The Influence of Shapes of Fuzzy Sets Membership Functions on Fuzzy System Performances

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Abstract: - In the paper the influence of the fuzzy sets membership functions shapes on the response and performances of fuzzy systems is considered. One of the designer's choices in fuzzy system design can be the selection of membership functions of fuzzy sets involved by the system. The fuzzy system considered in the paper is designed as a fuzzy proportional –derivative (FPD) rule-based controller. The FPD controller is described. On the software simulation of such fuzzy system, the experiments were organized with the shapes of input and output fuzzy sets membership functions. We have experimented with linear (triangular and trapezoidal) and nonlinear (bell shaped and flatten bell shaped) membership functions, and with the combinations of them. Some experiment results are given. The best membership functions choice according to chosen performance criterion, has been found. Conclusions are given: from the experiments we have concluded that the response of the considered fuzzy system and its performance are very sensitive to the shape of the membership functions. Some aspects of fuzzy system's response can, also, be tuned by the appropriate membership functions selection.

Key-Words: - soft computing, control systems, fuzzy logic, membership functions, fuzzy controller, tuning
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1 Introduction

Fuzzy sets theory was introduced by Zadeh [1] in 1965. This theory is powerful modelling tool for systems dealing with uncertainties and non-linearity. Fuzzy sets can be used in models of systems with gradual properties or soft constraints whose satisfaction is a matter of degree. Fuzzy sets can be used, also, in modelling information pervaded with imprecision and uncertainty. Those characteristics make fuzzy models useful in great variety of applications. Fuzzy logic with neural networks, probabilistic reasoning, belief networks, genetic algorithms, chaos theory and parts of learning theory makes complementary partnership of disciplines and technologies know as soft computing, which gives methods for solving complex problems in designing intelligent systems with the ability to exploit the tolerance for imprecision, uncertainty and partial truth, to achieve tractability, robustness and low solution cost.

The most popular area of applications of fuzzy sets is fuzzy control [2], which has found industrial applications. The popularity of fuzzy controllers is due to the fact that they do not necessarily require a theoretical model of the process which is to be controlled.

In the paper the influence of the fuzzy sets membership functions shapes on the response and performances of fuzzy systems is considered

The structure of this paper is as follows. Section 2 deals with a theoretical basis of fuzzy control of interest for the paper. In Section 3 some aspects of a fuzzy PD controller design are considered. Section 4 deals with experiments and results. Conclusions are given .

2 A Theoretical Basis of Fuzzy Control

In fuzzy control, as in many of fuzzy sets applications, expert knowledge is encoded in the form of fuzzy rules, which describe recommended actions for different classes of situations represented by fuzzy sets. An interpolation mechanism provided by the fuzzy control methodology is then at work. A fuzzy control unit implicitly defines a numerical function tying the control variable and the observed variables together. The fuzzy logic approach suggests that the control law can be built starting from the expertise of a human operator. PID controllers can attain only linear control law, while fuzzy controller may capture nonlinear laws. Any kind of control law can be modeled by the fuzzy control methodology, provided that this law is expressible in terms of ‘’ if – then ...‘’ rules, just like in the case of expert systems, [6]. Although a linguistic rule-based control is modeled , the function simulated by the control part remains continuous just like in classical automatic control, [3]. In the context of complex processes, it may turn out to be more practical to get the knowledge from an expert operator than to calculate an optimal control, due to modelling costs or because a model is out of reach. These are some of the facts which make fuzzy control practically important area.

The control algorithm is represented by fuzzy rules, [5]. A multivariable fuzzy system with two inputs and one output with the linguistic description of the process is given by rule base:

$$\mathbf{R_i}: \text{IF } X_1(i) \text{ AND } X_2(i) \text{ THEN } Y(i)\ , \qquad (1)$$

where X_1(i) and X_2(i) are fuzzy sets of input variables defined in the universes of discourse X_1 and X_2 respectively. $Y(i)$ is the output fuzzy set defined in the universe of discourse Y, and $i = 1, \dots, m$, where m is the number of rules.

The fuzzy relation R of the system is expressed as follows

$$R = \bigvee_{i=1}^{m} \left[X_1(i) \wedge X_2(i) \wedge Y(i) \right], \qquad (2)$$

where $\vee$ is the aggregation operator and $\wedge$ is the implication operator. For each rule a fuzzy relation $R(i)$ is constructed. To obtain the fuzzy controller relation R, fuzzy relations $R(i)$ are aggregated. To obtain new fuzzy output Y' for given the current fuzzy inputs X_1' and X_2', the compositional rule of inference is used

$$Y' = X_1' \circ X_2' \circ R\ , \qquad (3)$$

where $\circ$ is compositional operator of fuzzy relations.

3 Fuzzy PD Controller Design

As a fuzzy system, we are considering a fuzzy PD controller.

The basic idea of the discrete PID controller is to choose the control low by considering an error $e(kT)$, change-of-error

$$ce(kT) = (e(kT) - e((k\text{-}1)\ T))/\ T,$$

and the numerically approximated integral of error

$$ie(kT) = ie((k\text{-}1)T) + T\ e((k\text{-}1)T),$$

where T is the sample period.

The PID control law is

$$u_{\mathrm{PID}}(kT) = K_{\mathrm{p}} * e(kT) + K_{\mathrm{D}} * ce(kT) + K_{\mathrm{I}} * ie(kT) \qquad (4)$$

where K_{p} is proportional constant, K_{D} is a differential constant and K_{I} is an integral constant, all of them defined by characteristics of the process. For a linear process the parameters K_{p}, K_{D}, and K_{I} are designed in such a way that the closed loop control is stable. In the case of the nonlinear processes which can be linearized around the operating point conventional PID controllers also work successfully. However, the PID controller with constant parameters in the whole working area is robust, but not optimal. Hence, tuning of PID parameters has to be performed.

Fuzzy PID controller, [4], starts from the same assumptions which are decisive for the conventional PID controller:

- process is either linear or non-linear, but can be piece-wise linearized;
- process can be stabilized taking into account selected criteria.

The output of the fuzzy controller $u(kT)$ is given by

$$u(kT) = F\ (e(kT), ce(kT), ie(kT)) \qquad (5)$$

where $F(.)$ is a nonlinear function determined by fuzzy parameters.

We are considering the case when control goal is to regulate some process output around the setpoint or reference. In that case we have single output-single input control.

A type of those controllers is fuzzy PD controller, whose input is the error

$$e(kT) = y_{\mathrm{sp}}(kT) - y(kT) \qquad (6)$$

(and the change_of_error $ce(kT)$), where y_{sp} is setpoint value, and $y(kT)$ is the process output at $t = kT$. The structure of a fuzzy PD controller is given at Figure 1. A block denoted by f at Figure 1., is a rule base, and g_1, g_2 and gu are gains, which correspond to K_p, K_D, and K_I parameters in (4).

The linear PD controller can be viewed as a linearized fuzzy controller and looks like this:

$$u_{PD}(kT)=(g_1 * e(kT) + g_2 * ce(kT))*gu \qquad (7)$$

The fuzzy controller replace the expression in the parenthesis by a rule base, that can be nonlinear.

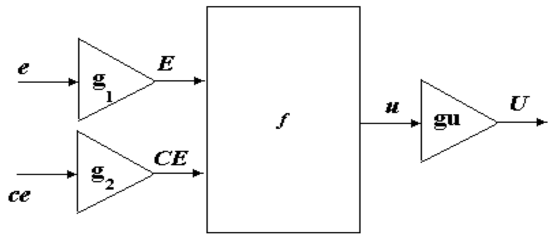

Figure 1. Fuzzy PD controller

On the software simulation of fuzzy PD controller, the experiments were organized with the shapes of input and output fuzzy sets membership functions.

4 Experiments and Results

For the fuzzy PD controller from Figure 1, whose gains are $g_1 = 230$, $g_2 = 45$, $gu = 0.5$, the following rule base describes some process:

R$_1$: IF *e is* N AND *ce is* N THEN *u is* SNO, (8)
R$_2$: IF *e is* N AND *ce is* Z THEN *u is* MNO,
R$_3$: IF *e is* N AND *ce is* P THEN *u is* ZO,
R$_4$: IF *e is* Z AND *ce is* N THEN *u is* MNO,
R$_5$: IF *e is* Z AND *ce is* Z THEN *u is* ZO,
R$_6$: IF *e is* Z AND *ce is* P THEN *u is* MPO,
R$_7$: IF *e is* P AND *ce is* N THEN *u is* ZO,
R$_8$: IF *e is* P AND *ce is* Z THEN *u is* MPO,
R$_9$: IF *e is* P AND *ce is* P THEN *u is* SPO.

In the rule base (8) *e* is error, (6), *ce* is the change_of_error. The labels N, Z and P are labels of input fuzzy sets (N for Negative, Z for Zero, and P for Positive). The labels SNO (Strong Negative Output), MNO (Moderate Negative Output), ZO (Zero Output), MPO (Moderate Positive Output), and SPO (Strong Positive Output) are the labels of output fuzzy sets, singletons, as in Figure 3. For singletons as output fuzzy sets there is no difference between min-implication and product implication. The inference is sum-product. The defuzzification is done using Center of Gravity with Singletons (Height) method.

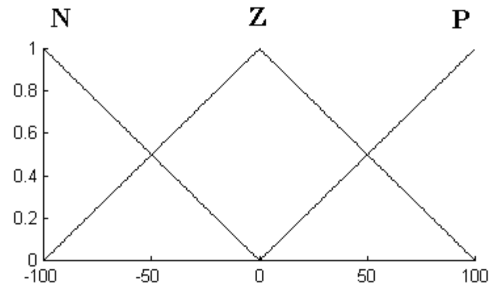

Figure 2. Input fuzzy sets (triangular).

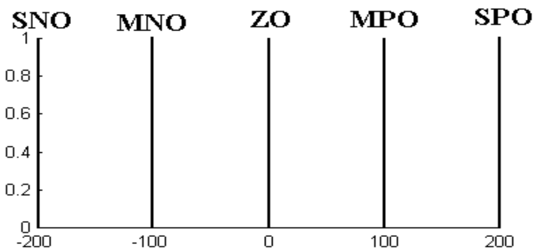

Figure 3. Output fuzzy sets as singletons

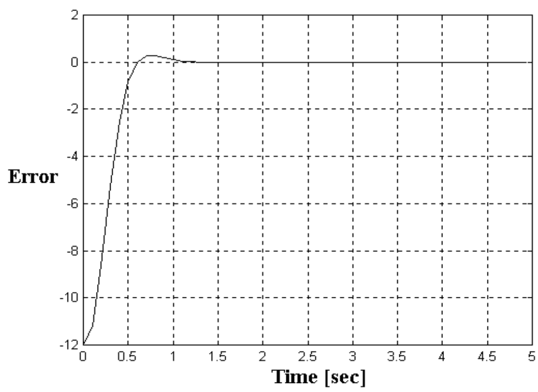

Figure 4. Error of the output relative to the setpoint value, for triangular fuzzy sets.

As the measure of quality of systems response usual performance index is used, Integral Absolute Error, IAE,

$$\text{IAE} = \frac{1}{N}\sum_{k=1}^{N}\left|e(kT)\right|. \qquad (9)$$

The smaller IAE is, the better performace of the system is. The index IAE is used to compare transient responses, for the same stop time.

For the described controller parameters, and input fuzzy sets membership functions given by Figure 2., the error *e* is obtained as given by Figure 4. It has small overshoot ,and the performance index is IAE = 4.1392.

In experiments we have changed the shapes of membership functions of input fuzzy sets. For the trapezoidal input fuzzy set, Figure 5., the error diagram is very similar to one given by Figure 4., with relatively smaller overshoot and better performance index IAE= 4.086.

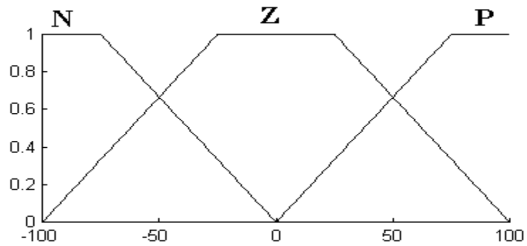

Figure 5. Trapezoidal input fuzzy sets.

For the nonlinear (bell shaped) input fuzzy set, Figure 6., the error diagram is given by by Figure 7., with much worse characteristics and IAE= 14.4621.

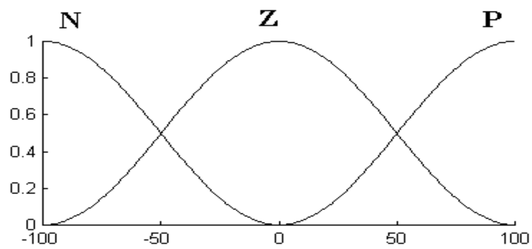

Figure 6. Nonlinear input fuzzy sets.

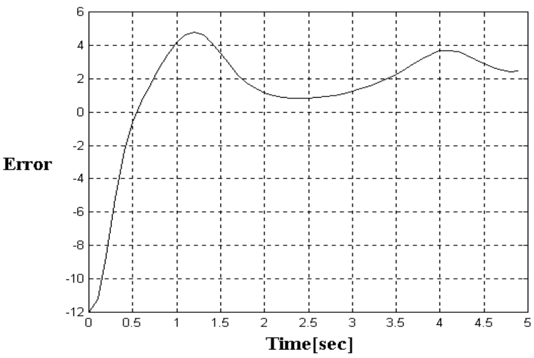

Figure 7. Error of the output relative to the setpoint value, for the bell shaped input fuzzy sets.

For the nonlinear (flatten bell shaped) input fuzzy set, Figure 8., the error diagram is given by Figure 9., with IAE= 5.9692.

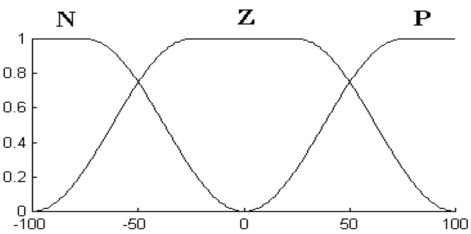

Figure 8 Flatten bell shaped input fuzzy sets.

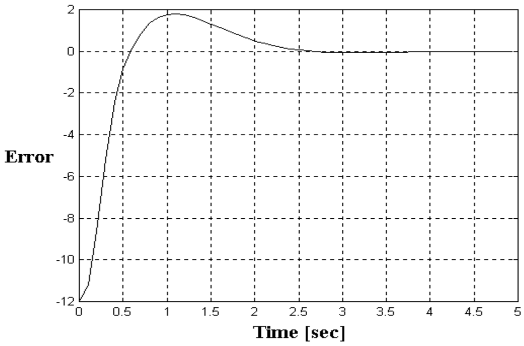

Fig 9 Error of the output relative to the setpoint value, for flatten bell shaped input fuzzy sets.

For combined linear and nonlinear input fuzzy sets, Figure 10., the error diagram is similar to the one given by Figure 4, IAE = 4.2018.

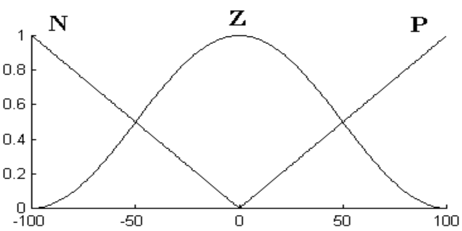

Figure 10 Combined linear and nonlinear input fuzzy sets.

For combined linear and nonlinear input fuzzy sets, Figure 10., and changed output fuzzy

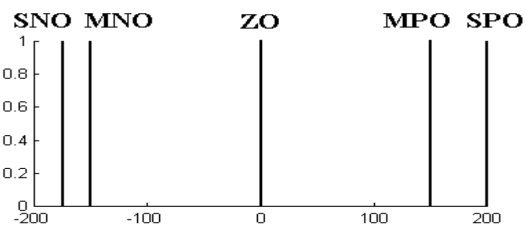

Fig 11 Different singletons.

sets, singletons, Figure 11., the error diagram is smooth, nice, with no overshoot, Figure 12., and that controller results with the best IAE = 4.0855, among the cases we have experimented with.

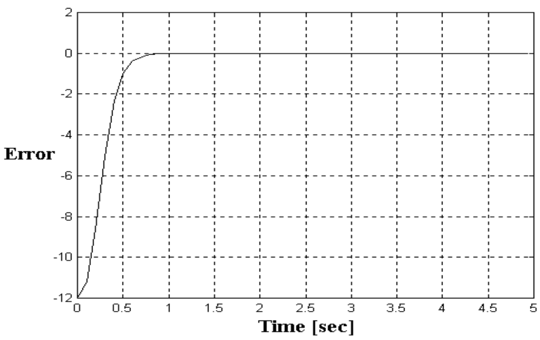

Figure 12. Error of the output relative to the setpointvalue, for the combined input fuzzy sets and changed output singletons.

5 Conclusions

The response of the considered fuzzy system and its performance are sensitive to the changes of the shapes of the membership functions. Some aspects of fuzzy system's response and performance can be tuned by the appropriate membership functions selection, for defined the other parameters of the system.

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