Time Series Model Application to Multiple Seasonality Acoustical Noise Levels Data Set

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Abstract: - The monitoring of polluting agents and the prediction of their behaviour is a very important issue in urban areas. Inhabitants of those environments are bound to be exposed to air pollution, noise, electromagnetic fields, among other types of hazardous elements. Therefore, with high probability those inhabitants may suffer some damage to their health. Acoustical noise produced by human activities has been largely studied in literature, especially concerning the complexity of predicting its behaviour. The models usually adopted, in fact, are often barely efficient. In this paper, the noise problem is approached by means of time series analysis of a noise levels dataset (sound level meter measurements). The model considered here assumes that measurements form a time series that can be decomposed into three parts: trend (long term behaviour), seasonality (periodic component) and irregularity (random variations). Observing the data set, a multiple seasonality is evidenced. In particular, a short range (weekly) and a long term (125 days, i.e., about 4 months) periodicities are highlighted and implemented in the seasonal component, validating the resulting model on a 44 days dataset.

Key-Words: - Time Series, Acoustics, Noise Control, Predictive Model.

1 Introduction

Population of large urban areas have their health often affected by several adverse effects caused by various forms of pollution [1-3]. The constant monitoring of these pollutants is generally expensive and not always easy to be implemented [4]. In addition, mitigation actions on the sources are usually consequential to periods in which the level of pollution has been particularly high and therefore has already affected the citizens’ health. These considerations point out the need to implement predictive models that can provide a reliable assessment of pollution levels (see for instance [5-28]). These models can induce possible mitigation measures, acting also on the sources, before the pollution begins to affect the population.

In this paper, the focus is on the prediction of acoustical noise in urban areas, that is mainly generated by anthropogenic activities, in particular vehicular traffic and other transport infrastructures.

Most of the existing forecasting models used to estimate pollution levels are based on the study of correlations or causal effects derived from the sources. However, regarding the acoustical noise, it is very difficult to predict the effects in a limited area by studying only the sources because of the nature of the physical phenomenon. That can be heavily influenced either by the architecture of the area where measurements are taken or by other environmental interferences highly variable over time.

The forecasting model considered here is based on Time Series Analysis (TSA) [29-32], applied to sound level measurements. The model can predict the evolution of noise levels for a certain time interval, in a specific area of interest, i.e. the area, in which the data used for the estimation of the parameters (tuning) of the forecasting function, have been collected. This function is known in its general form, but it can be adapted to the specific data to get more accurate forecasted values. In particular, the simplest functional forms of the model can be used with success when there are few measurements available to estimate the parameters. Moreover, if a
large tuning data set is available, it is possible to implement more complex models to reduce the forecast error.

In particular, in this study a set of noise measurements recorded at night in the city of Messina, in South Italy, is used. These data consist of equivalent sound pressure levels ($L_{A,eq}$), averaged on the eight night hours (from 10pm to 6am), and they are defined as follows:

$$L_{A,eq,T} = 10 \log \left[ \frac{1}{\sum_{t}^{n}} \left( \frac{L_{A,eq,i}}{10} \ t_i \right) \right], \quad (1)$$

where $T_i$ is the exposition time (from 10pm to 6am), $t_i$ is the single period of the series, i.e. the single day, $L_{A,eq,i}$ is the equivalent level measured in the $i$th period. The “A” index means that the A-weighting curve has been applied to the data, as required by European regulations [33].

The first step consists in constructing a simple but useful model working on the first 321 measurements available. Then, with this model, a 26 missing measurements are filled with forecasted values. That is made in order to have a time series without missing values. This series of 500 data may be used to produce a more sophisticated model. Finally, in order to validate the model, a comparison between actual and forecasted data is performed on the last 44 available measurements. Let us call attention to the fact that the measurements used in the validation have not been used to estimate the parameters. A detailed description on this kind of approach and on the sensibility of the model to the tuning data set, is reported in [34].

2 Methods

The procedure adopted to build the model is based on general Time Series Analysis (TSA) approach. This procedure is used in several research areas, such as Economics, Physics, Engineering, Mathematics, among others (see for instance [35-37]).

TSA models reproduce the slope, as a function of time, of a given data series and may be used to predict its values on a certain future time interval. The width of the prediction interval depends on the reliability of the model and on the variability of the series.

The basic assumption of these models is that a time series may be decomposed into three parts: a trend and a seasonality, that are predictable, and an irregular component, not foreseeable, which generates the prediction error:

$$A_t = F_t + e_t \quad , \quad (2)$$

where $F_t$ represents the forecasted value at a certain time $t$, and $e_t$ is the irregular component.

The ways these parts are composed, for instance by multiplying or adding the components, represent the different types of models. In this paper, the multiplicative approach has been pursued, resulting in the following formula:

$$F_t = T_t \ S_i \quad , \quad (3)$$

where $F_t$ represents the point forecast, $T_t$ the trend (with $t$ varying over the total number of periods) and $S_i$ the seasonal effect (with $i$ varying from 1 to $k$) at a given time $t$, averaged on the $i$th periods. In particular, for a given $t$, if $t<k$, the value $i$ is the remainder of the ratio between $t$ and $k$; if $t=k$, then $i=k$; if $t>k$, the value $i$ is the remainder of the ratio between $t$ and $k$.

The trend component can be evaluated by means of regression techniques, for instance linear regression on the actual data or, after having removed the seasonality by moving average method. In this paper, a linear regression on actual data has been used to calculate the trend. The width of the interval on which the centred moving average is evaluated, depends on the periodicity of the data, also known as lag. This lag is strongly related to the phenomenon under study and its features. In some cases, a multiple periodicity can be highlighted.

In the following sections, it is shown how a TSA model performance improves when a multiple lag is detected and implemented to calculate the forecasted values.

The seasonality is evaluated as the mean, calculated on all the homologous periods, of the ratio between the actual value and the centred moving average in a given period $t$.

If more than one periodicity is detected, the forecast is affected by another component of seasonality:

$$F_t = T_t \ S_{1,i} \ S_{2,j} \quad . \quad (4)$$

In order to remove the effects of short period seasonality from the data, a centred moving average with width $k_i$ (first lag detected) can be used. Then, it is possible to evaluate the recurring effect on the single day by the ratio between the actual data at time $t$ and the centred moving average at the same $t$.

Finally, evaluating the mean of these effects $S_{1,i}$, on $m_{1,i}$ homologous periods, the seasonal coefficient $S_{1,i}$ is obtained, i.e., for
where \( M_{(k_1)t} \) is the centred moving average with width \( k_1 \), at the period \( t \).

At this point, it is possible to clean up the values of the first moving average from the effect of the second seasonality with lag \( k_2 \). That is done using a second centred moving average process, with width \( k_2 \) (second lag detected). As in the previous step, the effect of the second seasonality for each period \( (S_{2,t}) \) can be calculated, and a second seasonal coefficient can be evaluated with a mean on \( m_{2,j} \) homologous periods:

\[
S_{2,t} = \frac{M_{(k_1)t}}{M_{(k_2)t}},
\]

\[
\bar{S}_{2,j} = \frac{\sum_{i=0}^{m_{2,j}-1} S_{2,i+t+k_2}}{m_{2,j}},
\]

where \( M_{(k_2)t} \) is the centred moving average with width \( k_2 \), at the period \( t \).

Having assumed the presence of an irregular component, indicated by \( e_t \), its evaluation is given by the difference between actual data and point forecast:

\[
e_t = A_t - F_t.
\]

This procedure is possible when the actual data are available, thus it may be performed in the calibration phase. Once the “error” distribution is obtained, its mean \( (m_e) \) can be used in the final forecast of the model and the standard deviation can be related to the width of a prediction interval ([32], [34]). Thus the point forecast can be evaluated improving formula (4) as follows:

\[
F_t = T_t \bar{S}_{1,t} \bar{S}_{2,j} + m_e.
\]

A validation process may be performed, comparing actual data with model forecasted values, in a data range not used in the calibration phase.

To evaluate the effectiveness of the model is useful to implement a statistical analysis of the errors. This test is performed both in the calibration phase described above and in the validation process. A relevant goal, in order to optimize the model, is to minimize both the absolute value of the mean and the standard deviation of the error distribution.

2.1 Detection of the presence of a lag

In order to detect the presence of a periodicity in the series, the Ljung-Box (LB) or the Box-Pierce (BP) test can be adopted ([38], [39]). These tests verify if the data have an autocorrelation and they may exclude the presence of fully random data fluctuations. Both tests adopt the autocorrelation coefficient that may be evaluated according to the following formula:

\[
r(k) = \frac{\sum_{i=1}^{n-k}(x_i-x)(x_{i+k}-x)}{\sum_{i=1}^{n}(x_i-x)^2},
\]

where \( x_i \) is the data in each period, \( \bar{x} \) is the mean of all the data, \( n \) is the total number of periods, \( k \) is the lag hypothesis under test. Using this coefficient, the LB test can be performed according to the following formula:

\[
\chi^2_{LB}(h) = n(n + 2) \sum_{k=1}^{h} \frac{r^2(k)}{n-k},
\]

where \( h \) is a chosen integer, related to the number of autocorrelation coefficients under test, which varies according to the assumed lag.

If the null hypothesis is true (absence of autocorrelation), the LB statistics is distributed according to a random variable \( \chi^2 \), with \( h \) degree of freedom.

The BP test, instead, is based on the following formula:

\[
\chi^2_{BP}(h) = n \sum_{k=1}^{h} r^2(k),
\]

where, again, \( n \) is the total number of periods, \( k \) is the assumed lag and \( h \) is a chosen integer, related to the number of autocorrelation coefficients under test. The two tests differ only in the different weighting systems adopted, but asymptotically converge to the same distribution.

2.2 Selection of the lag coefficient

Once the presence of a periodicity is detected, the choice of the lag may be performed according to the maximum data autocorrelation coefficient.

A very useful tool to detect the periodicity and to evaluate the autocorrelation as a function of the lag, is the autocorrelation plot, also called correlogram. This plot reports the \( k \) value on the horizontal axis and the correspondent autocorrelation coefficient on the vertical axis. In this paper, since the correlogram has been plotted in the “R” software framework, the autocorrelation coefficients are evaluated according to formula (11).
Let us remind that formula (11) adopts an unique mean calculated on the whole range of data. It may be useful, however, when the time series has not a constant mean, to adopt the following formula:

\[
\rho(k) = \frac{\sum_{t=1}^{n-k}(x(t−x_{s1}))(x_{s2}−x_{s2})}{\sqrt{\sum_{t=1}^{n-k}(x(t−x_{s1})^2)\sqrt{\sum_{t=1}^{n-k}(x_{s2}−x_{s2})^2}}} ,
\]

where \( x_t \) is the measurement value at time \( t \), \( n \) is the number of periods, \( k \) is the lag and:

\[
\bar{x}_{s1} = \frac{\sum_{t=1}^{n-k}x_t}{n-k} ; \quad \bar{x}_{s2} = \frac{\sum_{t=1}^{n-k}x_{t+k}}{n-k} .
\]

These two means are calculated excluding respectively the first and the last \( k \) periods.

In addition, when the lag is particularly high, another possible approach is to evaluate the correlation coefficient between a subset of data and the same data shifted by \( k \) periods:

\[
r_{xy} = \frac{\sum_{i=1}^{n}(x_i−\bar{x})(y_i−\bar{y})}{(\sum_{i=1}^{n}(x_i−\bar{x})^2)^{1/2}(\sum_{i=1}^{n}(y_i−\bar{y})^2)^{1/2}} ,
\]

where \( y_i \) is equal to \( x_i + k \).

### 2.3 Error metrics

A measurement of model performance can be obtained by “Mean Percentage Error” (MPE) and “Error Variation Coefficient” (CVE). The first quantitative metric gives a measurement of the error distortion, i.e. MPE is able to describe if the model overestimates or underestimates actual data. CVE considers the variation from the reality in absolute value, in other words it provides the error dispersion. Those metrics are evaluated according to the following formulas:

\[
MPE = \frac{\sum_{i=1}^{n}(\hat{A}_{t}−P_{t})}{A_{t}}100
\]

and

\[
CVE = \frac{\sum_{i=1}^{n}(P_{t})^2}{A_{t}}\frac{1}{n−1}
\]

where \( \hat{A} \) is the mean value of the actual data in the considered time range.

### 3 Data Analysis and Results

The basic data set used in this paper is related to the Messina’s long term field measurements used in [34] and in [40]. The local government of Messina, a city in the South of Italy of about 240000 inhabitants, adopted a continuous monitoring of noise in certain critical areas, in particular in proximity of the commercial dock, where a very high traffic flow and several industrial settlements occur. These data have been made available on a web platform [41].

In [34] the measurements taken during day time in "Viale Boccetta" street, were used. In this paper, the authors adopt the night measurements taken in the same site and, partially, in the same months. In particular, four data sets have been chosen. The first one goes from the 11th of May 2007 to the 26th of March 2008 (321 days). The second one goes from the 27th of March 2008 to the 21st of April 2008 (26 days; these data are missing in the data set). The third data set goes from the 22nd of April 2008 to the 21st of September 2008 (153 days). Finally, the fourth data set, used in the validation phase, goes from the 22nd of September to the 4th of November 2008 (44 days).

The night level is the equivalent level, with A weighting [33], evaluated in the time range \( T \), that goes from 10pm to 6am (8 hours), defined as follows:

\[
L_{Aeq,T} = 10 \log \left[ \frac{1}{T} \int_{0}^{T} \frac{P_{A}(t)}{P_{0}} \, dt \right] .
\]

The first aim of the analysis is to use a forecasting model, tuned on the first series of 321 measurements, to calculate the 26 missing data of the second data set defined above. Then, once the “data hole” has been filled, the same model is tuned on a data set of size 500, composed by the first 321 measurements, plus the 26 reconstructed ones, plus the next 153 measurements. In this way, the last 44 data (from the 501st observed measurement to the 544th) have been left for the validation of the model. The choice of this data set allows the implementation of a multiplicative model with a double seasonal component, that will exploit, in addition to the weekly seasonality \( (k_1 = 7) \), a second “long term” periodicity \( (k_2 = 125) \).

The summary statistics of the entire data set are given in Tab. 1.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>68.77</td>
<td>1.19</td>
<td>69.0</td>
<td>66.0</td>
<td>72.0</td>
</tr>
</tbody>
</table>

### 3.1 Seasonality detection and data set filling

In order to evaluate the presence and the value of the periodicity, the first step was the application of Ljung-Box (LB) and Box-Pierce (BP) tests, defined...
in formulas (12) and (13). These tests highlight the presence of autocorrelation in the data. The tests have been implemented in the “R” software framework and the results are given in Table 2.

<table>
<thead>
<tr>
<th>Type of test</th>
<th>$\chi^2$</th>
<th>$h$</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box</td>
<td>584.749</td>
<td>10</td>
<td>$&lt; 2.2\times 10^{-16}$</td>
</tr>
<tr>
<td>Box-Pierce</td>
<td>1514.589</td>
<td>50</td>
<td>$&lt; 2.2\times 10^{-16}$</td>
</tr>
</tbody>
</table>

The small $p$-value in both tests, i.e. the very small probability to observe the sample if the null hypothesis is true, indicates that the hypothesis of absence of autocorrelation in the data must be rejected.

In a first approach, the periodicity has been investigated in the first 321 days, by means of the autocorrelation plot approach, implemented in “R” software. The correlogram is reported in Fig. 1.

The highest autocorrelation value is obtained for a lag of 7 and its value, calculated by means of formula (11), is 0.79. Thus, it is evident that there is a weekly periodicity in the noise. This result is reasonable, because the data are strongly related to traffic flows, typically increasing during the working days and decreasing during the weekend.

At this point, a first toy model (Single Seasonality Model, SSM) has been implemented considering this lag, using the procedure described in section 2 (see also [34]), i.e. centred moving average for the trend and seasonal coefficient according to a periodicity 7. This model allowed to fill the hole in the data set, from day 322 to 348, making available a data set of 500 measurements. In Fig. 2 is possible to notice how the data hole has been filled and how the model roughly approximate the actual observed time series.

On this larger dataset (500 days), a second seasonality, with a frequency smaller than the previous one, is hidden and can be highlighted. The autocorrelation of the centred moving average values (with lag 7) has been studied by means of correlogram plot (Fig. 3).
the first 50 days and the ones between the 125th and the 175th, has been calculated. The result is 0.55.

In Fig. 4 the auto dispersion plot is reported, considering a lag of 125. The cluster of data along the bisector seems to confirm the presence of autocorrelation in the data.

![Auto dispersion plot](image)

**Fig. 4:** Auto dispersion plot. The moving average with span 7 is plotted as a function of the same moving average considering each data shifted by 125 days.

### 3.2 Double seasonality model design and results

After having established the presence of two seasonal effects, it is possible to remove these periodicities from the data and to evaluate two different seasonal coefficients. Thus, the improved model takes into account the effects of the high frequency seasonality, with a lag of 7 days, but also of the low frequency one, with a lag of 125 days.

In Fig. 5, three curves are reported: the actual data (in black), in red the first moving average (span 7), in blue the second moving average (span 125).

In this figure, it is possible to appreciate the combination of the two centred moving averages, that eliminates the double seasonality effects.

The first moving average curve (red curve) highlights the presence of four peaks and valleys. This is an empirical confirmation of the presence of a seasonality of about 500 measurements over 4 peaks/valleys, which is exactly 125.

In Fig. 6, a comparison between the actual data (black line) and the forecasted values of the Double Seasonality Model (DSM) (red line) is presented.

![Graph of the two centred moving averages combination](image)

**Fig. 5:** Graph of the two centred moving averages combination. In black the actual data, in red the first moving average (span 7), in blue the second moving average (span 125).

![Comparison between the forecasted values, obtained by DSM model, and the 500 calibration data](image)

**Fig. 6:** Comparison between the forecasted values, obtained by DSM model, and the 500 calibration data.

### 3.3 Models validation

Both the SSM and the DSM have been validated on the 44 days data set, from the 501st to the 544th observations. A graphical comparison has been performed in Fig. 7 and Fig. 8, respectively comparing the SSM and the DSM results with actual data.

![Comparison between the forecasted values, obtained by SSM model, and the validation with the actual data](image)

**Fig. 7:** Comparison between the forecasted values, obtained by SSM model, and the validation with the actual data.
Fig. 8: Comparison between the forecasted values obtained by DSM model, and the validation with the actual data.

A quantitative validation analysis of the models performance has been pursued calculating the difference between actual data and forecasts of the SSM and the DSM. In addition, the distortion and dispersion, measured by the MPE and CVE (see section 2), have also been evaluated.

The statistics of the error distribution, reported in Tables 3 and 4, show a relevant improvement in the forecasts obtained with the DSM, with respect to SSM results. The absolute values of the mean error strongly decreases, even if the standard deviation is practically the same. In addition, the DSM error distribution better approximate a normal distribution, considering the decreasing (in absolute value) of skewness and kurtosis.

The MPE and CVE results, reported in Table 5, confirm the better performance of DSM. Recall that the calibration error metrics have been evaluated excluding the 26 data obtained by the application of the first model, and considering only the days in which the actual data were available.

Both the graphical and quantitative comparisons between the models show that the DSM has a better performance on the considered set of data, with respect to the SSM.

<table>
<thead>
<tr>
<th>Model</th>
<th>Dataset</th>
<th>MPE</th>
<th>CVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSM</td>
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<td>0.013</td>
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<tr>
<td>SSM</td>
<td>validation</td>
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<td>DSM</td>
<td>tuning</td>
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<td>0.011</td>
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<tr>
<td>DSM</td>
<td>validation</td>
<td>-0.1</td>
<td>0.012</td>
</tr>
</tbody>
</table>

4 Conclusions

In this paper, the attention was focused on the noise pollution monitoring and prediction problem in urban areas. The statistical analysis of a noise levels dataset, obtained from a measuring station in Messina (South Italy), has been performed. These measurements have been adopted for the implementation and validation of a time series analysis model. The aim was to predict noise level exposure. This method assumes that measurements are the results of the composition of three parts: a long term behaviour (trend), that is function of time and is obtained by smoothing the raw data, a seasonal component (seasonality), that describes the periodicities in the phenomenon, and an irregularity, that is not deterministic, but can be probabilistically evaluated. The adopted model is multiplicative between trend and seasonality, and additive when considering the irregularity.

A first set of 321 data has been considered, and, thanks to the application of statistical tests, the presence of periodic fluctuations has been evidenced. Then, a toy model has been implemented on this dataset, considering a weekly periodicity, highlighted by a strong autocorrelation corresponding to a value 7 for the lag.

With the application of this model, a subset of 26 missing measurements has been filled and a 500 data set has been obtained and analysed. The evaluation of the correlogram on this enlarged dataset confirmed the weekly periodicity ($k_1 = 7$). Once the trend has been evaluated, by means of centred moving average (with span equal to 7), the correlogram has been computed on the moving average dataset and a second periodicity has been evidenced. This time, the periodicity is related to a longer term period ($k_2 = 125$, i.e., about 4 months). Thus, the “Single Seasonality Model” (SSM) has been improved, considering this multiple periodicity evidenced on the entire large dataset, resulting in the “Double Seasonality Model” (DSM). Both SSM and DSM have been validated by comparing their forecasted values with a 44 actual measurement dataset (not used in the calibration phase). These validation data have been also used for a
quantitative comparison between the performance of the two models, by means of error (difference between actual value and forecast) distributions. The Double Seasonality Model showed better performance, in terms of lower standard deviation and closer to zero mean value of the error distribution.

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