Mathematical Modeling of Basic Parts of Heating Systems with Alternative Power Sources

PETR MASTNY, JAN MORAVEK, JIRI PITRON
Brno University of Technology
Department of Electrical Power Engineering
Technicka 12, 616 00 Brno
CZECH REPUBLIC
mastny@feec.vutbr.cz, jan.moravek@phd.feec.vutbr.cz, xpitro00@stud.feec.vutbr.cz

Abstract: - In connection with the development and the growing integration of renewable energy sources into energy systems, it is still important to work on options for optimization of energy processes. The issue of energy systems for low energy buildings is currently the basic trend in reducing energy demands of buildings and is in accordance with the requirements of sustainable development. There are defined the conditions for correct approach in the design of energy systems with heat pumps and solar systems and there are presented selected results from the modeling of operational states of this sources.

Key-Words: - Energy, Heating, Solar collector, Heat pump, Modeling, Matlab, Alternative energy

1 Introduction

Efficient energy use in buildings is one of the primary aims of the current situation in the use of alternative energy sources. One possible way to achieve this set target is to increase the share of renewable energy sources (RES) in buildings. Sustainable systems using renewable power sources can have a positive effect on increasing the reliability of energy supplies and reducing greenhouse gas emissions. Using combined systems consisting of renewable power sources and efficient implementation of related measures in the construction and technology projects, there can be reduced operating and capital costs. [1]

Heat supply systems, using alternative energy sources come to the fore mainly due to the possibility of its perspective application regarding the limits of fossil fuels, decentralization of thermal energy production. Given the fact that the prediction of power supply from systems with alternative energy sources because of its dependence on the operating conditions becomes very difficult, it is necessary regarding design of such systems to follow certain principles. As it is typical for every energy source also renewable energy sources and have its advantages and disadvantages. It is not possible to evaluate such sources in the absolute scale because local conditions may completely exclude some sources, although, according to economic indicators such sources would be favorable. In some cases, the high investment costs offset an extremely low operating costs and minimal maintenance. Subsidies play an important role for sources with high specific investment costs and lower annual utilization. The operational experience shows that capital-intensive sources generate heat at a favourable price, because they have very high annual utilization. [2]

Regarding energy balance of building structures the decisive factor is the amount of energy needed for heating and hot service water. This is influenced by the fact that to cover the requirements for heating and hot service water the energy consumption is up to 80% from total energy demands of buildings. The remaining amount of energy is consumed in common operation. Regarding the requirement to reduce energy consumption and with when parallel introducing new technologies in the field of power sources, the requirements for technological and structural design of new buildings change. Building materials and constructions change and increasingly there emerge concepts of low-energy and passive houses. The most frequently used alternative power sources in our country are solar systems and heat pumps. [1]

1.1 Conception of Heating Systems

As was mentioned the actual design of a thermal system with alternative power sources will also depend on the design of the building and used construction materials. Different principles apply to the designs of systems in masonry buildings and different in wood-based constructions. The research results showed that for light construction (timber

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construction) it is advisable to choose a heating system, hot water (low temperature) or direct heating, hot air heating systems in these buildings appear to be unsuitable because of excessive reduction in indoor air relative humidity. Such heating system should be controllable as quickly as possible. General requirements for control measurements in these structures are much more significant issue than in buildings with heavy building materials. In contrast, the brick building is convenient to choose hot or hot-air heating system, while direct heating is unsuitable for this type of buildings because the main disadvantage of such system is the high operating costs primarily associated with large accumulation areas (heavy building materials). Within the modern building constructions the great emphasis is put on ventilation. Where natural ventilation is inadequate, then it is necessary to apply forced ventilation. Units of forced air heating and ventilation may be in these buildings (with heavy constructions) well suited to keep the relative indoor air humidity in standardized and therefore wholesome values. Results of research in the design of energy systems for low energy buildings define the basic conditions for the design.

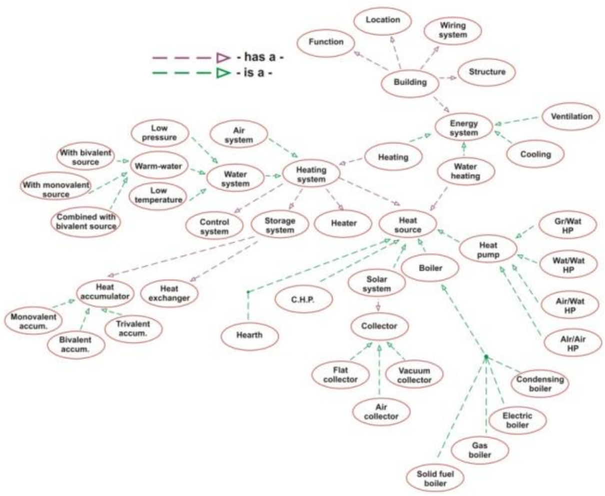

Fig. 1 Taxonomy of basic classes related with the design of energy system [1]

These conditions are based on the defined relations between the building (construction materials, location, local conditions, building orientation) and the energy system, including relations between its components (source type, type of heating system, storage system, control system) – see Fig. 1. The results show that there is a very close correlation between the choice of energy system and the type of low-energy building. The choice of the energy system in the context of the type of construction has an impact not only on the internal microclimate, but also on the economics of the operation of the energy system. [2]

2 Mathematical Simulation Parts of Heating System

The following sections describe the mathematical models of the solar collector and heat pump prepared for the possibility of verification and optimization of their operating conditions.

2.1 Model of Solar Collector

In this part is described the mathematical model of simple flat plate solar collector which works with the divided heat exchanger.

Solar heat systems present the alternative ecological way of heat production. The main focus of this research is aimed on heat produced by the solar thermal collectors, therefore their functions and types are further described.

Simulation of the different states in the heat systems is the important part of their design. Matlab Simulink software is suitable for solving the differential equations describing the dynamic states and behavior of the system. Each element of the system can be taken and described with the use of mathematical relations. Based on that, simulations blocks can be created and further connected. This allows the additional changes of the input parameters, determination of the effects of the linked blocks and estimation of the output parameters without need of the real device. [5]

2.1.1 Mathematical Description of Solar Collector

Solar collectors are designed to absorb solar radiation and convert it into thermal energy. It is necessary to point out that even the collector is important component, the entire system's efficiency depends on the quality of all the components.

An *absorber* is the part of the collector which receives the solar radiation. This is usually black or dark-colored surface. Additional special coating helps to reduce the re-emittance – the amount of the radiation that is lost as the absorber gets hot. These special coating are called the selective coatings. The collector's efficiency can be optimized by a special combination of dark and selective surface and lower absorber temperature. [4]

Absorbers contain manufactured *pipes* or *passageways* in which flows the fluid. The construction of the absorber affects the rate at which is the heat transferred to the fluid. When the fluid

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moves through pipes, the natural resistance from the sides of the pipes causes friction. The higher the resistance, the slower the fluid will move and will potentially require a bigger pump.

Next part of the collector is the *cover* (made from glass or plastic) which eliminates the heat losses to the air and protect the selective surfaces from moisture, contaminants, wind and mechanical damage.

For mathematical modeling we have chosen a flat plate collector, where the fluid absorbs the heat from the sun reduced by the heat loss to the ambient air. Schematic diagram is shown on the Fig. 2. [6]

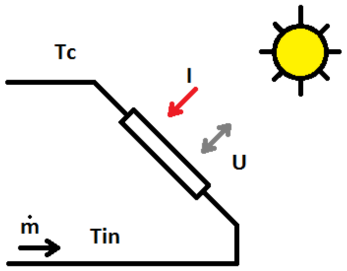

Fig. 2 Schematic diagram of a flat plate collector

Heat absorbed from the Sun can be described by equation (1) and heat losses of collector by equation (2): [5], [7]

$$Q_S = I \cdot A \cdot \tau \cdot \alpha \quad (W) \tag{1}$$

$$Q_L = U \cdot A_C \cdot (T_{abs} - T_a) \quad (W) \tag{2}$$

where

- τ (−) and α (−) are the constants reflecting the material properties of the absorber,
- I is irradiance in plate of the collector ($W \cdot m^{-2}$),
- A_C is aperture surface of the collector (m^2),
- U is the heat loss coefficient of collector ($W \cdot m^{-2} \cdot K^{-1}$),
- T_{abs} is the temperature of the surface of the absorber plate ($°C$),
- T_a is the ambient temperature ($°C$).

In case of the flat plate collector, the heat loss from the top side is prevailing, so we can neglect the other ways.

Heat transferred to the fluid is expressed by the equation (3):

$$Q_F = \dot{m} \cdot c \cdot (T_c - T_{in}) \quad (W) \tag{3}$$

where

- $\dot{m}$ is the mass flow rate of the fluid ($kg \cdot s^{-1}$),
- c is the specific heat capacity of the fluid ($J \cdot kg^{-1} \cdot °C^{-1}$),
- T_c is the outlet temperature of the collector (°C),
- T_{in} is the inlet temperature of the collector (°C).

With the use of the equations (1)–(3) can be written the differential equation for the outlet temperature of the collector:

$$\rho \cdot c \cdot V \frac{dT_c}{dt} = I \cdot A \cdot \tau \cdot \alpha - U \cdot A_c \cdot (T_{abs} - T_a) + \dot{m} \cdot c \cdot (T_c - T_{in}) \tag{4}$$

It is presumed, that the density of the fluid ρ, specific heat capacity of fluid c and volume $V$of the fluid in the collector are constant and don't depend on the temperature.

Simplified temperature of the absorber's surface is calculated as the average according the equation (5):

$$T_{avg} = \frac{T_{in} + T_c}{2} \quad (W) \tag{5}$$

The correction factor [−] reflects the radiative and convective heat loss from the absorber plate.

Equation (4) than can be rewritten into equation (6):

$$\rho \cdot c \cdot V \frac{dT_c}{dt} = I \cdot A \cdot \tau \cdot \alpha \cdot K - U \cdot A_c \cdot K \cdot \left(T_{avg} - T_a\right) + \dot{m} \cdot c \cdot (T_c - T_{in}) \tag{6}$$

Following substitution is performed:

- $\tau \cdot \alpha \cdot K$ is substituted with the optical efficiency of the collector η_o (−),
- U·K is substituted with the overall heat loss coefficient of the collector U_L ($W \cdot m^{-2} \cdot K^{-1}$),
- ρ·c·V is substituted with the overall heat capacity of the fluid C ($J \cdot °C^{-1}$),
- $\dot{m}$ is substituted with the volumetric flow rate of collector and the density of the fluid $F_c \cdot \rho$ ($kg \cdot s^{-1}$).

The equation (6) can be rewritten into the equation (7) using the substitution:

$$\frac{dT_c}{dt} = \frac{I \cdot A \cdot \eta_o}{C} - \frac{U_L \cdot A_c}{C} \cdot \left(T_{avg} - T_a\right) + \frac{F_c}{V} \cdot (T_c - T_{in}) \tag{7}$$

2.1.2 Block diagram in Matlab Simulink

In the Fig. 3 is shown the created model according the equation (7). Input parameters are the following:

- T_{in} – inlet temperature of the collector,
- I – irradiance in plate of the collector,
- T_a – ambient temperature,
- F_c – volumetric flow rate of the collector.

These parameters can be changed directly in the program model. Other constants are loaded from the additional file "*parameters.m*". The constants are:

- A_c – aperture surface of the collector,

- η_o – optical efficiency of the collector,
- C – overall heat capacity of the fluid,
- U_L – overall heat loss coefficient of the collector,
- V_c – volume of the fluid in the collector.

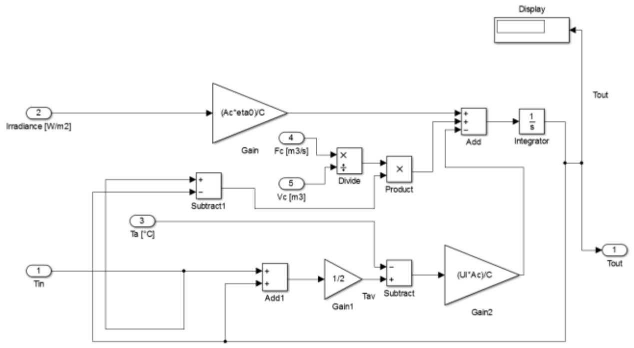

Fig. 3 Block diagram of a flat plate collector

The result from the simulation is the outlet temperature of the collector T_c.

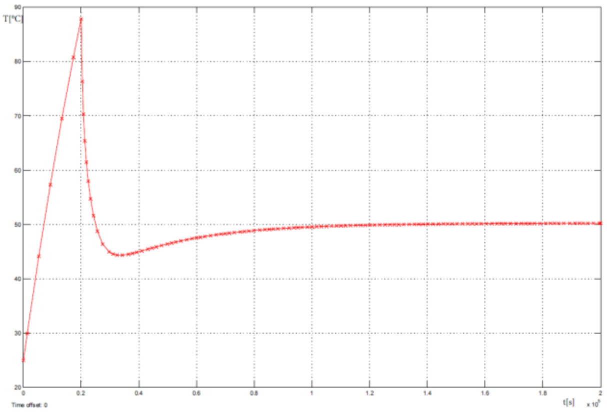

Fig. 4 Simulation result of the T_c

In the Fig. 4 can be seen the simulation result in time from 0–200 000 s. In time 0–20 000 s is the temperature rising because of the zero flow in the collector (F_c = 0 $m^3 \cdot s^{-1}$). In the time 20 000 s has been turned on the circulation pump and the fluid started to transfer the heat to the heat exchanger.

2.1.3 Model of Solar System

Simple solar system consisting of the solar collector and the heat exchanger described by the equations (8) and (9) is presented. Block diagram is shown in the Fig. 5.

$$\frac{dT_1}{dt} = \frac{F_c}{V_1} \cdot (T_c - T_1) - \frac{U \cdot A}{\rho \cdot c \cdot V_1} \cdot (T_1 - T_2) \qquad (8)$$

$$\frac{dT_2}{dt} = \frac{F_1}{V_2} \cdot (T_d - T_2) - \frac{U \cdot A}{\rho \cdot c \cdot V_2} \cdot (T_1 - T_2) \qquad (9)$$

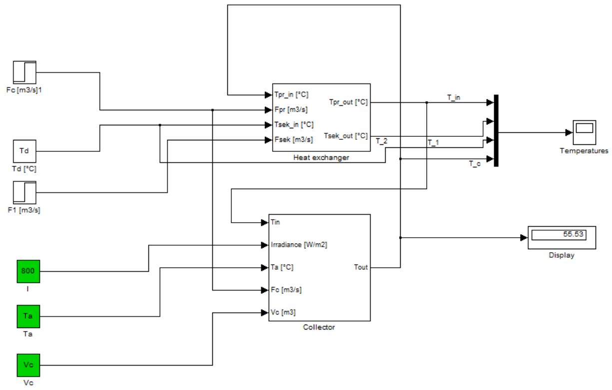

Fig. 5 Block diagram of solar system

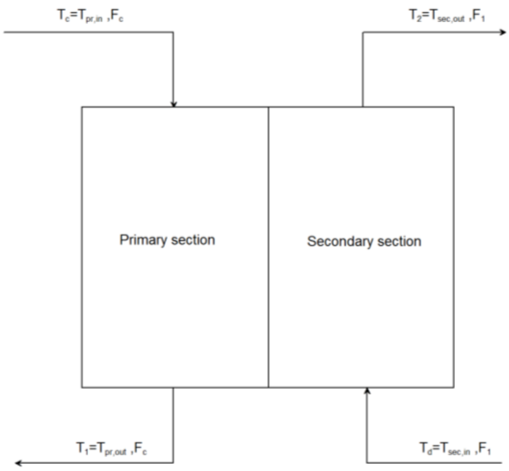

Fig. 6 Diagram of heat exchanger

Heat exchanger is in this case considered as the tank divided into two parts (Fig. 6). Between these partitions is the heat transferred. This simple solution is used in the initial phase of the simulation testing and will be further improved to the accumulation tank that allows the use of multiple energy sources.

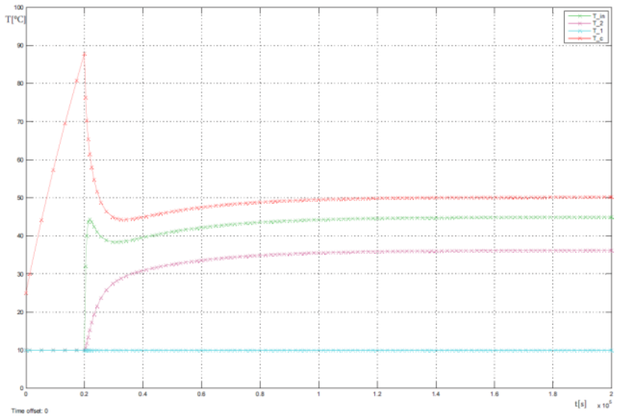

Fig. 7 Simulated results of temperatures in solar system

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Fig. 7 shows the results of simulated temperatures in the inlet T_{in} and outlet T_c of the collector and also the temperatures in the output T_1 of the primary and output T_2 of the secondary section of the heat exchanger. It is presumed, that the cold water entering the heat exchanger on its secondary side has 10 °C. For the simulation purposes is set the following initial conditions $(0) = T_a = 25\ °C$, $T_1(0) = T_2(0) = T_{in} = 10\ °C$.

2.2 Model of Heat Pump

The main function of the HP is to pump heat from low temperature heat sources to high temperature heat sinks, thus providing both comforts heating and cooling. Conventional HP includes basic items such as evaporator, compressor, condenser and expansion (throttling) valve. Those components have different parameters and possess different operational characteristics, particularly under transient conditions. It is conceivable, that the transient behavior of the integrated HP system is quite different from those of the single components. However, it is possible to force the heat from a source at a lower temperature to a sink at a higher temperature using a relatively small quantity of mechanical work.

2.2.1 Mathematical Description of Heat Pump

In Fig. 8 is shown the basic monovalent connection of heat pump in energy system with thermal energy accumulation.

The basic criterion for evaluating the operational characteristics of heat pump is heating factor. Heating factor (*COP*) is defined as the ratio of the heat Q_{HP} (heating capacity) and the energy required for hot swapping of W_{EP}. It expresses how many times we get more energy than we bring in the form of drive energy (electricity). Energy heating factor can then be defined by a simple equation (10). [3]

$$COP = \frac{Q_{HP}}{W_{EP}} \quad (-) \qquad (10)$$

Suggestion of heat pump's simulation has been created with validation on a specific heat pump. However, a description of all the changes which take place in the heat pump circuit specially describing the refrigerant is considerably more difficult. Therefore simulation considers some simplifications.

- In the entire heat pump system is considered a constant mass flow of refrigerant q_{ref}.
- The simulation calculates the energy value of the sub-components. When some value is changed, the impact on the other calculations in compliance with precision of the calculation has to be considered.

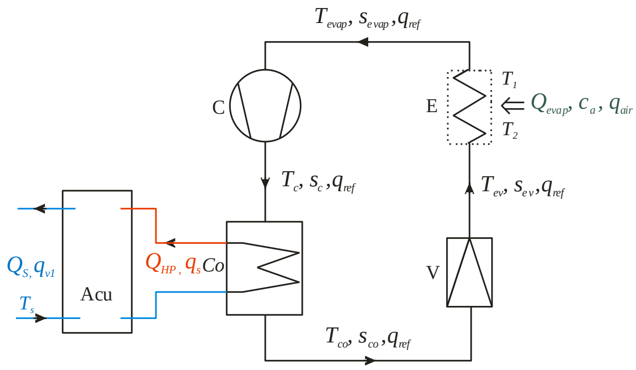

Fig. 8 Technological scheme of heat pump

In this simulation it is necessary to know temperatures between individual sub-components and then enthalpy of the refrigerant can be determined.

The energy of refrigerant entering into an evaporator:

$$Q_{ev} = q_{ref} \cdot s_{ev} \cdot T_{ev} \quad (W) \qquad (11)$$

The heat flow received from the environment through the evaporator can be expressed:

$$Q_{evap} = q_{air} \cdot c_a \cdot (T_1 - T_2) \quad (W) \qquad (12)$$

Thermal power delivered by the compressor into a refrigerant:

$$Q_{comp} = q_{ref} \cdot (s_c - s_{evap}) \cdot (T_c - T_{evap}) \quad (W) \qquad (13)$$

The energy input of refrigerant into the expansion valve:

$$Q_{cond} = q_{ref} \cdot s_{co} \cdot T_{co} \quad (W) \qquad (14)$$

Subsequently it can be expressed, that the heat flux input to the storage tank is reduced of the efficiency the heat exchanger, which is around 70-80%.

Equation (14) is used for the calculation as a simplification, because the s_{ev} and T_{ev} are not known. It presumes $Q_{cond} = Q_{evap}$.

$$Q_{HP} = \left(\frac{Q_{exp} + Q_{evap} + Q_{comp} - Q_{cond}}{q_{ref} \cdot \frac{s_{co} - s_c}{2}} \cdot c \cdot q_s \right) \cdot Q_z \quad (W) \qquad (15)$$

The simulation uses increments of energy obtained by the heat pump. In this simulation the energy inputted to an accumulation tank means that this energy is not dependent on the temperature of water output from the accumulation tank into the condenser. The calculation of mass flow of hot water into the accumulation tank is determined:

$$q_s = \left(\frac{q_{ref} \cdot \frac{s_{co} - s_c}{2}}{c} \right) \quad (kg^{.}s^{-1}) \qquad (16)$$

For equations (11) – (16) are defined the following variables:

- q_{ref} , q_{air} - mass flow of refrigerant, air entering into an evaporator ($kg^{.}s^{-1}$)
- q_s - mass flow of water input to the accumulation tank ($kg^{.}s^{-1}$)
- s_{ev} , s_c - entropy of refrigerant behind the expansion valve, the compressor ($J^{.}kg^{-1.}K^{-1}$)
- s_{evap} , s_{co} - entropy of refrigerant behind the evaporator, the condenser ($J^{.}kg^{-1.}K^{-1}$)
- T_{ev} , T_{evap} - temperature of refrigerant behind the expansion valve, the evaporator (K)
- T_c , T_{co} - temperature of refrigerant behind the compressor, the condenser (K)
- T_1, T_2 - input and output temperatures of the air (K)
- c_a , c - specific heat capacity of air, of water ($J^{.}kg^{-1.}K^{-1}$)
- η_c - the efficiency of the compressor (-)

3 Conclusion

Previously, the theoretical parameters of the solar collector with addition of their simplified mathematical description were described. On this basis were created the mathematical models in Matlab Simulink. Initially, the behavior of the model was briefly demonstrated and is suitable for further validation. For the described model were neglected some of the parameters. For example it is presumed the constant density and specific heat capacity of the fluid. Furthermore were neglected the heat losses of the bottom and sides of the collector, because the heal loss of the top side is the prevailing. In the following research will continue the integration of the solar collector into the model of a solar system with additional energy source. The model of the collector can be extended with other solar heat collectors types, as has been described previously. After that, the difference between the simulated results and values measured on the real device can be compared and evaluated.

By comparing the results obtained from mathematical modeling with real values measured on a physical model of heat pump, it can be stated that the chosen mathematical description of the heat pump is very close to the real parameters. This mathematical model could then be usable in a practical design for determining the type and the performance of heat pump for a particular object.

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