the criteria: C₂, C₄, C₁, C₆, C₅, C₃.

It should therefore be a revising of the criteria, which is shown in table 4.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>X</td>
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<td></td>
<td></td>
<td>C₂</td>
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<tr>
<td>C₂</td>
<td>X</td>
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<td>C₄</td>
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<td></td>
<td></td>
<td></td>
<td>X</td>
<td>C₆</td>
</tr>
</tbody>
</table>

According to table 4, we obtained the matrix of comparison:

\[
A = \begin{pmatrix}
1 & 1 & 2 & 7 & 1 & 5 & 6 \\
2 & 1 & 8 & 1 & 4 & 7 \\
1 & 1 & 1 & 1 & 9 & 5 & 3 \\
3 & 2 & 9 & 1 & 6 & 8 \\
1 & 1 & 5 & 6 & 1 & 7 \\
1 & 1 & 4 & 3 & 1 & 8 & 7 & 1
\end{pmatrix}
\]

The following results are obtained:

- Maximum eigen value: \( \lambda_{\text{max}} = 6.6239 \)
- Consistency Ratio: \( CR = \frac{\text{CI}}{\text{RI}} = \frac{\frac{\lambda_{\text{max}} - n}{n-1}}{\frac{n}{n-1}} = \frac{6.6239 - 6}{6.6239 - 6} = 10.06\% \)

These parameters suffered an obvious improvement, so that the correct weights \( W_i \) shall be deemed the above ones, resulting from revising the criteria. It can be seen that the order of importance of the analyzed criteria remained unchanged.

Finally we can calculate the \( K_B \) - jamming resistance quality coefficient:

\[
K_{KB} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j
\]

where:
- \( i \) - is the number of protection devices in radiolocator
- \( j \) - is the number of types of jamming affecting the radiolocator
- \( w \) - is the number of individual types of jamming.

For example, if \( i \in \{2, 3\} \), it results:

\[
K_{KB} = w_1 + w_2 + w_3 = 0.1999 + 0.2536 + 0.0240 = 0.4775
\]

We can calculate the probability of killing taking into account the jamming resistance quality coefficient:

\[
P_{01} = \left[ 1 - \prod_{i=1}^{n} (1 - P_i) \right] \cdot K_{KB}
\]
A final remark now: because it is not very easy to compute exactly the priority vector in Matlab (to do this we have to obtain the comparison matrix and to input it in Matlab each time we have to perform a job like this), we consider that a procedure in this matter is worthy. So we tried to conceive such a procedure, whose logical diagram is presented in figure 1.

It is clear that the procedure builds the comparison matrix step by step (it asks the user to input the result of each comparison), and finally it compute the priority vector.

5. Conclusions and contributions.

The importance, efficiency and effectiveness of the ADIS derived from its mission.

This article has proposed an analysis of the ECM (Electronic Counter Measures Capabilities) that influences actual possibilities of upgrading the ADIS, characterized by high complexity. Understanding the factors that are involved, the risks, the proposed project costs is vital for the selection of this project, because in this case a complex set of constraints at the level of performances and costs is involved.

Also it is important to understand the proper project management to obtain the proposed results in compliance with the performance requirements.

AHP has many strength points, the main ones being the following:

- AHP is an analytical method: This is the main advantage of AHP: it involves mathematical and logical reasoning to get the decision. In this way it turns into quantitative items the human qualitative judgments.
- AHP organizes data as a hierarchical structure and this is a human natural approach. In this way complex problems are transformed into several sub-problems to be solved sequentially. Psychological studies concluded that human beings can compare \(7 \pm 2\) items simultaneously, and this fundaments the Saaty’s scale.
- The AHP defines a procedure for making decisions, formalizing this process and placing it in a scientific context.

As a weak point of AHP, the possibility of getting an inconsistent comparisons matrix is the main one, especially when comparing a lot of items.

Using AHP method in the proposed application, the weights of criteria \(C_1, C_2, C_3, C_4, C_5, C_6\), were computed and assumed. It was shown also an interesting analysis of the study consistency, namely the consistency of comparison matrix, based on the fact that comparative judgment is consistent only if respects the principle of transitivity.

The study results show that 3 from 6 criteria are influencing the single shoot kill probability till 50%. Also the application provides a suggestive overview that enables the project manager to have an efficient management at low cost and superior performance.

It would be interesting a similar approach in the future, using AHP method, to other performance characteristics of special integrated aerospace systems.

References:


