Modeling and analysis of general reservoir operation problems by using
the causal feedback loop diagram of system dynamics

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Abstract: In the past, the system diagram and flowchart always serve as the blueprints of a model development for system analysis. However, neither of these two diagrams is enough to provide all the information required for program development, because the system diagram presents the structure of system components and the flowchart describes the computation procedures in the model. With a causal feedback loop diagram of system dynamics, the casual interactive relationships among model variables and parameters can be revealed to elucidate the nature of impact dynamics and feedback, portraying information feedback in a system. Therefore, in this paper, the causal feedback loop diagram of system dynamics was briefly described and the feedback characteristics within a system was identified by using the technique of causal feedback loop diagram through
two case studies of the general reservoir operation problems. According to scenario results, the proposed design of causal feedback loop has the ability to show the causal relationships among variables in the system and effectively analyzes the feedback of system to reservoir operation problems.

**Key-Words:** - Causal feedback loop diagram; System dynamics; System analysis; Reservoir operation; Flowchart; Modeling.

1 Introduction
Reservoirs, the most important hydraulic facilities in a water resources supply system, can have a significant impact on regional water conservation. The conventional system analysis approaches for reservoir operation problems have been applied to simulate, optimize, or choose a compromise alternative solution based on trade-offs between conflicting objectives (e.g. Oliveira and Loucks, 1997; Chang and Yang, 2002; Soncini-Sessa et al., 2003; Reddy and Kumar, 2007; Yang et al., 2007; Mitchell et al., 2008; Chang, 2009; Wang, 2010). In the past, the system diagram and flowchart always serve as the blueprints of a model development for system analysis. However, neither of these two diagrams is enough to provide all the information required for program development, because the system diagram presents the structure of system components and the flowchart describes the computation procedures in the model. Besides, the casual interactive relationships among model variables and parameters are not revealed from these two diagrams such that it is difficult for a programmer to grasp immediately all the contents and relationships merely by viewing both diagrams.

Furthermore, a feedback refers to the situation of X affecting Y, and Y in turn affecting X, perhaps through a chain of causes and effects. If dynamic behavior arises from feedback within the system, it is likely that problems might worsen over time. This is similar to how reservoir levels vary, which can be represented as a problematic trend over time. Finding an effective mode of operation usually requires understanding the system feedback structure. Nevertheless, the characteristic of feedback to reservoir operation problem can’t be showed within the system diagram and flowchart.

A causal feedback loop diagram of system dynamics, initially developed by Jay W. Forrester (Forrester, 1961), provides an understanding of the nature of impact dynamics and feedback. It shows the technique to portray information feedback in a system. Therefore, it is appropriate to design and analyze a causal feedback loop diagram of system dynamics for a reservoir operation problem. *Causal* refers to a cause-and-effect relationship. The presentation of such a relationship consists of variables, abbreviations and arrows. The arrows link variables as shown in Fig.1, wherein the cause-and-effect relationships can be clearly indicated by adding “+” or “-”. A “+” on an arrow connecting two variables indicates a positive correlation, which means as the variable at the tail of the arrow increases, the variable at the head of the arrow also increases. A “-” indicates a negative correlation; as the variable at the tail of the arrow increases, the variable at the head of the arrow decreases.

![Fig.1. Causal relationship between variables.](image-url)
The word feedback loop refers to a closed chain of cause-and-effect and a change in one variable among the loop feeds back to reinforce or slow down the initial change. There are two types of feedback loops. One is called positive, shown in Fig. 2 (Yang and Yeh, 2014), indicated by a “+” sign, if it contains an even number of negative causal links. The other is called negative, shown in Fig. 3, indicated by a “−” sign, if it contains an odd number of negative causal links. Positive causal feedback loops generate growth, amplify deviations, and reinforce change. This behavior in mathematics is called “dispersion”, shown in Fig. 4. Negative causal feedback loops seek balance, equilibrium, and stasis. Also, negative causal feedback loops act to bring the state of the system closer to a goal or desired state. This behavior in mathematics is called “convergence”, shown in Fig. 5 (Sterman, 2000). From above, using the technique of causal feedback loop diagram in reservoir operation problem is easy to present the characteristic of feedback within a system. The details of the causal feedback loop diagram applied to a reservoir operation problem are explained in the following two case studies.
2 Case studies

2.1 Case study one

Shown in the left side of Fig.6, the system diagram of a hypothetical water system is basically comprised of a reservoir ($S$, represented by an inverse-triangle), an inflow into the reservoir ($I$, represented by an arrow) and a water supply from the reservoir ($O$, represented by an arrow.) Within a time period, five operational steps to compute the water supply of the reservoir are formulated in the flowchart, the right side of Fig.6. Meanwhile, those computation steps also illustrate how the variables in Fig.6 change their values during the system operation.

Fig.6. Visual connective arrow between system diagram and flowchart drawing.

Step1. Setting the input data and initial conditions.

Conventionally, setting input data and initial conditions is always the first step in the flowchart. In this case, the inflow of the reservoir at every time period ($I_t$), initial reservoir storage volume at time $t=1$ ($S_1$) and the coefficient of water supply ($C$) are given as input data. Then an unknown decision variable, the state variable and the related variable can be estimated.

Step2. Calculating the variable of available water.

The available water at time $t$ is the sum of reservoir storage and inflow at time $t$.

$$AV_{It} = I_t + S_t \quad t = 1 \ldots n \quad (1)$$

Where, $AV_{It}$ is the available water of system at time $t$; $S_t$ denotes the storage of the reservoir at time $t$; $I_t$ represents the inflow of reservoir at time $t$; and $n$ is the number of simulated periods.

Step3. Determining the water supply decision variable.

The determination of the water supply from the reservoir ($O_t$) is assumed to be a linear function, denoted by equation (2), consisting of the available water supply and the coefficient of water supply. If the value of the coefficient is 0.5, it means that only
half the volume of available water can be released to meet demand at time \( t \); the rest must be stored in the reservoir.

\[
O_t = C \times AVI_t \quad t = 1 \ldots n
\]  

(2)

Step 4. Calculating the reservoir storage state variable after releasing the water supply.

Equation (3) is the transition function of reservoir storage during time interval \([t, t+1]\). This implies that reservoir storage at time \( t+1 \) \((S_{t+1})\) depends on the storage at time \( t \) \((S_t)\), the inflow of reservoir at time \( t \) \((I_t)\), and the water supply of reservoir at time \( t \) \((O_t)\).

\[
S_{t+1} = S_t + I_t - O_t \quad t = 1 \ldots n
\]  

(3)

Step 5. Judging whether the final simulation time period has been reached.

The simulation stops when \( t = n \), otherwise it returns to step 2 and continues the procedures for the next time period.

From Fig.6, the system diagram displays the relation among the components, the structure, in a water supply system, while the flowchart demonstrates the operating/computing procedures of the system. The values of three variables \((I, S, O)\) at every time period in the system diagram must always be defined by the flowchart. However, the interaction between the system diagram and flowchart is not presented within neither of these two diagrams, and those hidden information are usually essential for implementing a program.

For this reason, this study tries to draw several visual connective arrows between the system diagram and flowchart to illustrate their underlying hidden relationships. To set the input data and initial conditions in the first rectangular box of flowchart, two values required for the system diagram, \( I_t \) and \( SI \), are provided. Thence, two visual connective arrows from the first rectangle into the inflow \((I)\) and storage of reservoir \((S)\) of the system diagram were added to show the relationship. To calculate the available water \((AVI_t)\) in the second rectangle of flowchart based on the formula \( AVI_t = I_t + S_t \), the inflow data \( I_t \) at every time step is given directly from the first rectangle of the flowchart. However, only the initial storage volume of reservoir at time \( t = 1 \) \((SI)\) is from the first rectangle; the storage in every subsequent time step, i.e., \( S_t, t > 1 \), has to be specified from the system diagram such that a visual connective arrow displays how \( S_t \) is moved from the system diagram into the flowchart. In the other hand, the water supply decision variable \((O_t)\) in the third rectangle of the flowchart is calculated based on the results from previous rectangles, therefore, a visual connective arrow from the this box into the water supply \((O)\) of the system diagram is utilized to reveal the relationship. To obtain the reservoir storage state variable after releasing the water supply, the value of \( S_{t+1} \) is found through the equation in the fourth rectangular box of the flowchart. Consequently, another visual connective arrow from this fourth box is linked into the storage of reservoir \((S)\) of the system diagram. In conclusion, the design of the five visual connective arrows clearly displays the relationships between the system diagram and the flowchart in Fig.6.

Next, reference to above variables of \( I, S, O, AVI \) and \( C \), the causal feedback loop herein is a negative causal feedback loop, as shown in Fig.7. The loop shows how the storage of the reservoir, the available water for supply, and the actual volume of water supplied all change and affect one another in each simulated time step. When the storage of the reservoir at time \( t \) increases, the available water at time \( t \) also increases. This will cause the water supply at time \( t \) to rise, while the available water at time \( t \) increases. As the water supply increases, the storage of the reservoir at time \( t+1 \) will decrease. The negative sign in the center of this loop means
that the storage of the reservoir will gradually approach steady levels over time, a behavior which in mathematics is called convergence. Furthermore in Fig.7, the inflow and the coefficient of water supply are not internal variables of this loop, thus they are called external variables. They will strongly affect the time spent on and scale of convergence. When the inflow at time \( t \) increases, the available water at time \( t \) and the storage of the reservoir at time \( t+1 \) also increases. As the coefficient of water supply increases, the water supply will increase.

With these components, the simulation model design (also called stock-flow diagram) is identified as a negative feedback loop, as shown in Fig.8, where the rectangle is a stock that represents the storage of water present within a reservoir. The inflow and water supply belong to the object of flows. The purpose of the system diagram is to display physical water flow between components, so the stock and flow of system dynamics objects are applied to rebuild the system diagram. The converters of available water and the coefficient of water supply are the rules of controlling the stocks and flows in the model. This stock-flow diagram displays significantly which variables belong to the variables of system diagram, which variables are generated by the operating/computing consideration of problem solving and links between variables. Compared with Fig.6, the stock-flow diagram is a fusion of a causal feedback loop and system diagram and has the ability to clearly present the structure of a system, the interaction of all system elements and all hidden information required for solving a problem.
models in Vensim are constructed graphically and all related data or functions can directly build within objects.

Fig. 9. Simulation model implemented by Vensim software.

There are two kinds of scenario simulations conducted in this case. The first one is the change of inflow under $C = 0.8$ and $SI = 50$ m$^3$, while the second one is the scale change of the coefficient of water supply under $It = 100$ m$^3$ and $SI = 50$ m$^3$. In the first scenario, reservoir storage levels shown in Fig.10 will stabilize over time irrespective of the scale of inflow. The simulation results in the second scenario are presented in Fig.11, indicating that levels will take longer to level off as $C$ decreases. Reservoir levels will stabilize over time irrespective of $C$. In conclusion, the above scenario results prove that the feedback loop adequately details the characteristics of the system over time, and that the external variables ($It$ and $C$) are not capable of changing the behavior of convergence in a negative feedback loop. Instead, the external variables only strongly affect the time spent on or scale of convergence.

2.2 Case study two

In reality, the determination of the water supply is strongly related to water demand, which should be met to the fullest extent possible. Based on this premise, case study two uses a variable of water demand ($Dt$) instead of the coefficient of water supply ($C$). A modified causal feedback loop is also shown in Fig.12, wherein the volume of available water ($AVIt$) is compared to water demand ($Dt$). If the volume of available water is greater than the volume of water demand ($AVIt > Dt$), this indicates that the reservoir not only has enough water to meet water demand, but also that the rest can be stored in the reservoir. In this situation, the causal relationship between the available water and the water supply is inactive because the volume of water supply is always equal to the volume of water demand whatever the variation of available water ($Ot = Dt$). Therefore, the causal feedback loop in Fig. 12 should be modified as shown in Fig.13.
There is no negative causal feedback loop in Fig.13 due to an inactive link between the available water and the water supply, preventing convergence. Otherwise ($AVI_t \leq D_t$), the maximum water supply of a reservoir is equal to the volume of available water ($O_t = AVI_t$) such that the causal relationship between the available water and the water supply is active. Therefore, a negative causal feedback loop exists and convergence takes place.

From the above, the determination of the water supply from the reservoir ($O_t$) must obey an if-then-else rule as denoted by equation (4),

$$\text{If } AVI_t > D_t \text{ Then } O_t = D_t \text{ Else } O_t = AVI_t \quad t = 1 \ldots n \quad (4)$$

Where $AVI_t$ is the available water of system at time $t$; $D_t$ denotes the water demand at time $t$; $O_t$ is the water supply of a reservoir at time $t$; and $n$ is the number of simulated periods.

Fig.13. Causal feedback loop under $AVI_t > D_t$ in case study two

Next, the model can be designed as presented in Fig.14 with stocks, flows, converters and connectors in reference to the causal feedback loop in Fig.12. The simulation model can then be developed and implemented using Vensim software according to the above model design.

Fig.14. Simulation model design in case study two

There are two kinds of scenario simulations conducted in this case. The first one is the change of inflow under $D_t = 120 \text{ m}^3$ and $S1 = 500 \text{ m}^3$, while the second one is the change of water demand under $I_t = 100 \text{ m}^3$ and $SI = 500 \text{ m}^3$.

In the first scenario, reservoir storage levels shown in Fig.15 will stabilize over time when the volume of inflow is less than the volume of water demand ($I_t \leq D_t$). Given $I_t \leq D_t$, the water deficit between $I_t$ and $D_t$ must be compensated for by using reservoir storage ($S_t$), which inevitably decreases over time whether the initial storage ($SI$) is large or small. Therefore, the condition of $AVI_t \leq D_t$ ($AVI_t$ consists of $I_t$ and $St$) may occur at any one time step during the model simulation. This will result in the volume of water supply in the reservoir equaling the volume of available water ($O_t = AVI_t$) from equation (4). Thus, the causal relationship between the available water and the water supply is active. Then a
negative causal feedback loop, displayed in Fig. 12, exists and also systematically converges. On the other hand, reservoir storage levels shown in Fig. 15 will grow constantly over time when the volume of inflow is larger than the volume of water demand \( (I_t > D_t) \). Given \( I_t > D_t \), the volume of available water \( (AVI_t) \) consists of \( I_t \) and \( S_t \) is always greater than the volume of water demand \( (AVI_t > D_t) \). According to equation (4), the volume of water supply is always equal to the volume of water demand \( (O_t = D_t) \). Next, the condition of \( O_t = D_t \) creates an inactive causal relationship between the available water and the water supply as presented in Fig. 13. Thus, a negative causal feedback loop does not exist such that convergence will not occur. Instead, the surplus water between \( I_t \) and \( D_t \) is stored in the reservoir which explains why reservoir storage levels will grow constantly over time.

![Fig.15. Results of first scenario in case study two \((D_t = 120 \text{ m}^3)\).](image)

Simulation results in the second scenario are presented in Fig. 16, indicating that reservoir storage levels will stabilize over time when the volume of inflow is less than the volume of water demand \( (I_t \leq D_t) \). Fig. 16 also indicates that reservoir storage levels will grow constantly over time when the volume of inflow is larger than the volume of water demand \( (I_t > D_t) \). This explanation for this is the same as in the first scenario.

![Fig.16. Results of second scenario in case study two \((I_t = 100 \text{ m}^3)\).](image)

### 3 Conclusions

Through two case studies, this investigation has achieved its goal of applying causal feedback loop diagram of system dynamics to solve a reservoir operation problem systematically and effectively. It produced the following findings: (1) In case study one, the hidden relationship between the system diagram and flowchart is not presented within these two diagrams, but it is required for implementing a program. It is therefore essential that a programmer comprehends fully the information presented in both the system diagram and flowchart before going on to actual problem solving. (2) According to scenario results, our proposed design of causal feedback loop has the ability to show the causal relationships among variables in the system and analyzes effectively the feedback of system to reservoir operation problems.

In conclusion, it is hard to realize completely the structure and behavior of a system just from a system diagram, flowchart, or the outcomes of a software simulation. A causal feedback loop diagram of system dynamics is one approach that can help decision maker to better grasp the structure and characteristics of a system. It uses a perspective based on information feedback and mutual or recursive causality to elucidate the dynamics of concerned system. Therefore, this study
demonstrates two simple cases that adopt the causal feedback loop diagram of system dynamics to design and analyze reservoir operation problems more easily understood by general water resources engineers. It is also believed that the study can serve as a useful reference to general water resources engineers who would like to apply the causal feedback loop of system dynamics in their works.

References: