Theoretical Estimation and Frequency Performance Analysis of AlGaN/GaN Single Quantum Well Short Wavelength Transistor Laser

BEHZAD HAKKARI¹, HASSAN KAATUZIAN¹*, IMAN TAGHAVI², HASSAN RAHBARDAR MOJAVER³

¹Photonics Research Laboratory, Electrical Engineering Department, Amirkabir University of Technology, Tehran, IRAN
²Electrical and Computer Engineering Department, Kashan University, Isfahan, IRAN
³Electrical and Computer Engineering Department, Concordia University, Montreal, Quebec, CANADA

*Email: hsnkato@aut.ac.ir, (corresponding author), http://www.aut.ac.ir/hsnkato

Abstract, The authors investigate device performances and characteristics of an AlGaN/GaN, single quantum well transistor laser with 336 nm wavelength at room temperature. A charge control model based on coupled rate equations is utilized in order to obtain optical frequency response, carrier and photon lifetimes, threshold current density, optical gain, transparency electron density, the K-factor, current gain, differential quantum efficiencies and output power. Our simulations show that we have a 10.6 current gain and 3.46 mw output for our proposed structure Also we show that a resonance peak of 5 dB with an optical bandwidth of 30 GHz is achievable in the case of 15 nm quantum well width and a cavity length of 500 µm.


1 Introduction

Although the idea of extracting laser emission from a transistor backs to 1980, the first “Quantum Well Transistor Laser” was successfully demonstrated in 2004 [1, 2]. The transistor laser (TL) is a 3-port optoelectronic device realized by a Heterojunction Bipolar Transistor (HBT) with embedded quantum well (QW) within its highly-doped base region. The TL operates as a laser and as a transistor simultaneously, by converting an electrical input signal into two coupled outputs, i.e. electrical output (from collector) and optical output from QW-base region [3].

Miscellaneous studies have been yet done on transistor lasers at near infrared (1 µm) as well as long wavelength (1.55 µm) [4-8]. Fabrication details and performances of the heterojunction bipolar ultraviolet light-emitting transistors have been reported before [9]. However, we focus on short wavelength transistor lasers in this paper (ultraviolet region) for the first time. Our proposed structure is similar to a previously-fabricated diode laser [10, 11].

Figure 1 shows the epitaxy structure of our TL. Under forward biasing, electrons are injected from the emitter into the base region. A portion of these injected electrons are captured in the QW, where most of them are radiatively recombined with holes. The rest of injected electrons transport across the base region, mainly by diffusion, injected to the collector region and construct the transistor current gain (electrical output). The described carrier transport mechanism is shown in Fig.2.
2 MODELING

We used the charge control model developed by Zhang and Leburton for our simulations [12]. This model describes the dynamics of carriers, photon and charge densities by solving coupled rate equations:

\[
N_{p0} = n_0 \frac{\tau_q}{\tau_{qw}} \left( \frac{J_u}{J_{th}} - 1 \right)
\]

(1)

\[
n_0 = n_u + \frac{1}{\Omega \tau_p}
\]

(2)

where \(N_{p0}\) and \(n_0\) are the steady state photon and electron densities, \(\tau_p\) is the photon lifetime, \(\tau_{qw}\) is the recombination lifetime, \(J_{th}\) is the threshold current density, \(n_u\) is the transparency electron density and \(\Omega\) is the differential gain factor.

Fig. 1 Cross sectional view of proposed Transistor Laser

The threshold current \(J_{th}\) is estimated as:

\[
J_{th} = \frac{q n_0 \tau_{cap}}{v \tau_{qw} \tau_{rb}} \left[ 1 + \left( \frac{1}{v} - 1 \right) \frac{\tau_{cap}}{\tau_{rb0}} \right]
\]

(3)

\(\tau_{cap}\) is the electron capture time, \(v\) is the geometry factor and \(\tau_{rb0}\) is overall base charge (base bulk charge) lifetime.

Fig. 2 Carrier transport in TL under base-emitter forward bias and base- collector reverse bias condition

Small signal analysis of the TL can be obtained from coupled rate equations by superimposing a small sinusoidal to the DC current where we have:

\[
S(\omega) = \frac{\Delta N_p(\omega)}{\Delta J(\omega)} = \frac{1}{1 + j \omega \tau_{rb}} \frac{A}{\omega^2 + j \omega \gamma}
\]

(4)

where \(S\) is the small signal modulation response, \(\omega = 2\pi f\), \(f\) is the frequency and also we have:

\[
A = \frac{\Omega \nu N_{p0} \tau_{cap}}{q \tau_{cap}}
\]

(5)

\[
\omega_i = \frac{\Omega N_{p0}}{\tau_p}
\]

(6)

\[
\gamma = \frac{1}{\tau_{qw}} + \Omega N_{p0}
\]

(7)

3 SIMULATION PARAMETERS

Primary parameters of interest are the carrier recombination lifetime in QW, photon lifetime, carrier recombination lifetime in waveguide and differential gain factor. All of these parameters can be calculated for our proposed structure as follows.
3.1 Recombination lifetime with spontaneous emission in the QW ($\tau_{qw}$)

Lifetime for intersubband spontaneous emission $\tau_{qw}$ is calculated as [7, 13]:

$$\tau_{qw} = \frac{4\varepsilon mc^2W_{qw}}{q^2\omega O_{if}} \tag{8}$$

where $c$ is the speed of light in vacuum, $W_{qw}$ is the QW width, $\varepsilon$ is the permittivity, $\omega$ is the photon angular frequency and $q$ is the free electron charge. $O_{if}$ is the oscillator strength and equals to:

$$O_{if} = \frac{2mc^2}{\hbar} |\langle \psi_i | z | \psi_f \rangle|^2 \tag{9}$$

where the term $|\langle \psi_i | z | \psi_f \rangle|^2$ is the transition moment integral that determines the strength of the transition. $\psi_i$ and $\psi_f$ are initial and final state wave functions, respectively.

3.2 Electron capture time in base QW ($\tau_{cap}$)

The electron capture time, that is the time required for electrons to be transported across the base region, can be calculated as follows:

$$\tau_{cap} = \frac{x_{qw}^2}{2D} \tag{10}$$

where $x_{qw}$ is the distance from emitter to QW and $D$ is the diffusion constant, which is assumed uniform throughout the base.

3.3 Photon lifetime in QW ($\tau_p$)

The photon lifetime is a time that describes the growth or decay of energy in a cavity and can be obtained by:

$$\tau_p = \frac{n_{qw}}{c \left( \alpha_i + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \right)} \tag{11}$$

$$\alpha_i = (\Gamma_{qw} + \Gamma_{wg}) k_p N_b \tag{12}$$

In (11) and (12), $n_{qw}$ is the refractive index of QW, respectively, $R_1$ and $R_2$ are the facet reflectivities, $L$ is the cavity length, $N_b$ is the base region doping concentration and $k_p$ is the intervalence band absorption (Asada et al. 1984). The refractive index of Al$_{x}$Ga$_{1-x}$N is estimated by empirical equation $n=2.5067-0.43x$ [14]. Optical confinement factor for QW ($\Gamma_{qw}$) and waveguide ($\Gamma_{wg}$) can be calculated by [8]:

$$\Gamma_{qw} = \frac{D^2}{D^2 + 2} \frac{n_{qw} W_{qw}}{W_b} \tag{13}$$

$$\Gamma_{wg} = \frac{D^2}{D^2 + 2} \frac{n_{wg} 2L_{wg}}{W_b} \tag{14}$$

$$D' = \frac{2\pi}{\lambda} W_b \sqrt{n_{eff}^2 - n_{cladding}^2} \tag{15}$$

where $n_{wg}$ is the refractive index of waveguide, $L_{wg}$ is the length of waveguide and the effective refractive index($n_{eff}$) can be estimated by $n_{eff} = (2L_{wg} n_{wg} + W_{qw} n_{qw})/W_b$. The base width is equal to $W_b = 2L_{wg} + W_{qw}$. As shown in Fig. 3(a), intrinsic optical loss($\alpha_i$) is larger for wider base region. The photon lifetime depends on both optical confinement factor ($\Gamma$), optical loss and base doping. Figure 3(b) displays the dependence of $\tau_p$ on base doping ($N_b$).

3.4 Optical gain (g), Transparency electron density ($n_t$) and Differential gain factor ($\Omega$)

Optical gain is described as the optical amplification in semiconductors due to stimulated emission associated with light emission created by recombination of carriers. Optical gain caused by photon-induced transitions of electrons from the conduction band to the valence band and can be calculated by [7, 15]:

$$g = g_{\max} \left( 1 - e^{-N_{qw}/N_c} - e^{-N_{qw}/N_v} \right) \tag{16}$$

$$g_{\max} = \frac{1}{\lambda h^2} \frac{1}{2m^* W_{qw}} \tag{17}$$

$$N_{qw} = \frac{\tau_{cap}}{\tau_{esc}} N_0 \tag{18}$$

where $h$ is the Plank constant, $\lambda$ is the wavelength, $N_c$ and $N_v$ are the density of state for conduction and valence bands, respectively, $N_{qw}$ is the bounded carrier density in QW, $\tau_{esc}$ ($\tau_{cap}$) is the carrier thermionic escape (capture) lifetime to (from) the QW and $N_0$ is the average unbounded carrier density around the QW. Also, transparency carrier density can be obtained by setting
g(n_e)= 0 yielding:
\[ n_{tr} = - N_c \ln(1 - e^{-n_{tr}/N_e}) \]  
(19)

An approximate evaluation for the differential gain (\( dg / dn \)) of QW can be obtained using (16) yielding:
\[ a = g_{\text{max}} \left( \frac{e^{-n_{tr}/N_c}}{N_c} + \frac{e^{-n_{tr}/N_v}}{N_v} \right) \]  
(20)

Also we can write \( \Omega = \Gamma_{qw} V_s a \), where \( \Gamma_{qw} \) is the QW optical confinement factor and \( V_s \) is the group velocity.

![Graph](image1.png)

**Fig. 3 (a)** Base width effects on intrinsic optical loss  
**Fig. 3 (b)** Photon lifetime in QW (\( \tau_p \)) versus base doping (\( N_b \)) for different base region width

### 3.5 Recombination lifetime of carriers in the waveguides region (\( \tau_{\text{rbo}} \))

Recombination lifetime of carriers in the waveguides or base charge lifetime is defined as the average time required for carriers (electrons) to be recombined with holes in waveguide obtained from:
\[ \tau_{\text{rbo}} = \frac{1}{B_{\text{rad}} N_b} \]  
(21)

where \( B_{\text{rad}} \) is the bimolecular radiative recombination coefficient.

#### 3.6 K-factor and maximum possible modulation bandwidth (\( f_{\text{max}} \)):

Another key factor for high speed modulation systems is the K-factor, which is usually utilized to determine the maximum possible frequency [16]:
\[ K = 4\pi^2 \left[ \frac{\tau_p}{\varepsilon_g} + \frac{\varepsilon_g}{V_s a / \chi} \right] \]  
(22)

\[ \chi = 1 + \frac{\tau_{\text{cap}}}{\tau_e} \]  
(23)

where \( \varepsilon_g \) is the gain compression factor, \( \chi \) is the transport factor and \( \tau_e \) is the emission lifetime, which can be calculated by:
\[ \tau_e = \frac{2\pi m^* W_{qw}^2}{k_B T} e^{E_b/k_B T} \]  
(24)

In (28) \( k_B \) is the Boltzmann constant, \( T \) is the temperature and \( E_b \) is the effective barrier height in QW, which can be calculated by \( E_b = \pi^2 \hbar^2 / 2m^* W_{qw}^2 \).

High frequency performance of the device is typically characterized by maximum possible modulation bandwidth or maximum oscillator frequency. By definition, \( f_{\text{max}} \) is the frequency at which the maximum possible power gain as well as the unilateral power gain of the device is unity. The expression for \( f_{\text{max}} \) was formulated by Kapon in 1991 [17] as:
\[ f_{\text{max}} = \frac{2\sqrt{2\pi}}{K} \]  
(25)

where \( f_{\text{max}} \) (maximum frequency of oscillation) is a figure-of-merit. Our simulations predict a maximum frequency of 53.4 GHz for our proposed TL.
All physical quantities and parameters used in simulation are summarized in Table 2.

### Table 1: Modeling parameters for the TL

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{qw}$</td>
<td>Recombination lifetime with spontaneous emission in QW</td>
<td>Sec</td>
</tr>
<tr>
<td>$\tau_{rb0}$</td>
<td>Base bulk charge lifetime</td>
<td>Sec</td>
</tr>
<tr>
<td>$\tau_{rb}$</td>
<td>Overall base charge lifetime</td>
<td>Sec</td>
</tr>
<tr>
<td>$\tau_{cap}$</td>
<td>Electron capture time in QW</td>
<td>Sec</td>
</tr>
<tr>
<td>$\tau_{p}$</td>
<td>Photon lifetime</td>
<td>Sec</td>
</tr>
<tr>
<td>$\Gamma_{qw}$</td>
<td>QW optical confinement factor</td>
<td>unitless</td>
</tr>
<tr>
<td>$\Gamma_{wg}$</td>
<td>Waveguide optical confinement factor</td>
<td>unitless</td>
</tr>
<tr>
<td>$W_b$</td>
<td>Base region width</td>
<td>M</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Differential gain factor</td>
<td>m$^2$s$^{-1}$</td>
</tr>
<tr>
<td>$n_{tr}$</td>
<td>Transparency electron density</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$J_{th}$</td>
<td>Threshold current density</td>
<td>m$^2$A</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Geometry Factor</td>
<td>-</td>
</tr>
<tr>
<td>$V_g$</td>
<td>QW Group velocity</td>
<td>ms$^{-1}$</td>
</tr>
</tbody>
</table>

### 3.7 Current gain or electrical output of TL ($\beta$)

In HBTs current gain ($\beta$) defined as $\beta = I_c/I_b = \tau_n/\tau_{rb}$ where $\tau_n$ is the base recombination lifetime and $\tau_{rb}$ is the base transit time. In the case of TLs $\beta$ similarly can be defined as:

$$\beta = \frac{\tau_{rb}}{\tau_{TL}}$$  \hspace{1cm} (26)

where $\tau_{TL}$ is the effective base recombination lifetime and we assume that $\tau_{TL} \approx \tau_{rb}$ [18] and $\tau_{rb}$ is the overall base recombination lifetime which can be wrote as $\tau_{rb} = W_b^2/2D$. With simulations we reached to $\beta = 10.6$ for our TL.

### 3.8 Internal (external) differential quantum efficiency and output power:

Internal differential quantum efficiency (IQE) which is the fraction of the excess of the injection current over the threshold current that results in stimulated emission [19]:

$$\eta_{int} = \frac{qN_p W_{qw}}{J_{th} \tau_{rb}}$$  \hspace{1cm} (27)

where $J_{th}$ are stimulated threshold current density. The power emitted by stimulated emission from quantum well of transistor laser equals to:

$$P_e = \eta_{int} A(J_0 - J_{th}) \frac{hc}{\lambda q}$$  \hspace{1cm} (28)

Part of this power dissipated inside the resonant cavity and the rest is coupled out through the output end. The output power of a TL (or a laser diode) can be defined as:

### Table 2: Physical parameters of the short wavelength TL

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{qw}$</td>
<td>QW width</td>
<td>15</td>
<td>nm</td>
</tr>
<tr>
<td>$n_{qw}$</td>
<td>QW refractive index</td>
<td>2.4809</td>
<td>unitless</td>
</tr>
<tr>
<td>$n_{wg}$</td>
<td>Waveguide refractive index</td>
<td>2.4379</td>
<td>unitless</td>
</tr>
<tr>
<td>$n_{cladding}$</td>
<td>Cladding refractive index</td>
<td>2.3777</td>
<td>unitless</td>
</tr>
<tr>
<td>$N_b$</td>
<td>Base doping</td>
<td>$2 \times 10^{18}$</td>
<td>cm$^3$</td>
</tr>
<tr>
<td>$N_c$</td>
<td>Density of states of conduction band</td>
<td>2.454 x 10$^{14}$</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$N_v$</td>
<td>Density of states of valance band</td>
<td>$2.379 \times 10^{19}$</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
<td>336</td>
<td>nm</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>300</td>
<td>K</td>
</tr>
<tr>
<td>$L$</td>
<td>Cavity length</td>
<td>500</td>
<td>µm</td>
</tr>
<tr>
<td>$\beta_{rad}$</td>
<td>Bimolecular radiative recombination coefficient</td>
<td>$1.5 \times 10^9$</td>
<td>cm$^3$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Relative static permittivity</td>
<td>8.78</td>
<td>unitless</td>
</tr>
</tbody>
</table>
\[ P_{\text{out}} = P_e \frac{\ln(1/R)}{\ln(1/R) + q \cdot L} \]  

(29)

where \( R \) is the reflectivity of cavity facet (\( R_0 = R_0 = 0.2 \)). Other used parameters were defined before. Above lasing threshold the external differential quantum efficiency is the ratio of the increase in photon output rate to the increase in the injection rate. We obtain it as:

\[ \eta_{\text{ex}} = \frac{q \lambda}{hcA} \frac{dP_{\text{out}}}{dJ_0} \]  

(30)

As could be predicted, the output power is directly proportional to the injection current above the threshold, which means that the modulation of the output light will be directly proportional to injection current with negligible distortion.

Simulations in this section shows that a 3.46 mw output power and 3.7% external differential quantum for our proposed TL. These values are in good agreement with experimental data for diode laser [10, 11].

4 RESULTS AND DISCUSSIONS

First we analyze the photon and electrons steady states carrier density. Results are shown in Figure 4. Threshold current density of our proposed TL is also shown in Fig. 5, where it is minimized for 73 nm base width.

As can be seen, \( J_{\text{th}} \) has a minimum value of 3.1, 5.06 and 7.4 KA/cm\(^2\) for different cavity lengths (400, 500, 660 micron), respectively.

Table 3 gives a comparison between threshold current density of our proposed AlGaN/GaN single quantum well short wavelength transistor laser and experimental data for other transistor lasers. As can be seen, threshold current density of our work is comparably higher than others hence high breakdown field of GaN-based devices allows high bias conditions.

At last, optical frequency response of TL using calculated parameters has been analyzed. Figure 6 shows the optical frequency response as a function of frequency for different base width. Results show that we can reach 30 GHz bandwidth for AlGaN/GaN transistor lasers with 15nm QW width.

A resonance peak of less than 5 dB is also anticipated at \( f_r = 20 \) GHz. The lower resonance peak will definitely improve the frequency response as it eliminates the ringing effects. As shown in Fig. 6, frequency response can be even improved further, i.e. lower \( f_r \) and larger \( f_{3dB} \), by increasing the base width.
**Fig. 5** Base width effect on threshold current density for different cavity lengths.

**Table 3** Comparison of calculated (this work) and experimental values of threshold current density

<table>
<thead>
<tr>
<th>Sample</th>
<th>λ (nm)</th>
<th>Temperature (K)</th>
<th>Area (µm²)</th>
<th>Jth (kA/cm²)</th>
<th>current gain (β)</th>
<th>reference</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1006</td>
<td>298</td>
<td>850 x 2.2</td>
<td>2.13</td>
<td>-</td>
<td>[2]</td>
</tr>
<tr>
<td>2</td>
<td>1005</td>
<td>288</td>
<td>400 x 10</td>
<td>5.5</td>
<td>1.2</td>
<td>[20]</td>
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<td>3</td>
<td>964</td>
<td>213</td>
<td>450 x 10</td>
<td>1.6</td>
<td>4.5</td>
<td>[21]</td>
</tr>
<tr>
<td>4</td>
<td>1330</td>
<td>298</td>
<td>500 x 1.8</td>
<td>1.89</td>
<td>0.01</td>
<td>[22]</td>
</tr>
<tr>
<td>5</td>
<td>1550</td>
<td>288</td>
<td>400 x 10</td>
<td>8.75</td>
<td>-</td>
<td>[5]</td>
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<tr>
<td>6</td>
<td>336</td>
<td>300</td>
<td>500 x 10</td>
<td>12.3</td>
<td>10.6</td>
<td>Our work</td>
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</tbody>
</table>

**Fig. 6** Normalized optical frequency response of our proposed TL as a function of frequency.

**Fig. 7** (a) shows the resonance peak versus \( W_b \). It can be seen that the curve has a maximum at 225 nm. Also Figure 7(b) shows a comparison between the normalized optical frequency responses of Long wavelength (1550 nm) [7, 8] and our work. For the sake of better comparison, QW width and cavity length is the same in all cases. The resonance peak for our work is comparably lower (less than 5dB) than others hence our device is more stable than long wavelength TL. More ever, the intrinsic bandwidth for our short wavelength device is significantly higher. This wideband, low-resonance frequency response could be of special interest for optoelectronic integrated circuits.

**Conclusion**

In this article, we introduced an AlGaN/GaN single quantum well transistor laser, which can operate in short wavelength. We used a previously-developed rate equation charge control model to analyze the optical frequency response of ultra-violet single quantum well transistor laser. Optical confinement factors, carrier and charge lifetimes, threshold current density and differential gain factor were first calculated. Then, we investigated base width effects on the device performances such as optical frequency response, bandwidth, threshold current density,
electron and photon density, photon lifetime and carrier recombination lifetime. Our simulation results predict a resonance peak of less than 5dB and an optical bandwidth of 30 GHz for optical frequency response achievable in the case of 15 nm QW width and a cavity length of 500 µm. Also we have a 10.6 current gain and 3.46 mw output for our transistor laser.

Finally, we compared our proposed structure for short-wavelength transistor lasers with others (i.e. long wavelength and near infrared TLs), where we found that our designed TL can bring us a ratherly smoother optical frequency response with reasonable bandwidth and small enough resonance peak.

References