Blind Identification of SIMO FIR Systems

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Abstract: In this paper a new blind identification method for estimating the impulse responses of single-input multiple-output (SIMO) FIR channels is proposed. This method, that relies only on second-order statistics, allows to deal with channels affected by different unknown amounts of additive noise. Numerical simulations, based on models already considered in the literature, show the robustness of the method even with data characterized by poor signal–to–noise ratios.

Key-Words: Blind identification, linear systems, errors-in-variables models. *CSCC'99 Proceedings,* Pages:1641-1645

1 Introduction

The blind identification of dynamic systems is of great relevance in many fields of signal processing . The term "blind" denotes the impossibility of measuring the input of the system and the purpose of blind identification is the reconstruction of the transfer function of a transmission channel starting from noisy measurements performed on its output.

This problem has received an increasing attention from the researchers working in the area of signal processing, and particular emphasis has been put on the multi–output case. In fact, real processes of this type are present in many engineering applications in telecommunications, sismology, radioastronomy, biomedical signal processing, etc.

Blind identification techniques usually rely on a linear model of the process that is described by a set of parallel channels driven by an unknown sequence. The models of the channels are often characterized by a finite impulse response (FIR) which allows the implementation of fast numerical algorithms [1, 2].

Two different approaches can be used for solving the blind identification problem. The first relies on optimization techniques based on higher order statistics [3], while the second group of methodologies (such as maximum likelihood and cross–correlation techniques) uses only second order statistics [4-6]. These methods, however, give good results only when the process is characterized by high signal–to–noise ratios and the channels are affected by the same amount of noise.

In this work a new method for the blind identification of FIR multichannel systems is presented. This procedure, that can be considered as a generalization of cross–correlation methods, has been developed on the basis of the results obtained by the authors in the identification of errors–in–variables models [8, 9]. This approach gives good results also when the system is characterized by poor signal–to–noise ratios and the output measurements are affected by different amounts of noise.

The paper is organized as follows. Section 2 describes the mathematical setup of the blind identification problem for multiple–FIR–channels. The problem is then redefined as an errors–in–variables identification scheme and in this context, Section 3 proposes a new solution. Section 4 reports the results obtained in testing the procedure on a multichannel FIR model already described in the literature. Finally, some concluding remarks are reported in Section 5.

2 Statement of the problem

Let us consider a linear, discrete, time–invariant single–input multi–output (SIMO) system described by M finite impulse responses with order L (FIR's) $h_1(k), \dots, h_M(k)$ $(k = 0, 1 \dots, L)$. Let us denote with $s(\cdot)$ the input sequence and with $x_1(\cdot), \dots, x_M(\cdot)$ the corresponding M output sequences.

In absence of noise the model of the process can be described as follows

$$
\begin{aligned}
x_1(k) &= h_1(k) * s(k) \\
x_2(k) &= h_2(k) * s(k) \\
\vdots & \qquad \vdots \\
x_M(k) &= h_M(k) * s(k)
\end{aligned} \tag{1}
$$

where "$*$" denotes the linear convolution operator.

This multiple–FIR–channels model is useful to describe the case of a single unknown source in presence of multiple spatially and/or temporally distributed sensors; this is a situation common to many applications of signal processing. Also the assumption of FIR channels is, in general, very common in practical applications, since any infinite impulse response (IIR) system can be well approximated by a FIR representation, by assuming L large enough.

It can be noted, by following the idea suggested in [4], that the outputs of every channel pair are related by the corresponding channel responses. In fact, by considering relation (1) for two different channels

$$
\begin{aligned}
x_i(k) &= h_i(k) * s(k) \\
x_j(k) &= h_j(k) * s(k)
\end{aligned} \tag{2}
$$

we obtain

$$
\begin{aligned}
h_j(k) * x_i(k) &= h_j(k) * [h_i(k) * s(k)] \\
&= h_i(k) * [h_j(k) * s(k)] \\
&= h_i(k) * x_j(k) \; .
\end{aligned} \tag{3}
$$

It is thus possible to write an overdetermined set of linear equations involving the unknowns $h_i(k)$ and $h_j(k)$ which, under suitable identifiability conditions, can be uniquely determined up to a scalar factor.

More specifically, relation (3) leads to $N-L$ linear equations (where N denotes the number of samples) in the unknowns $h_i(k)$ and $h_j(k)$ ($k = 0, \ldots, L$):

$$
\left[X_i(L) \;\; | \;\; X_j(L) \right] \begin{bmatrix} c_j \\ -c_i \end{bmatrix} = 0 \; , \tag{4}
$$

where

$$
c_m = \left[\, h_m(L), \ldots, h_m(0) \, \right]^T \tag{5}
$$

and

$$
X_m(L) = \begin{bmatrix} x_m(0) & \ldots & x_m(L) \\ x_m(1) & \ldots & x_m(L+1) \\ \vdots & & \vdots \\ x_m(N-L-1) & \ldots & x_m(N-1) \end{bmatrix} , \tag{6}
$$

with $m = i, j$. In the following, for the sake of simplicity, we will consider a multiple–FIR–channels model with only two outputs, $x_1(\cdot)$ and $x_2(\cdot)$.

By introducing the covariance matrix of the bivariate process $[x_1(\cdot) \mid x_2(\cdot)]^T$, defined as

$$
\hat{\Sigma}_L = \lim_{N \to \infty} \frac{1}{(N-L-1)} \left[X_1(L) \mid X_2(L) \right]^T \times \left[X_1(L) \mid X_2(L) \right] \tag{7}
$$

relation (4) can be rewritten as

$$
\hat{\Sigma}_L \begin{bmatrix} c_2 \\ -c_1 \end{bmatrix} = 0 \; . \tag{8}
$$

It can be proved [4, 7] that the coefficients $h_1(k)$ and $h_2(k)$ can be uniquely determined (up to a scalar factor) under the assumption that the polynomials

$$
\begin{aligned}
H_m(z) &= h_m(0)\, z^L + h_m(1)\, z^{L-1} + \ldots \\
&\quad + h_m(L-1)\, z + h_m(L) \qquad m = 1, 2
\end{aligned} \tag{9}
$$

are coprime and the input $s(k)$ is persistently exciting of sufficient order.

When the data are corrupted by additive noise, the channel outputs $x_1(\cdot)$ and $x_2(\cdot)$ are not directly accessible and only the noisy signals

$$
\begin{aligned}
y_1(\cdot) &= x_1(\cdot) + n_1(\cdot) \\
y_2(\cdot) &= x_2(\cdot) + n_2(\cdot)
\end{aligned} \tag{10}
$$

can be measured. With reference to model (10) we will introduce the following assumption.

Assumption 2.1

- the processes $n_1(\cdot)$ and $n_2(\cdot)$ are zero–mean, mutually uncorrelated white noises, with unknown variances $\sigma^*_{n_1}$ and $\sigma^*_{n_2}$, respectively.
- the processes $n_1(\cdot)$ and $n_2(\cdot)$ are uncorrelated with the unknown input $s(\cdot)$ and, therefore, with the noise–free signals $x_1(\cdot)$ and $x_2(\cdot)$.

Under these assumptions, the blind identification problem can be stated as follows.

Problem 2.1 – Given N noisy observations of the channel outputs $y_1(\cdot)$, $y_2(\cdot)$, determine the variances $\sigma^*_{n_1}$ and $\sigma^*_{n_2}$ of the noises and the coefficients $h_1(k)$ and $h_2(k)$ ($k = 0, \ldots, L$).

Of course, Problem 2.1 is a basic step for channel equalization, i.e. for the reconstruction of the unknown input $s(\cdot)$.

The block diagram of the unknown multiple–FIR–channels model is shown in Figure 1.

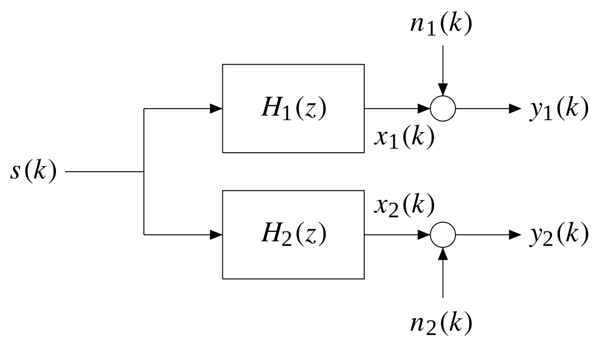

Figure 1: Block structure of the process

In the noisy case, relation (8) does no longer hold since the covariance matrix Σ_L of the noisy bivariate process $[y_1(\cdot) \mid y_2(\cdot)]^T$ is non singular (positive definite); under Assumption 2.1 the following relation holds

$$\Sigma_L = \hat{\Sigma}_L + \tilde{\Sigma}_L^* , \quad (11)$$

where

$$\tilde{\Sigma}_L^* = \mathrm{diag}\left[\sigma_{n_1}^* I_{L+1},\ \sigma_{n_2}^* I_{L+1}\right] \quad (12)$$

is the covariance matrix of the bivariate process $[n_1(\cdot) \mid n_2(\cdot)]^T$.

3 Blind identification

Let us consider the covariance matrices Σ_ℓ related to different orders, ℓ, of the FIR models, built according to relations (6) (7).

When Assumption 2.1 is satisfied and the length of the sequences $N \to \infty$, the solution of Problem 2.1 can be obtained analyzing the properties of the sequence of increasing-dimension matrices $(\Sigma_1, \Sigma_2, \ldots)$. Let us consider, for this purpose, the family of matrices $\tilde{\Sigma}_\ell = \mathrm{diag}[\sigma_{n_1} I_{\ell+1},\ \sigma_{n_2} I_{\ell+1}]$ such that

$$\hat{\Sigma}_\ell = \Sigma_\ell - \tilde{\Sigma}_\ell \geq 0 \quad (13)$$

It has been proved [8] that, for every ℓ, the compatible matrices $\tilde{\Sigma}_\ell$ are defined by the points of a convex curve, belonging to the first quadrant of the noise plane, whose concavity faces the origin. Every point $P = (\sigma_{n_1}, \sigma_{n_2})$ on this curve satisfies the relation

$$\hat{\Sigma}_\ell(P) = \Sigma_\ell - \mathrm{diag}\left[\sigma_{n_1} I_{\ell+1},\ \sigma_{n_2} I_{\ell+1}\right] \geq 0 \ . \quad (14)$$

The related FIR models $c_1(P)$, $c_2(P)$ can be obtained from the relation

$$\hat{\Sigma}_l(P)\, v_l(P) = 0 \ , \quad (15)$$

where $v_l(P) = [c_2^T(P)\ \ -c_1^T(P)]^T$.

It has also been shown that every curve includes all subsequent ones i.e. those associated with higher values of ℓ.

When Assumption 2.1 is satisfied and $N \to \infty$, the point $P^* = (\sigma_{n_1}^*,\ \sigma_{n_2}^*)$, corresponding to the actual variances of the noises, belongs to all curves associated with models with order $\ell \geq L$. The models corresponding to this point are characterized by the actual coefficients (up to a scalar factor) $c_1(P^*)$, $c_2(P^*)$. In this theoretical context, the determination of the common point in the noise plane leads to the solution of Problem 2.1.

When Assumption 2.1 is not satisfied and/or the length N of the sequences is finite, the curves corresponding to orders $\ell \geq L$ do not present any common point. Their distance should however decrease in the neighbourhood of P^*. This property can be used to obtain an estimate for the order L of the channels. Once that this order has been choosen, a single solution for the identification problem can be obtained by introducing a suitable criterion. Possible procedures can be developed by using, for istance, the rank deficiency properties of the matrices $\hat{\Sigma}_\ell(P^*)$ $(\ell \geq L)$. In fact, when Assumption 2.1 is satisfied and $N \to \infty$ the following properties hold:

i) if $\ell \geq L$ the dimension of the null space of $\hat{\Sigma}_\ell(P^*)$ and, consequently, the multiplicity of its least eigenvalue, is equal to $(\ell - L + 1)$;

ii) for $\ell > L$ all linear dependence relations between the vectors of the matrices $\hat{\Sigma}_\ell(P^*)$ can be described by the same sets of coefficients c_1, c_2.

For example, when $\ell = L + 1$ it can be easily verified that

$$\ker[\hat{\Sigma}_{L+1}(P^*)] = \mathrm{im}\begin{bmatrix} c_2 & 0 \\ 0 & c_2 \\ -c_1 & 0 \\ 0 & -c_1 \end{bmatrix} . \quad (16)$$

When Assumption 2.1 is not satisfied and/or the length N is finite this condition does no longer hold; in this case the models and the noise variances can be estimated as follows [9]. Let us consider two points $P = (\sigma_{n_1}, \sigma_{n_2})$ and $\bar{P} = (\bar{\sigma}_{n_1}, \bar{\sigma}_{n_2})$ on the curves of order L and $L+1$, respectively, linked by the following relation

$$\frac{\sigma_{n_1}}{\sigma_{n_2}} = \frac{\bar{\sigma}_{n_1}}{\bar{\sigma}_{n_2}} \ . \quad (17)$$

P and $\bar{P}$ are thus points belonging to the straight line with slope–rate (17) from the origin. It is possible to prove [10] that the coordinates of $\bar{P}$ are given by

$$\bar{\sigma}_{n_1} = \frac{\sigma_{n_1}}{\lambda_M} \qquad \bar{\sigma}_{n_2} = \frac{\sigma_{n_2}}{\lambda_M} , \tag{18}$$

where

$$\lambda_M = \max \text{eig} \left(\Sigma_{L+1}^{-1} \, \text{diag}[\sigma_{n_1} I_{L+2}, \sigma_{n_2} I_{L+2}] \right) . \tag{19}$$

It is then possible to search among the solutions $v_L(P) = [c_2^T(P) \; -c_1^T(P)]^T$ satisfying condition (15) the one which minimizes the cost function

$$J(P, \bar{P}) = \text{trace} \left(\begin{bmatrix} c_2(P) & 0 \\ 0 & c_2(P) \\ -c_1(P) & 0 \\ 0 & -c_1(P) \end{bmatrix}^T \times \hat{\Sigma}_{L+1}(\bar{P}) \begin{bmatrix} c_2(P) & 0 \\ 0 & c_2(P) \\ -c_1(P) & 0 \\ 0 & -c_1(P) \end{bmatrix} \right) . \tag{20}$$

Note that when Assumption 2.1 is satisfied and $N \to \infty$ we have $J(P^*) = 0$, i.e. the minimum of the cost function (20) is achieved in correspondence of the point associated with the actual noise variances.
On the basis of previous considerations, the following algorithm can thus be devised for the real cases.

Algorithm 3.1

1) Start from a generic point P on the curve of order L and compute the model $v_L(P) = [c_2^T(P) \; - c_1^T(P)]^T$;

2) Compute the corresponding point $\bar{P}$ on the curve of order order $L+1$ by means of relation (18);

3) Compute the value of the cost function (20).

4) Use a search procedure on the curve of order ℓ to obtain the point associated with the minimum of the function (20).

4 Numerical results

In this section the performance of the proposed method is illustrated by means of simulations. For this purpose the following two–FIR–channels model, extracted from [5], has been considered

$$\begin{aligned} H_1(z) &= -1.1836\,z^5 + 0.4906\,z^4 - 0.3093\,z^3 \\ &\quad + 0.4011\,z^2 + 0.1269\,z - 1.8522 \\ H_2(z) &= 1.2965\,z^5 + 0.0525\,z^4 + 0.3410\,z^3 \\ &\quad - 0.0260\,z^2 + 0.3991\,z + 0.8817 \, . \end{aligned}$$

Note that $H_1(z)$ has 4 nonminimum phase zeros. The input sequence $s(\cdot)$ is a coloured stochastic process with unity variance and lenght $N = 500$. The output sequences $x_1(\cdot)$ and $x_2(\cdot)$ have been corrupted by adding white noises $n_1(\cdot)$ and $n_2(\cdot)$ with standard deviations $\text{std}(n_1)$ and $\text{std}(n_2)$ ranging from 10% to 40% of the standard deviations of the noise–free signals. Note that the noiseless signals are characterized by different standard deviations ($\text{std}(x_1) = 2.35$ and $\text{std}(x_2) = 1.56$) so that the same percent amount of noise actually corresponds to different amounts of noise on the channels.

In correspondence to every percent amount of additive noise, one–hundred different simulations and identifications have been performed. The results are summarized in Figures 2–3, that report the true zeros of the channels and the estimated ones for noise levels ranging from 10% to 40%.

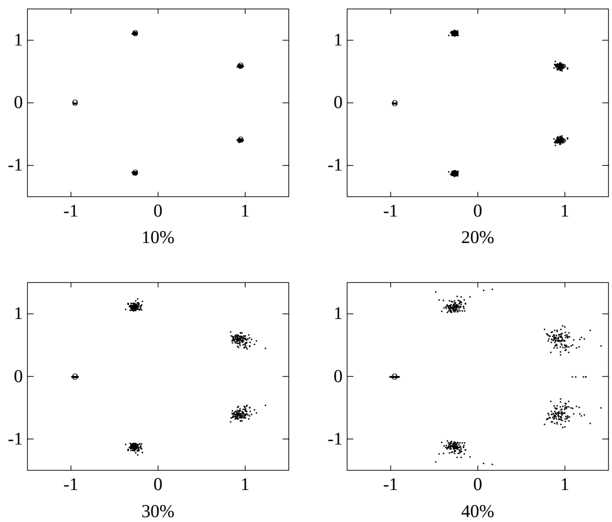

Figure 2: True and estimated zeros of $H_1(z)$

The performance has then been expressed by means of the normalized root–mean–square–error (NRMSE) defined by

$$\text{NRMSE} = \frac{1}{\|c\|} \sqrt{\frac{1}{R} \sum_{i=1}^{R} \|\hat{c}_i - c\|^2} \tag{21}$$

where R is the number of runs and $\hat{c}_i$ is the i–th estimate of the coefficients $c = \begin{bmatrix} c_1^T & c_2^T \end{bmatrix}$. Figure 4 shows the NRMSE versus the signal–to–noise ratio (SNR)

$$\text{SNR} = 20 \, \log \frac{\text{std}(\text{x}_\text{i})}{\text{std}(\text{n}_\text{i})} \; (\text{dB}) \qquad i = 1, 2 \, . \tag{22}$$

The obtained results show good estimates of the the FIRs coefficients even with low signal–to–noise ratios.

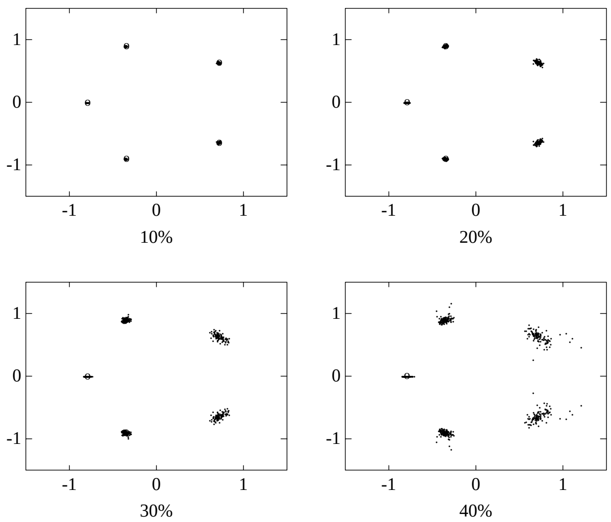

Figure 3: True and estimated zeros of $H_2(z)$

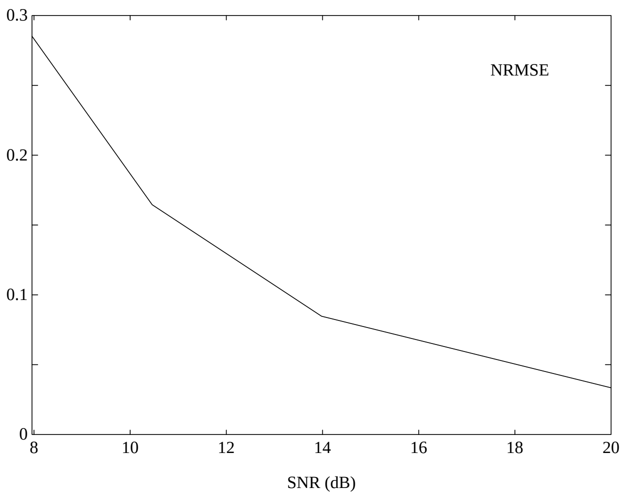

Figure 4: NRMSE versus SNR for 100 runs

5 Conclusion

In this work the blind identification of multiple–FIR–channels from noisy output measurements has been approached by extending an identification procedure developed by the authors in the context of error–in–variables identification.

This method allows to deal with output measurements affected by different amounts of noise on the channels. This unique feature of the proposed method is not shared by any of the presently available procedures.

The approach has been tested on a simulated system taken from the literature and has shown a superior performance also in presence of data characterized by poor signal–to–noise ratios.

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