Topology Optimization of Multi-material with Heat Conduction Problem

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Abstract: This study presents a numerical approach of topology optimization with multiple materials for the heat conduction problem. The multiphase level set model is used to implicitly describe the geometric boundaries of material regions with different conductivities. Finally, the proposed algorithm provides a general framework to extend the traditional binary phase topology optimization solvers for the solution of multiphase topology optimization of the heat conduction problems.

Key-Words: Topology optimization, Multi-material, Heat conduction

1 Introduction

For the heat conduction problem, Cho et al. (1969) developed a topology optimization approach using the level set model. The minimization of thermal energy was taken as the optimization model. They derived the weak form of the governing equation for the steady heat conduction problem. They made use of the Fréchet derivative of the weak form and introduced the adjoint variable to obtain the optimality conditions. Li et al. (1999) developed topology optimization for the steady heat conduction problem by the evolutionary structural optimization (ESO) method.

In the present work, a proposed method will be showed by MATLAB-based (Sigmund, 2001; Lee et al., 2014) computationally evaluating optimal topology and shape of heat conduction problem with multi-material. The advantages of this method are providing a general framework to solve the multiphase problems.

2 Governing equation

Following Zhou and Wang (2007), the SIMP modified version of linear interpolation is used within thermal conductivity tensor K:

\[ K(\alpha) = \sum_{i=1}^{n} \alpha_i K_i \]  

where \( K_i \) is the conductivity tensor corresponding to the phase i-th. For a given material distribution \( \alpha \), the PDE operator \( E \) could be expressed as follow:

\[ \nabla \cdot (K(\alpha)\nabla u) = f(x) \text{ in } \Omega \]  
\[ u(x) = u(x) \text{ on } \Gamma_u \]  

where \( u \) denotes the temperature field which is analog to the state variable \( U \) in our abstract formulation, \( f \) denotes the thermal heat source, \( u \) denotes the prescribed temperature on boundaries \( \Gamma_u \), \( q \) denotes the prescribed heat flux on \( \Gamma_q \) and \( \partial \Omega = \Gamma_u \cup \Gamma_q \). The corresponding objective functional is defined as follows:

\[ T(\alpha, u) = \frac{1}{2} \int_{\Omega} K(\alpha)\nabla u \cdot \nabla u dx \]  

3 Numerical applications

Figure 1 shows design conditions and space adapted from Bendsoe and Sigmund (2003).

Table 1 describes design parameters of three cases. As can be seen in Fig. 2, Case 1, 2, and 3 are closed to analytical solution (Bendsoe and Sigmund, 2003).
Table 1. Design parameters

<table>
<thead>
<tr>
<th>Problem</th>
<th>(n_x)</th>
<th>(n_y)</th>
<th>(\rho)</th>
<th>(e)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>50</td>
<td>50</td>
<td>3</td>
<td>([4\ 2\ 1])</td>
<td>([0.2\ 0.5\ 0.3])</td>
</tr>
<tr>
<td>Case 2</td>
<td>50</td>
<td>50</td>
<td>4</td>
<td>([6\ 4\ 2\ 1])</td>
<td>([0.2\ 0.3\ 0.2\ 0.3])</td>
</tr>
<tr>
<td>Case 3</td>
<td>50</td>
<td>50</td>
<td>5</td>
<td>([8\ 6\ 4\ 2])</td>
<td>([0.2\ 0.2\ 0.2\ 0.2\ 0.2])</td>
</tr>
</tbody>
</table>

Fig. 2 Optimal results of heat conduction

(a) Case 1                             (b) Case 2

(c) Case 3              (d) Analytical solution

4 Conclusions

In this article, the topology optimization of multiple materials for the heat conduction problem has been discussed. The multiphase level set model is used to represent the multiple material regions with complex geometric shape and topology. The accuracy and the efficiency of computation can be further improved by finer finite element model and the effective numerical technique for solving the level set equation.

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