Uncertainty costs on an international duopoly with tariffs

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Abstract: We consider two firms located in different countries selling the same homogeneous good in both countries. In each country there is a tariff on imports of the good produced in the other country. We show that the expected welfare of the countries increase with the variances of the production costs of both firms.

Key–Words: Game Theory; International Duopoly, Uncertainty

1 Introduction

We consider a usual duopoly international trade model with incomplete information, where there are two countries and a firm in each country that sells in its own country and exports to the other one (see [6, 10, 13, 16]).

The international trade model has two stages: first, the governments simultaneously choose tariff rates; secondly, the firms observe the tariff rates and simultaneously choose quantities for home consumption and for export. The decision of the governments to impose or not a tariff can be interpreted as a game where the utilities are the expected welfares of the countries. We show that under some appropriate conditions this game is like the Prisoner's Dilemma (see [13]).

2 International duopoly model

In this section, we introduce the relevant economic quantities of the international duopoly model.

The *home consumption* h_i is the quantity produced by the firm F_i and consumed in its own country X_i. The *export* e_i is the quantity produced by the firm F_i and consumed in the country X_j of the other firm F_j, where $i, j \in \{1, 2\}$ with $i \neq j$. The *tariff rate* t_i is determined by the government of country X_i on the import quantity e_j.

The *aggregate quantity* Q_i sold on the market in the country X_i is

$$Q_i \equiv Q_i(h_i, e_j) = h_i + e_j \ .$$

The *inverse demand* p_i in the country X_i is

$$p_i \equiv p_i(h_i, e_j) = \alpha - Q_i \ ,$$

where $\alpha \geq 0$ is the *demand intercept*.
The *payoff* π_i of firm F_i is

$$\begin{aligned}\pi_i &\equiv \pi_i(h_i, e_i, h_j, e_j; t_j)\\ &= (p_i - c_i)h_i + (p_j - c_i)e_i - t_j e_i \ ,\end{aligned}$$

where $c_i \geq 0$ is the firm F_i's *unitary production cost*. The *custom revenue* CR_i of the country X_i is given by

$$CR_i \equiv CR_i(e_j; t_i) = t_i e_j \ .$$

The *consumer surplus* CS_i in the country X_i is given by

$$CS_i \equiv CS_i(h_i, e_j) = \frac{1}{2}Q_i^2 \ .$$

The *welfare* W_i of the country X_i is

$$W_i \equiv W_i(h_i, e_i, h_j, e_j; t_i, t_j) = CR_i + CS_i + \pi_i \ .$$

3 Costs uncertainty

In this section, for every pair of tariffs, we compute the home and export quantities practiced by both firms at the Bayesian-Nash equilibrium for the second stage game.

We suppose that each firm has two different technologies L and H and uses one of them according to a certain probability distribution. The use of one or the other technology affects the unitary production cost

$$c_i : \{L, H\} \to \mathbb{R}_0^+ \ ,$$

where $c_{i,L} < c_{i,H}$ for $i \in \{1, 2\}$. For $k \in \{L, H\}$ and $i \in \{1, 2\}$, let $\theta_{i,k}$ be the probability of the firm F_i to use technology k. Hence, $\theta_{i,H} \geq 0$, $\theta_{i,L} \geq 0$ and $\theta_{i,H} + \theta_{i,L} = 1$.

The home quantity h_i and the export quantity e_i of firm F_i are random variables

$$h_i : \{L, H\} \to \mathbb{R}_0^+ \qquad \text{and} \qquad e_i : \{L, H\} \to \mathbb{R}_0^+ \,.$$

The *ex-ante profit* $\pi_i^A : \{L, H\} \to \mathbb{R}_0^+$ of firm F_i is

$$\begin{aligned}
&\pi_i^A(h_i, e_i, h_j, e_j; t_j)(k_i) \\
&\quad = E_j\Big(\pi_i(h_i, e_i, h_j, e_j; t_j)\Big)(k_i) \\
&\quad = \textstyle\sum_{k_j \in \{H,L\}} \pi_i(h_i(k_i), e_i(k_i), h_j(k_j), e_j(k_j); t_j) \,.
\end{aligned}$$

Let $E_i \equiv E(c_i)$ be the expected cost of the firm F_i. The *cost difference* $\Delta_i : \{L, H\} \to \mathbb{R}$ is

$$\Delta_i(k_i) = c_i(k_i) - E_i \,.$$

The *complete maximal tariff* of the government of state X_i is denoted by

$$T_i^E \equiv T_i(E_i, E_j) = \frac{\alpha + E(c_i) - 2E(c_j)}{2} \,.$$

The *incomplete maximal tariff* $\bar{T}_i$ of the government of state X_i is

$$\bar{T}_i = T_i^E - \frac{3}{4}\Delta_i(H) \,.$$

Assumption 1: For all $i, j \in \{1, 2\}$ with $i \neq j$, we have $\bar{T}_i > 0$ and

$$0 \leq t_i \leq \bar{T}_i \,.$$

The Bayesian-Nash equilibrium of the second stage game is determined by the home quantities and the export quantities that maximize the ex-ante profit of both firms.

Theorem 1 *Let $(t_i, t_j) \in [0, \bar{T}_i] \times [0, \bar{T}_j]$. Under assumption 1, the Bayesian-Nash equilibrium of the second stage game for the home consumption $h_i^B(t_i) : \{L, H\} \to \mathbb{R}_0^+$ is*

$$h_i^B(k_i; t_i) = \frac{1}{3}\big(2T_j^E + t_i\big) - \frac{1}{2}\Delta_i(k_i) \,;$$

and for the export quantity $e_i^B(t_j) : \{L, H\} \to \mathbb{R}_0^+$ is

$$e_i^B(k_i; t_j) = \frac{2}{3}\big(T_j^E - t_j\big) - \frac{1}{2}\Delta_i(k_i) \,,$$

for $i, j \in \{1, 2\}$ with $j \neq i$.

Proof: (see [3]).

From Theorem 1, we obtain the following expected economic quantities. The ex-post Bayesian-Nash profit

$$\pi_i^B(t_i, t_j) : \{L, H\}^2 \to \mathbb{R}$$

is

$$\begin{aligned}
\pi_i^B(k_i, k_j; t_i, t_j) &\equiv \pi_i^B(c_i(k_i), c_j(k_j), E_i, E_j; t_i, t_j) \\
&= \big(p_i^B(k_i; t_i) - c_i(k_i)\big)h_i^B(k_i; t_i) \\
&\quad + \big(p_j^B(k_j; t_j) - c_i(k_i) - t_j\big)e_i^B(k_k; t_j) \\
&= \tfrac{1}{9}\big[(2T_j^E + t_i)^2 + 4(T_j^E - t_j)^2\big] \\
&\quad + \tfrac{4T_j^E + t_i - 2t_j}{6}\,\Delta_j - \tfrac{1}{2}\Delta_i\Delta_j + \tfrac{1}{2}\Delta_i^2 \,.
\end{aligned}$$

Let $V_i \equiv V(c_i)$ be the variance cost. The expected Bayesian-Nash profit is

$$E(\pi_i^B(t_i, t_j)) = \frac{(2T_j^E + t_i)^2 + 4(T_j^E - t_j)^2}{9} + \frac{V_i}{2} \,.$$

The Bayesian-Nash custom revenue

$$CR_i^B(t_i) : \{L, H\} \to \mathbb{R}$$

is

$$CR_i^B(k_j; t_i) = \frac{2t_i(T_i^E - t_i)}{3} - \frac{t_i}{2}\,\Delta_j(k_j) \,.$$

The Bayesian-Nash consumer surplus

$$CS_i^B(t_i) : \{L, H\}^2 \to \mathbb{R}$$

is

$$\begin{aligned}
CS_i^B(k_i, k_j; t_i) &= \tfrac{1}{18}(2T_i^E + 2T_j^E - t_i)^2 \\
&\quad + \big(\tfrac{t_i}{6} - \tfrac{T_i^E + T_j^E}{3}\big)(\Delta_i + \Delta_j) \\
&\quad + \tfrac{\Delta_i\Delta_j}{4} + \tfrac{1}{8}(\Delta_i^2 + \Delta_j^2)
\end{aligned}$$

4 Welfare and the Prisoner's dilemma for tariffs

In this section, we find the subgame perfect equilibrium that it is characterized by the the Bayesian-Nash tariffs that maximize the welfare of the countries using the Bayesian-Nash home and export quantities found

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in the previous section. Then, we show that the decision of the governments to impose or not a tariff can be interpreted as a game that it is like the Prisoner's Dilemma.

The ex-post Bayesian-Nash welfare

$$W_i^B(k_i, k_j; t_i, t_j)$$

of the country X_i is

$$W_i^B(k_i, k_j; t_i, t_j) = \pi_i^B(k_i, k_j; t_i, t_j) + CR_i^B(k_i; t_i) + CS_i^B(k_i, k_j; t_i) \ .$$

The expected Bayesian-Nash welfare $E(W_i^B(t_i, t_j))$ of the country X_i is

$$E(W_i^B(t_i, t_j)) = \frac{2}{9}[(T_i^E + T_j^E)^2 + 4(T_j^E)^2 + (2T_i^E + T_j^E)t_i - 4T_j^E t_j)] - \frac{t_i^2}{2} + \frac{4t_j^2}{9} + \frac{1}{8}(5V_i + V_j) \ .$$

Hence,

$$\frac{\partial E(W_i^B)}{\partial t_i} = \frac{2}{9}(2T_i^E + T_j^E) - t_i \ .$$

and

$$\frac{\partial^2 E(W_i^B)}{\partial t_i^2} = -1 < 0 \ .$$

The subgame perfect equilibrium consists in finding the tariffs that maximize the expected Bayesian-Nash welfare of both countries. Hence, if $2T_j^E < 5T_i^E$ and $2T_i^E < 5T_j^E$ then, the Bayesian-Nash tariffs (t_i^B, t_j^B) are

$$(t_i^B, t_j^B) = \left(\frac{2}{9}(2T_i^E + T_j^E) \ , \ \frac{2}{9}(2T_j^E + T_i^E)\right) \ .$$

Theorem 2 *If* $T_i^E/T_j^E \in [0.64, 1.57]$*, then for the expected Bayesian-Nash welfares of the both counties we have*

$$E\left(W_i^B(t_i^B, 0)\right) > E\left(W_i^B(0, 0)\right) > E\left(W_i^B(t_i^B, t_j^B)\right) > E\left(W_i^B(0, t_j^B)\right) \ .$$

Therefore, the inequalities obtained in Theorem 2 for the expected welfares of both countries show that the decision of the governments to impose or not a tariff can be interpreted as a game that it is like the Prisoner's Dilemma.
Proof: (see [3]).

5 Conclusions

We proved that the expected profit of each firm increases with the variance of its production costs. We showed that the expected welfare of each government increases with the variances of both production costs.

We showed that the decision of the governments to impose or not a tariff can be interpreted as a game where the utilities are the expected welfares of the countries. We show that this game is like the Prisoner's Dilemma because the welfares of the countries are higher in the case where both governments do not impose tariffs than in the case where both governments decide to impose the Bayesian-Nash tariffs.

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