Methods for the Assessment of Historical Masonry Arches

LUCIO NOBILE, VERONICA BARTOLOMEO
DICAM Department-Campus of Cesena
University of Bologna
Viale Cavalcavia 61-47521 Cesena
ITALY
lucio.nobile@unibo.

Abstract: - The aim of this paper is to give an overview of the main analytical and numerical methods for the assessment of masonry arches , highlighting strengths and weaknesses. The methods are mainly three: i) the Thrust Line Analysis Method; ii) the Mechanism Method; iii) the Finite Element Method. The Thrust Line Analysis Method and the Mechanism Method are analytical methods and are based on two of the fundamental theorems of the Plastic Analysis, while the Finite Element Method is a numerical method that uses different strategies of discretization to analyze these structures.

Key-Words: - Masonry arch, Structural Models, Discrete limit analysis, Collapse Mechanism

1 Introduction

In his book [1] “La scienza delle costruzioni e il suo sviluppo storico”, Edoardo Benvenuto gave us the historical perspective of the first static theories regarding the masonry arch.
Between the seventeenth and eighteenth century, the geometric and the empiric rules reported in the ancient treatises were replaced by a real static theory on the stability of the arches.
Philippe De La Hire[2] was the first developing an innovative approach, which remained the same through the eighteenth century. The arch was considered as a series of rigid blocks of well-defined geometry and specific weight. However his model neglected the friction, which was taken into account by Coulomb Model
Only around the fifties of this century, the problem was taken up and dealt with a more congenial method. Attempts in the twenties to adapt the elastic theory to the masonry arch were not very successful. The weak points of these attempts were to assume the masonry material as elastic and to consider valid the results even if the thrust line was external to the core in some points. The turning point of the fifties was the introduction of the limit design and of its increasing applications in structural analysis. The theorems of limit analysis are admirably suited to the determination of the collapse load of masonry arches.
So nowadays the engineering methods of assessment for arch bridges mainly rely on the pioneering work by Pippard and Ashby[3] and Pippard [4]. They determined the load required, at a given location, to cause the formation of two additional hinges, and hence a mechanism, in a two hinged arch. These procedures guaranteed that an equilibrium configuration exists for the considered structural model but gave only rough estimates of the limit load. Following this approach and Drucker’s studies, Kooharian [5] published the first modern work on this topic in 1952, which was followed one year later by Onat and Prager’s [6] paper.
Another milestone was Heyman publication [7] in 1966, in which he explained for the first time the applicability of ultimate load theory for any masonry loadbearing structure.

2 Jacques Heyman and the “Safe Theorem”

Jacques Heyman introduced three hypotheses for the determination of the admissibility domain of the masonry material.
Heyman does not introduce anything new, but formalizes in a clear way some hypotheses on the material that formed the basis for the calculation of the arches in the XVIII and XIX century. These assumptions enable Heyman to frame the masonry action in the plastic theory and to formulate the famous safe theorem that will be explained later on.
The three hypotheses are: (i) the masonry has no tensile strength (Fig.1); (ii) the masonry has infinite compression strength (Fig.2); (iii) sliding failure doesn’t occur (Fig.3). The first assumption that does not always adhere to the reality, but it is a safety benefit. It is strictly true only if the masonry is made by dry-stone blocks or with weak mortar:

however, in most cases, the adherence between mortar and masonry blocks is negligible because the mortar may decay in time. Therefore, whatever is the ultimate tensile strength of the individual blocks, the masonry may be considered a non resistant tensile material (NRT material). The hypothesis of infinite compression strength is a valid approximation only if the ratio between the average compression stress and the masonry compression strength is a negligible value compared to the unit.

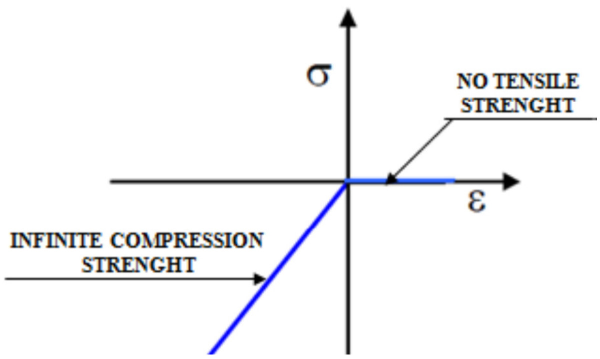

Fig.1 Heyman's first two hypotheses

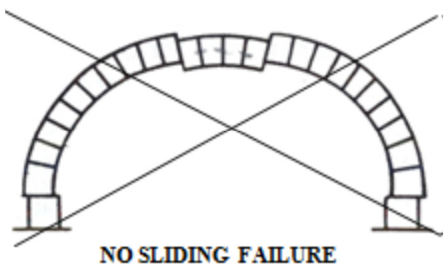

Fig.2 Heyman's third hypothesis

A reduction of the resistant section occurs in a NRT material with a consequent redistribution of the compression stresses leading to an increase of the peak values.

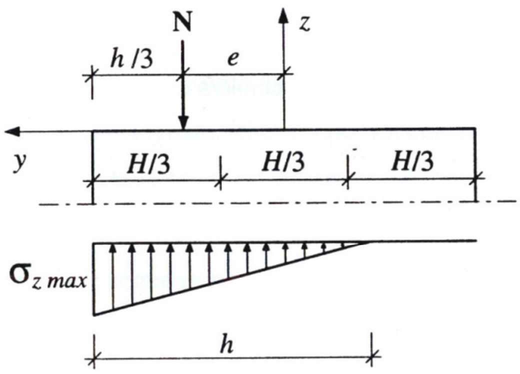

Fig.3 Reduction of the resistant section

In normal conditions of service, stresses are so low that any phenomenon of crushing failure does not occur.

The assumption of absence of sliding failure implies that the shear component of the stress exerted between two adjacent voussoirs can never exceed the friction resistance between them. In fact, low compression stresses allow developing high friction forces that prevent voussoirs from losing cohesion and sliding. The validity of this hypothesis can be verified considering the slope of the thrust line with respect to the joint lines: if the thrust line is perpendicular to the joints, there is no mutual sliding between the voussoirs. If it forms an angle minor than 90°, the voussoirs tend to slide downwards or upwards.

Concerning Heyman's hypotheses, the collapse mechanism of the arch is then identified by the progressive formation of hinges that coincide with the points where the thrust line is tangent to the intrados or extrados of the arch. The mechanism for formation of hinges is not the only possible for the arch, but the experimental studies of Hendry [8] show that it can be considered as the most likely collapse mechanism for arches well buttressed. The analogy between the rotation failure mechanics of the arch and that of the steel frames allows Heyman to apply the fundamental theorems of the plastic analysis, including the safe theorem:

"*If any equilibrium state can be found that is one for which a set of internal forces is in equilibrium with the external loads, and, further, for which every internal portion of the structure satisfies a strength criterion, then the structure is safe*".

The safe theorem allows remedying the vagueness connected to the true thrust line location between infinite numbers of possibilities: an arch is safe simply if a thrust line can be drawn inside his thickness.

The thrust line has not to go out of the masonry thickness: to this end, it is interesting to study its two extreme positions that represent two states still in equilibrium. In fact, when the thrust line touches the lower or the upper boundary of the arch, the masonry finds itself at the limit of the admissible states region and the eccentricity is such that promotes the formation of hinges. In particular, in the two extreme conditions, the thrust line gives the location of three hinges that open: in this way, the value of the horizontal abutment thrust can be calculated, as shown in Fig.4.

In the two extreme positions of the thrust line, the horizontal abutment thrust will be: a) minimum; b) maximum. The minimum horizontal thrust will be obtained when the arch acts on the environment: for

ISBN: 978-960-474-403-9

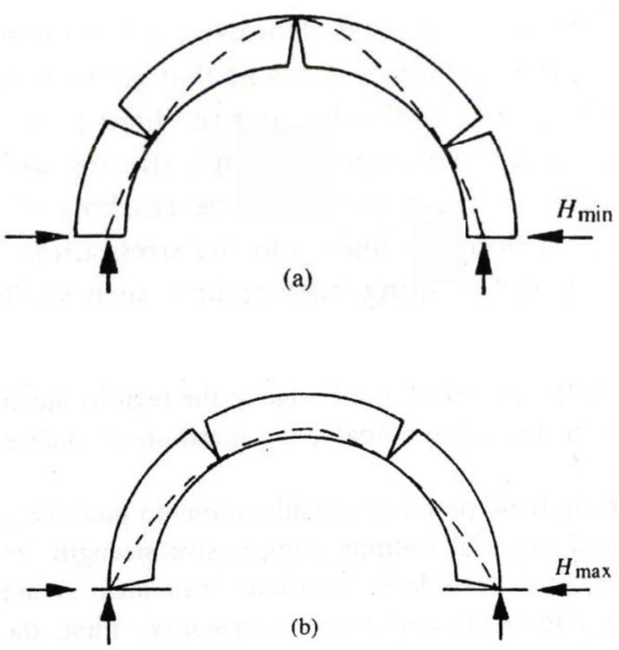

Fig.4 (a) Minimum abutment thrust (b) Maximum abutment thrust

example,byr removing the centring that supports the masonry, an arch will thrust on the abutments and these one will open slightly. In minimum thrust state, or passive state, the thrust line will have the greatest rise and the smallest clear span; it will touch the extrados at the key and intrados at the back. The maximum horizontal thrust will be obtained when the environment acts on the arch: for example, when two abutments move closer to each other, the arch span diminishes. In state of maximum thrust state, or active state, the thrust line will have the smallest rise and the greatest clear span; it will touch the extrados at the crown and the intrados down. Three hinges will open if one is at the key; on the contrary, four hinges form.

It is important to know the two extreme positions of the thrust line, because the real thrust of the arch can't be calculated, but the upper and the lower limits can be fixed.

The collapse of a masonry arch does not involve an absence of strength, but rather a loss of stability. In fact the collapse takes place when a thrust line can't be finding within the arch boundaries. The crisis is connected with the formation of a fourth hinge that transforms the stable arch in an unstable mechanism of collapse. The four hinges open in alternating way in the intrados and in the extrados, following a pattern that is function of the arch shape and the working loads. In case of symmetrical load, a fifth hinge can open, but generally slight geometrical failings make the structure to behave asymmetrically.

A masonry arch has to support two main types of loads: i) the self-weight; ii) the additional loads. The additional point loads have a thrusting nature and can cause collapses because their action move the thrust line out of the arch, generating the fourth hinge, as shown in Fig.5.

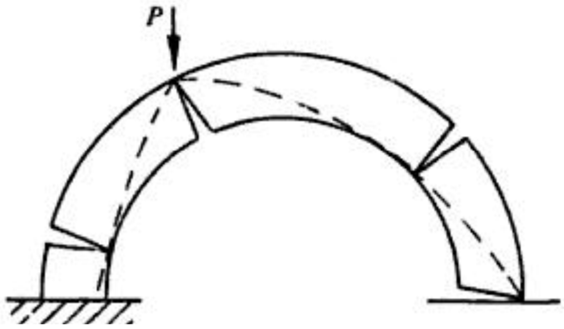

Fig.5 An additional point load generating the fourth hinge

On the contrary, the self-weight is the resistant load of any masonry structure and offers resistance to any mechanism of collapse.

2.1 Stability Check

The catenary is the arch true shape. Arches with other shape stand up because catenaries are included in their thickness. The thrust line shape is the mathematical catenary if the self-weight is equally distributed around the arch. There is a minimum thickness of semicircular arch that just contains a catenary. The limit arch has exactly this minimum thickness and is in unstable equilibrium. The ratio between the real arch thickness and the limit arch one defines the safety factor that is of geometric nature. Heyman suggests 2 as safe practical value: that is, if you're able to draw a thrust line in the middle half of the arch, the arch is safe, as shown in Fig.6. So the thrust line can be perceived as an index of the stability condition of the arch.

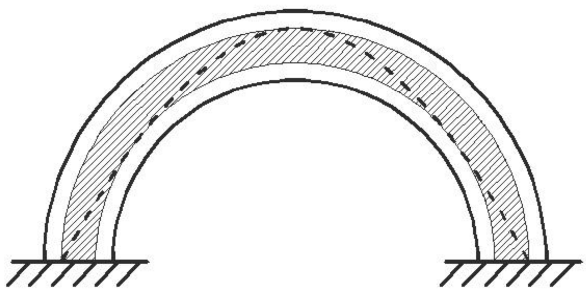

Fig.6 Geometry Safety factor of 2

2.2 Plastic Limit Analysis

The research of Professor Heyman highlights that an elastic analysis is problematic for masonry structures because there isn't a unique calculable equilibrium state. On the contrary, the limit analysis allows considering the structure only in relation to its ultimate state, using few material parameters and neglecting the initial stress state. Some of the principal methods for the assessment of masonry arch bridges are based on the fundamental theorems of Limit Analysis.

A summary of the basic rules that apply in the theory of plasticity can be found in the work of Horne [9]. In the context of masonry arches, there are fundamentally three main considerations to apply the theorems of plastic limit analysis: i) the internal actions must be in equilibrium with the external loads; ii) there must be a sufficient number of hinges to transform the structure into a mechanism; iii) the maximum stresses must be less than or equal to the material strength.

The three fundamental theorems of plastic analysis can be stated in simplified form as:

• **Static or lower bound theorem.** If the equilibrium and yield conditions are everywhere satisfied, then the load factor λl is less than or equal to the failure load factor λp;

• **Kinematic or upper bound theorem.** If the equilibrium and the mechanism conditions are everywhere satisfied, then the load factor λu is greater than or equal to the failure load factor λp;

• **Uniqueness theorem.** If the internal stress state is such that the three conditions of equilibrium, mechanism, and yield are satisfied then that load factor is the collapse load factor λp.

3 Methods for the assessment of the masonry arches

Structural analysis is a general term describing the operations to represent the real behavior of a construction. The analysis can be founded on mathematical models created on theoretical bases or on physical models tested in laboratory. In both cases, the models try to individuate the load carrying capacity of the structure, identifying the stress state, the strain and the internal forces distribution of the entire structure or of its parts. Besides, the models proposed for arch structures try to indicate the failure mode and the location of plastic hinges.

In this paper, analytical methods for the structural analysis of the masonry arch bridges are treated. In literature there are many types of theoretical methods that can be used. These methods can be divided into different categories concerning their origin, scope, and applicability and approximation level.

As previously seen, among the three fundamental structural criteria (strength, stiffness and stability), it is the stability that governs the life of the masonry arches because the average medium stresses are low and the strains are negligible (Fig.7).

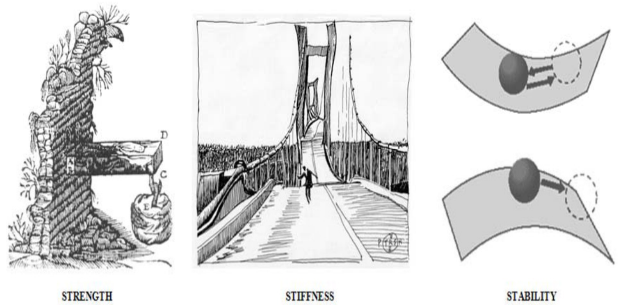

Fig.7 Methods of load carrying capacity assessment

So the most important methods for the evaluation of masonry arch bridges are derived from Heyman's theories and from the fundamental theorems of the Plastic Analysis. They are: i) the thrust line analysis method; ii) the mechanism method.

The Thrust Line Analysis Method is based on the lower bound theorem or "safe" theorem and defines the limits for the thrust line location. It uses a static approach and defines the limit load that ensures the equilibrium of the arch bridge analyzed. On the contrary, the Mechanism Method is based on the upper bound theorem and studies the number of plastic hinges needed to transform the arch in a mechanism. In this case, the stability of the arch is analyzed with regards to a kinematic approach. Both the methods are valuable: due to their different bases, the first one underestimates the structure strength, while the second overestimates it.

3.1 Thrust Line Analysis Method

This general method analyzes the arch stability, evaluating the location of the thrust line inside the cross section. The thrust line represents the *locus* of points along the arch through which the resultant forces pass. If all the arch voussoirs have the same size, the line of thrust has almost the shape of an inverted catenary.

"*As hangs the flexible, so but inverted will stand the rigid arch.*" wrote Robert Hooke in 1675. "*None but the catenaria is the figure of a true and legitimate arch.*" completed Gregory twenty years later, in

ISBN: 978-960-474-403-9

1697. These quotes describe the mechanics of the arch in a brief, but precise way. Fig.8 shows a simple example used by Heyman to explain this concept: a weightless string subjected to three forces. The funicular polygon inverted represents the thrust line.

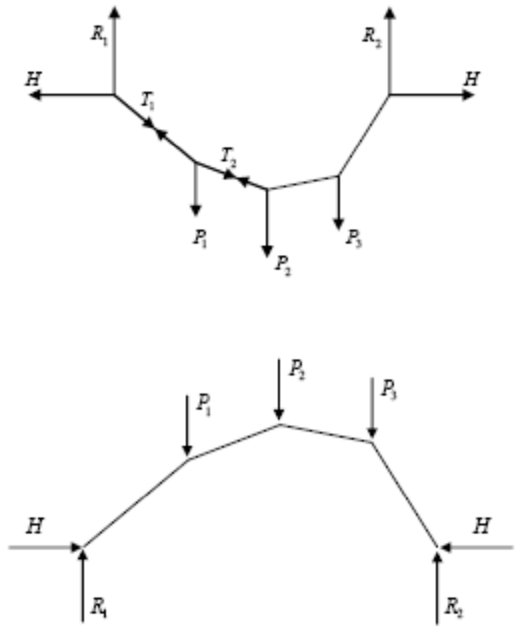

Fig.8 Inverted funicular polygon and the Thrust Line

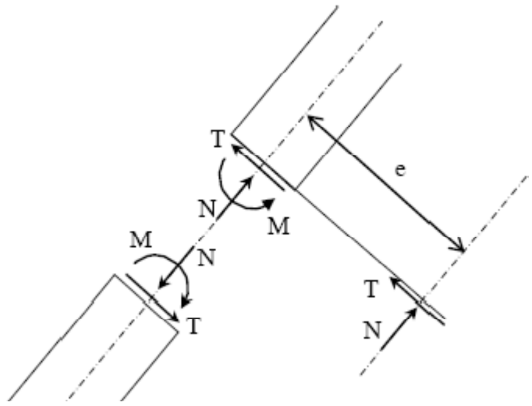

Fig.9 The eccentricity e

The thrust line may be located at the middle. The thrust line may be located at the middle of the section or very close to the edge. It depends from the resultant of inertial forces in a given cross section. If no moment and transverse force occur into the arch, the thrust line coincides with the centre-line of the section. In the other cases, the thrust line departs from the arch centre-line and so it is important to define the distance between the thrust line and the center of the mass, i.e. the eccentricity *e* (Fig.9).

The thrust line method analyzes the location and the slope of the thrust line inside the cross section through two parameters. The first one is the eccentricity of the forces resultant that describes the location of the thrust line in the cross section. The eccentricity is easy to calculate because it is a function of the normal force *N* and the bending moment *M* acting in the considered cross-section. The second important parameter is the relation between normal force *N* and shear force *T* that defines the slope of the thrust line.

Calculation of thrust line location can be performed using the equilibrium equation or by solving a linear programming problem.

So every thrust line is a possible equilibrium solution. Unfortunately the masonry arch is not a statically determinate structure and this solution is not unique. There are infinite possible lines of thrust. The equilibrium equations are not sufficient to obtain the inner forces.

The thrust line analysis method defines the load carrying capacity by limiting the zone where the resultant force can be positioned. This method presents some variants which differ from each other by the size of the limits. The limits depend on the theory and the material model assumed. The main approaches will be described below.

The first variant of this method is also the most ancient. The Middle Third Rule was anticipated by Thomas Young in 1817, worked out by Claude-Louis Navier in 1826 and applied to masonry arch by William Rankine in 1858. This rule states that the thrust line must lay within the middle third of the cross section that is it must lie within the kern to avoid any tensile stresses.

This criterion is based on the elastic theory. Until the forces resultant remains within the kern, there are only compressive stresses. When the force passes the middle third, the section undergoes also tensile stresses (Fig.9). However it is assumed that the masonry has not tensile strength, so in this case the section is not contributing entirely. Cracks may occur and this is wanted to avoid.

The middle third rule is an extremely safe approach for the determination of the collapse load. It is very difficult to satisfy because of this rigorous limit. It can be reach only: i) if it is considered in the design phase; ii) if the dead loads dominate considerably over live loads.

The difficulty to satisfy the previous criteria has led to apply a less conservative version of this method that is the middle half rule. This approach increases

ISBN: 978-960-474-403-9

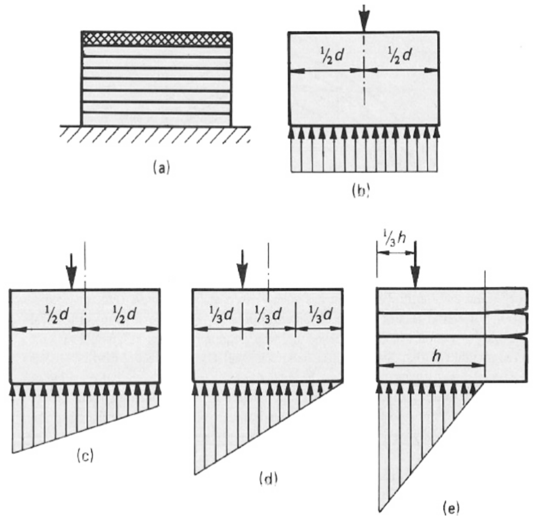

Fig.9 A pile of stone subjected to a compressive moving force.

the limits for the thrust line. In this case, the thrust line should lie within the central half of the arch section (Fig.10).

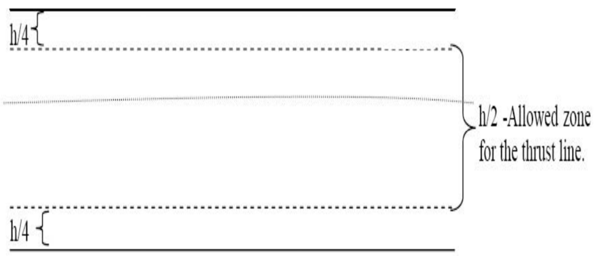

Fig.10 Middle Half Rule

Another variant of the thrust analysis method is proposed by Jacques Heyman. By employing *the safe theorem*, he assumes that an arch is safe simply if a thrust line can be drawn inside his thickness. An arch will collapse only if the thrust line reaches the arch edge at least in four points, converting the arch into a mechanism. This rule is surely the less conservative than the others because the whole cross section becomes the allowed zone for the thrust line. This approach includes an important assumption concerning the masonry behavior. Infinite compression strength is attributed to the masonry material. This enables the thrust line to stay at the edge of the cross section. The assumption is not realistic, but this method can be considered a good method to use because in the majority of the masonry arch bridges the stress level is quite low respect to the masonry compressive strength.
All the variant of the thrust analysis method can be summarized by the Heyman's concept of "geometric safety factor". For example the masonry arches that satisfy the middle third rule have a geometric safety factor equal to three.

3.1.1 Computer Based Application: Archie-M

Thrust line analysis together with Heyman's safe theorem can be used to elaborate computational strategies for the structural analysis of masonry arch bridges. For example, in 2006 Philip Block [10] developed an interactive computational procedure that uses the thrust lines to clearly visualize the forces within the masonry and to predict possible collapse modes.
The program lets the user to change the arch geometry, analyzing the different locations that can be assumed by the thrust line.
Between the specialized analysis programs based on this method, there is also Archie-M developed by Harvey and OBVIS Ltd12 in 2001. Archie-M is a computer program that analyzes multi-span arch bridges together with supports and backfill. It carries out a form of equilibrium analysis. That is to say it determines whether an arch will remain stable, without first considering how it will deform under load. In fact the software uses the thrust line analysis combined with a thrust zone to model the masonry finite crushing strength. In practice the program is based on the thrust zone analysis method. Calculations are carried out on a static scheme of a three hinges arch. The hinge positions are chosen as the most likely for the given load pattern. The program is easy to use because it shows graphically the position of a potential thrust-line and the formed hinges for any given loading regime. Until the thrust zone is within the cross section of the arch at every point, the structure is safe. When the thrust zone begins to touch the arch edge in a fourth point, a mechanism is created and the collapse state is reached.
Although the aim of Archie-M is to demonstrate whether an arch bridge can withstand a given load or not, the collapse load can be estimate by varying the load value until a sufficient number of hinges is formed.The program provides also the internal forces and the thrust zone position for each arch segment. The live load is distributed through the fill with a sine shape. The backfill is modeled as a continuous body that spreads the load and provides both active and passive soil pressure.

3.2 Mechanism Method

The Mechanism Method is a kinematical method, based on the upper bound approach. This method

ISBN: 978-960-474-403-9

belongs to the plasticity theory and was firstly used for steel structures. Later Heyman has applied it to masonry arch. The term *mechanism* refers to the possibility of structure to move in accordance to internal and external constraints. This Method assumes that a masonry arch becomes a mechanism when at least four plastic hinges open. Many experimental tests confirm this hypothesis. However position of hinges is unknown. First step is to assume the possible position of four hinges. In a simplified analysis with only a concentrated force on the arch, the first three hinges can be assumed to be located under the load and at the springing. It's reasonable to hypothesize hinges A and C on the intrados and hinges B and D on the extrados (Fig.11). The concentrated force W is applied on the arch with no dispersion through the fill. Self weights *Vi* include the weights of the backfill blocks and of the corresponding arch segment. The four unknowns are the reaction forces of the two abutments *H, Va, Vb* and the failure load *W*. The problem can be solved with the moment equilibrium equations at the hinges or with the equations of virtual works. In the first case, four equilibrium equations can be derived around the hinges and solved, giving the four unknowns.In the second case, the structure collapses if the total virtual work for at least one of the mechanisms allowable is positive.In order to find the best mechanism, it is necessary to repeat the analysis for each

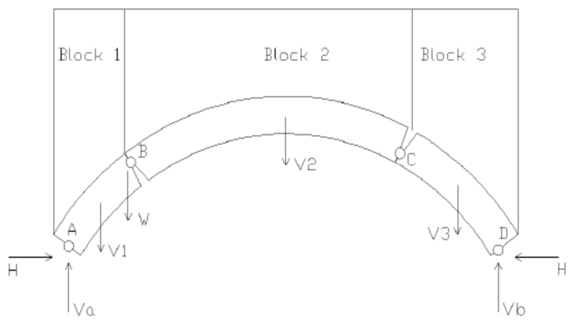

Fig.11 Arch with assumed hinges. Reproduced from Institution of Civil Engineers ICE (2008)

3.2.1 Rigid Blocks

This theory simplifies the masonry arch as an assemblage of plane blocks that are infinitely rigid and have an infinite strength [11].

The division into these blocks is regular, but doesn't respect necessarily the actual number of units of the original arch. Usually the blocks are slightly larger than the physical ones because the mortar joints are not explicitly modeled. The blocks can be also extremely larger than the actual ones in order to reduce the computational effort. In this case it must be careful that the discretization does not affect the expected mode of response. As checked experimentally, the number of blocks to obtain a sufficiently exact solution is about forty.

At the collapse, the blocks can either slide or rotate. The blocks movement can be calculated using the minimal energy for global deformation.

3.2.2 Rigid-Plastic Blocks

An important extension of rigid block analysis has been made by Gilbert [12]. As no real material can sustain infinite compressive stresses, this variant of the mechanism method assumes a finite compressive strength, redefining the failure domain of normal stress and moment . Also in this case, the failures are modeled in the contacts between the blocks, but the explained assumption constrains the hinges not to stay on the arch edges. In this way, the rotation point is brought back inside the arch that behaves as it would have a lower thickness. In the proximity of the hinges, the compressive force is carried by a rectangular stress block lying at the edge of masonry.

The passage to a finite compressive strength complicates the computation. In fact it transforms a linear problem to a non linear one. Gilbert solves the question applying an iterative solution that uses a Linear Programming solver. In this way it is possible to obtain a solution to the global problem and to approximate the constraints as a series of linear constraints. The rigid-plastic block analysis can be considered the basic model for understanding the fundamental behavior of the masonry arches.

3.2.3 Computer Based Application: Ring

The two-dimensional rigid-plastic analysis has been inserted by Gilbert and Melbourne into a software called RING, developed by the University of Sheffield spin-off company, LimitState Ltd. The program is able to analyze multi-span masonry arch bridges, built of arch barrels, supports and backfill. A particular feature of this software is the capacity to analyze multi-ring arches enabling separations between the various rings [13].

The program employs an efficient linear programming technique for the solution of virtual works equations. This mathematical optimization allows identifying the ultimate limit state, determining the percentage of live load that will lead to the collapse. As a result of the analysis, the minimum adequacy factor for live load is obtained, together with a graphic representation of the thrust line and the failure mode. Exact location of hinges is

ISBN: 978-960-474-403-9

indicated. The live load is distributed through a Boussinesq distribution with a maximum spread angle. The passive pressure is the only lateral pressure used.

3.3 Finite Element Method

Another method frequently used to describe the structural behavior of the masonry arch bridges is the Finite Element Method.

It starts from a completely different approach. Adopting different strategies of discretization, as micro-modeling or macro-modeling, the structure can be divided in a series of finite elements. Non linear analysis can be performed, assigning particular constitutive laws to the material. The results include the maximum stress and deformability analysis.

The Finite Element Method represents the most versatile tool for the numerical analysis of structural problems. However in the case of historic masonry, the peculiar nature of material leads to pay particular attention to the application of this method.

3.4 Elasto-Plastic Model

The last method presented in this paper deals with a particular closed-form approach developed by some Belgian researchers in the last years.

This method is based on the fundamental theorems of limit analysis and is used to determine the critical points with a relatively small modeling effort. To assure the stability of the masonry arch bridges, a model based on equilibrium equations and compatibility conditions is first developed. Next, the material properties are added to determine the formation of the hinges.

4 Conclusion

The methods for assessing historical masonry arches are mainly three: i) the Thrust Line Analysis Method; ii) the Mechanism Method; iii) the Finite Element Methods. The Thrust Line Analysis Method and the Mechanism Method are analytical methods and derived from two of the fundamental theorems of the Plastic Analysis, while the Finite Element Method is a numerical method, that uses different strategies of discretization to analyze the structure.

In the future, the next analysis step will be the comparison of the results obtained by these three methods applied to a case study.

Acknowledgements

This research has been supported by University of Bologna, Italy.

References:

[1] Benvenuto E. (1981**),** *La scienza delle costruzioni e il suo sviluppo storico*, Sansoni, Firenze

[2] De La Hire P. (1730), *Traitè de Mècanique*, Acts of Acadèmie des Sciences, Paris

[3] Pippard A.J.S., Ashby R.J. (1939**),** *An experimental study of the voussoir arch*, Inst. Civ.Eng., 10

[4] Pippard A.J.S. (1948**),** *The approximate estimation of safe loads on masonry bridges*, Civil engineer in war, 1, 365, Inst. Civ. Eng.

[5] Kooharian A. (1953), *Limit analysis of voussoir (segmental) and concrete arches*, Proc. Am.Concr. Inst., Vol. 49

[6] Onat E.T. and Prager W, (1953), *Limit Analysis of Arches*, Journal of Mechanics and Physics of Solids, Volume 1.

[7] Heyman J. (1966), *The stone skeleton. Structural Engineering of Masonry Architecture,* University of Cambridge, Cambridge

[8] Hendry A.W., Davies S.R. and Royles R. (1985**)**, *Test on a Stone, Masonry Arch at Bridgemill-Girvan,* Transport and Road Research Lab, Contractor Report 7, UK

[9] Horne M.R. (1979), *Plastic theory of structures*, 2nd edition, Oxford: Pergamon Press

[10] Block P., Ciblac, T. and Ochsendorf, J. (2006), *Real-time limit analysis of vaulted masonry buildings*, Computers & Structures, 84 (29-30), pp. 1841-1852.

[11] Livesley R.K. (1978), *Limit analysis of structures formed from rigid blocks*, International Journal for Numerical Method in Engeneering, 12, pp. 1853-1871.

[12] Gilbert M. (2007**).** *Limit Analysis Applied to Masonry Arch Bridges: State of the art and Recent Developments.* Paper presented at the ARCH'07 - 5th International Conference on Arch Bridges.

[13] Nobile L., Bartolomeo V.,Bonagura M.(2012), Structural Analysis of Historic Masonry Arch Bridges: Case Study of Clemente Bridge on Savio River, Key Engineering Materials, 488-489 (2012) pp 674-677.