Abstract: Nowadays, the real plastic zone shape and size at the crack tip cannot be described using the linear elastic fracture mechanics theory (LEFM). In fact, one of the basic principles of the LEFM theory is to consider the Plastic Zone Size (PZS) at the crack tip as negligible with respect to the crack length. Moreover, since the Plastic Zone Size (PZS) strictly depends on many variables, an exact analytical solution, such as to take into account all of these parameters is not available. Therefore, numerical simulations analyses are mandatory.

Within this work, an extensive numerical analysis, based on elastic-plastic fracture mechanics theory (EPFM), has been developed in order to study the plastic zone size both at the tip of a trough crack and at the tip of a pre-crack at the notch edge under MODE I loading condition. In particular, in this work, a parametric 3D finite element model has been carried out in order to show the influence of the crack size and of the component thickness on PZS.

Key-Words: EPFM, Large Scale Yielding, Plastic radius, Crack tip, Pre-cracked notch.

1 Introduction

The behaviour of damaged structures is usually studied through the Linear Elastic Fracture Mechanics (LEFM), which considers only plane stress-strain states at the crack front. The main advantage of two-dimensional theories is their analytical simplicity compared to the three-dimensional ones, but for Large Scale Yielding (LSY) phenomena they aren’t able to overcome some limits in describing the actual behaviour of the
material around the damage [1]. Therefore, one of the basic principles of the LEFM theory is to consider the Plastic Zone Size (PZS) at the crack tip as negligible with respect to the crack length, i.e. Small Scale Yielding (SSY) condition [2]. For this reason, the LEFM theory can’t describe the behaviour of “short cracks”, where the state of stress at the tip is generally characterized by a Large Scale Yielding (LSY) and then by high ratios of PZS to the crack length [1, 3, 4]. Several numerical and experimental investigations [5-9] have shown that such ratio is larger for short cracks than for long ones, for a given nominal Stress Intensity Factor (SIF) [10-13].

The difficulties encountered to describe the stress-strain state at the crack tip through the parameter of LEFM theory is leading to consider the Elastic-Plastic Fracture Mechanics (EPFM) theory’s parameters [14, 15], as the CTOD (Crack Tip Opening Displacement), the CTOA (Crack Tip Opening Angle), the COD (Crack Opening Displacement) and the J-integral [16, 17]. However, since PZS strictly depends on many variables (the material yield stress $\sigma_y$, the applied remote load $\sigma$, the crack size $a$ and the component thickness $t$) [18, 19], an analytical formulation for PZS such as to take into account all these parameters is not yet available [20]. For this reason, finite element analysis is mandatory.

The main scope of the study reported in the present paper is to describe the plastic zone size, which takes place around the crack tip under Mode I loading condition, by using the EPFM theory and a parametric 3D finite element model [21-24]. In particular, within this paper the structural behaviour of two different plates have been presented. The first one, characterized by a pre-cracked circular notch; the second one, characterized by a trough crack (without hole).

<table>
<thead>
<tr>
<th>W [mm]</th>
<th>L [mm]</th>
<th>a [mm]</th>
<th>t [mm]</th>
<th>R [mm]</th>
<th>$\sigma$ [MPa]</th>
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<tr>
<td>50</td>
<td>100</td>
<td>2.6/5</td>
<td>0.5+5</td>
<td>2.5</td>
<td>0÷352</td>
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2 Problem Formulation
Please, A plate with a trough crack in the middle transverse section and a plate with a pre-cracked circular notch, both subjected to a remote longitudinal stress (Mode I), whose value spans the range 1÷352 N/mm², have been numerically modeled (Fig. 1). A parametric analysis has been performed, whose allowed ranges of the considered geometrical and physical parameters are illustrated in Tab. 1.

Figure 2: Material properties.
The material properties have been assumed non-linear (Fig. 2); elastic-plastic analyses of the model have been performed by using Abaqus® ver. 6.11 code.

The FE models of both plate configurations, shown in Fig. 3, have been built with a number of nodes between 64826 and 180855 and a number of elements between 14520 and 42090, depending on the values assumed by the geometrical parameters. Symmetry conditions have been used for an efficient computation and therefore a quarter symmetric model has been adopted. The reduced integration 20-nodes brick elements (element type C3D20R by the Abaqus® elements' library) have been used. In all models, the element sizes have been kept accurately small to match with those necessary at the crack tip to reach the required resolution of the stress field (minimum average element length is about $1E-04$ mm).

A number of elements between 20 and 30 have been considered along the thickness, depending on the values assumed by the geometrical parameters, to resolve consistently the out of plane stress gradient.

![Figure 3: FE model detail at crack tip: trough crack (left); pre-cracked notch (right).](image)

![Figure 4: $r_p$ (a=5 mm $t=0,5\div5$).](image)

2 Analysis of results and conclusions

As matter of the fact, the plastic radius ($r_p$), i.e. the plastic zone size on the crack plane in the middle plane of the plate, has been evaluated through the...
von Mises yield criterion, by considering the distance from the crack front at which the von Mises stress, $\sigma_{vm}$, reaches the value of the material yielding stress, $\sigma_y = 503.15$ MPa.

For fixed thickness values ($t = 0.5, 1, 2.5$ and $5$ mm) and crack size ($a$) equals to $5$ mm (circular notch radius $R=2.5$ mm), the evolution of plastic radius obtained by both numerical models have been shown as function of applied load ($\sigma$) and correlated

Figure 5: $J$ ($a=5$ mm $t=0.5\div5$).

Figure 6: $r_p$ ($a=2.6$ mm $t=0.5\div5$).
between themselves (Fig. 4). In addition, the evolutions of J-Integrals have been correlated (Fig. 5).

For similar crack dimension and hole radius values, the curves for both cracked plate configurations are very close. Specifically, in accord to the graphs above, the plastic radius dimension and the J-Integrals values, related to the plate with pre-cracked notch, are slightly higher than those related to the plate without hole. It is more evident for higher values of stress and depends on the notch effect caused by the hole. However, for fixed hole radius size (R=2.5 mm), with shorter crack length values, for example a=2.6 mm, the aforementioned curves, obtained by both plates, are not in agreement (Fig. 6 for plastic radius evolution and from 7 for J-integrals evolution). It is likely to be caused by the difficulty to use the EPFM theory for describing the real plastic zone shape and size at the tip of a short-crack.

References:


