General Methodology for System Design

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Abstract: - The main step of some system design is the optimization strategy that minimizes of the objective function and determines the optimum values of all system's elements. The evaluation of the operations number for different strategies gives the possibility to select the optimum or quasi-optimum strategy that is needed of the minimum computer time.

From the optimization theory point of view the traditional design strategy can be determined as an optimization of some objective function with constraints. The model of the system is the constraints in that case. From the other hand it is possible to use the general design strategy or control theory for the system design.

The evaluation of operations number for the system design has been done for general design strategy. More general methodology for the system and circuit design was elaborated by means of optimum control theory formulation. By this theory the problem of the system design can be formulated as the classical problem of the optimal control for the minimum time. In that context the aim of optimal control is to result each right hand side of the main system of the differential equations $dx_i / dt = f_i (x_1,x_2,...,x_N,u_1,u_2,...,u_M)$ to zero for the final time t_{fin} and minimize the total computer time T. These equations include the special control functions $u_1,u_2,...,u_M$ that are introduced into consideration artificially to generalize the total design process. Optimum dependencies of these control functions u_j give us the minimum computer design time. This approach generalizes the design process and generates infinite number of the different design strategies. The problem of the optimal behavior definition of the control functions u_j can be solved adequately by means of the special optimization procedure or by means of Pontryagin's maximum principle.

The analysis of different types of electronic systems shows that the optimal strategy can be fined by control theory formulation and maximum principle. In that case it is possible to reduce the total operations number to many times and accelerate the design process.

Key Words: - Optimal system design, control theory approach, maximum principle.
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1 Introduction

The electronic system design by traditional methodology includes the formulation of the principal equation system, definition the number of independent variables K and the number of dependent variables M and using some type of optimization procedure. The principal equation system can be formulated as algebraic system or integral-differential system. The systems model can be determined as the equation system relation between independent and dependent variables. From the optimization problem point of view this system can be determined as the system of constraints for the objective function optimization.

On the other hand it is possible to use the idea of general optimization [1,2] for the electronic system design. On this way the independent variables vector includes arbitrary number of the systems components from K to $K+M$. In that case the objective function includes additional penalty terms that simulate the relation equations. This strategy can reduce the total computer design time.

In this paper one approach for the system design is proposed. This method is based on the optimum control theory formulation and serves as the generalization of different design strategies. It can reduce considerably the necessary computer design time.

2 The Operations Number Evaluation for General Design Strategy

For the computer time comparison of different kinds of design strategy and for optimal algorithm elaboration it is necessary to evaluate the operations number.

By general design strategy, in case when the number of independent parameters is variable and equal to $K+Z$ the following two systems are used:

$$\frac{dx_i}{dt} = -b \cdot \frac{d\ F(X)}{d\ x_i} \qquad (1)$$

$$j = 1,2,\ldots,K+Z$$

and

$$g_j\ (X) = 0 \qquad (2)$$

$$j = Z+1, Z+2, \ldots, M$$

where $F(X) = C(X) + \frac{1}{e} \sum_{j=1}^{Z} g_j^2(X)$.

In this case the total operations number N for the solution of the systems (1), (2) is equal to:

$$N = L\{K+Z+(1+K+Z)\{C+(P+1)Z+S\cdot[(M-Z)^3+(M-Z)^2(1+P)+(M-Z)P]\}\} \qquad (3)$$

when the Newton's method is used.

In case of $Z=0$ the formula (3) gives us the operations number for traditional design strategy and when $Z=M$ formula it is a modified traditional design strategy.

Sometimes the necessary operation number C for the objective function $C(X)$ calculation has no dependency from the independent parameters number $K+Z$, but for the majority of electronic systems is in proportion to the sum $K+Z$ ($C = c\ (K+Z)$). Formula (3) in this case is transformed into following expression:

$$N(Z) = L \cdot \{K+Z+(1+K+Z)\{c(K+Z)+(P+1)Z$$

$$+S\cdot[(M-Z)^3+(M-Z)^2(1+P)+(M-Z)P]\}\} \qquad (4)$$

Analysis of the operations number N as the function of Z by formula (4) gives us the conditions of the minimum computer time. In case when the system (2) is the linear one this general design strategy almost has no preference in computer time as shown in [1]. Formula (4) gives the optimum point Z_{opt} that is within the region $[0, M]$ for the nonlinear system (2).

In more general case, when the system's model can be separate on two parts as linear and nonlinear we have the following systems :

a) the nonlinear part is given by

$$g_j(X) = 0$$

$$j = 1,2,\ldots,r\ (M-Y) \qquad (5)$$

b) the linear part is given by

$$AX = B$$

where $r \in [0,1]$; A and B are matrices of the order $(1-r)\cdot(M-Z)$. For this case the formula for the operations number has the following form:

$$N(Y,Z) = L\ \{K+Y+Z+(1+K+Y+Z)$$

$$\cdot\{C+(M+1)Z+[M(1-r)-Z]^3+M(1-r)-Z+(P+1)Y$$

$$+S\cdot[(M\cdot r-Y)^3+(M\cdot r-Y)^2(P+1)+(M\cdot r-Y)P]\}\} \qquad (6)$$

Analysis by this formula shows that for the majority of the practice problems it is correct that the optimum point of the function $N(Y,Z)$ is within the dominion. This case is illustrated in Fig. 1.

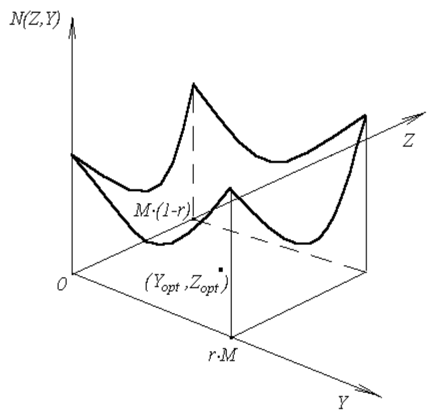

Fig. 1 Behavior of the function *N(Y,Z)*.

The optimum point (Y_{opt}, Z_{opt}) minimizes the necessary computer time for the large system design and has dependency from the electronic system size and topology. This optimal point can be fined by different methods, for example by ordinary gradient method.

The optimization of the space dimension number of independent parameters leads to reduction of the total operation number and therefore to reduction of the total computer time for electronic system design. The analysis of different types of electronic systems shows that the optimal space dimensions of independent parameters can reduce the total computer time in 10 - 50 times. This optimal space dimension has dependency from electronic systems' size and topology. In this work the problem of optimum order of the space dimension is solved by optimal control theory method. The total computer time is served as the objective function for the optimal algorithm finds.

3 Design Strategy by Control Theory Formulation

It is possible to define the problem of the optimum algorithm construction for more general case. We can determine the problem of a large system design as the problem of optimal control.

The principal equations system can be determined as:

$$\frac{dx_i}{dt} = f_i(X,U) \tag{7}$$

$$i = 0,1,\ldots,N$$

where $N=K+M$; X is the variables vector $X=(x_1,x_2,\ldots,x_K,x_{K+1},x_{K+2},\ldots,x_N)$; U is the vector of control variables $U=(u_1,u_2,\ldots,u_M)$; $u_j \in \Omega$; $\Omega = \{0;1\}$.
The sense of the variable u_j is presence (when u_j=0) or absence (when u_j = 1) of the equation number j in the system (2). The function $f_0(X,U)$ is determined as the necessary calculation time for one step of the system (7) integration. In this case the variable x_0 is determined as the total computer time for the electronic system design.

The functions of the right part of the system (7) are determined as:

$$f_i(X,U) = -b\left\{\frac{dC}{dx_i} + \frac{1}{e}\frac{d}{dx_i}\left[\sum_{j=1}^{M} u_j g_j^2(X)\right]\right\} \tag{8}$$

for $i = 1,2,\ldots,K$ and

$$f_i(X,U) = -b\cdot u_{i-K}\left\{\frac{dC}{dx_i} + \frac{1}{e}\frac{d}{dx_i}\left[\sum_{j=1}^{M} u_j g_j^2(X)\right]\right\} + \frac{(1-u_{i-K})}{dt}\left\{-x'_i + h_i(X)\right\} \tag{8'}$$

for $i = K+1, K+2, \ldots, N$;

where x'_i is equal to $x_i(t-dt)$; $h_i(X)$ is the implicit function $(x_i = h_i(X))$ that is determined by the system: $(1-u_j)g_j(X) = 0$; $j = 1,2,\ldots,M$.

In this case we determine the problem of some system design as the classical problem of the optimal control. In that context the aim of optimal control is to result each function $f_i(X,U)$ to zero for the final time t_{fin} , $f_i(X(t_{fin}),U(t_{fin})) = 0$ and minimize the total computer time x_0 . The minimum-time problem for the system (7) with non-continued or non-smoothed functions (8) can be solved most adequately by means of Pontryagin's maximum principle [3].

For the classical Pontryagin's form optimal control problem formulation it is necessary to define the conjugate system for the additional functions ψ_i :

$$\frac{d\psi_i}{dt} = -\sum_{l=0}^{N} \frac{\partial f_l(X,U)}{\partial x_i}\cdot \psi_l \tag{9}$$

$$i = 0,1,\ldots,N$$

Hamiltonian is determined as :

$$H(X,U,\Psi) = \sum_{i=0}^{N} \psi_i f_i(X,U) \tag{10}$$

This function has supreme value during the optimal trajectory with the Pontryagin's maximum principle:

$$M(X,\Psi) = \sup_{u\in\Omega} H(X,U,\Psi) \tag{11}$$

The main problem of the maximum principle application in that formulation is unknown vector Ψ_0 of initial values of the functions ψ_i. This problem has adequate solution only for linear functions $f_i(X,U)$, for example in [4]. For the nonlinear case it is possible to use one iterative algorithm for the solution of the problem (7) - (11).

This method in maximum principle formulation that is used for the solution of the optimal control problem (7)-(11) is based on the boundary problem solution for $2\times(N+1)$ order equations system (7), (9). The iteration process for the numerical integration of this system includes consecutive iterations of Cauchy problem solution.

The strategy of this method is that:

1. The initial value of vector X_0 has been given, because it is known ; $X_0 = (x_{10}, x_{20}, \ldots, x_{N0})$.
2. The initial value of vector Ψ_0 has been given arbitrary; $\Psi_0 = (y_{10}, y_{20}, \ldots, y_{N0})$.
3. The vector of control variables U is fined by the formulas (10), (11).
4. Two systems (7), (9) are solved in one time step Δt and new values of vectors X and Ψ are determined.
5. The conditions $|f_i(X,U)| < e$ are verified for all index i . If this conditions are right, in this case we pass to step number 6, if they are not right we return to step 3.
6. In that case we have the solution of the problem (7)-(11). We have the functions *X(t), U(t),* $\Psi(t)$ and the total computer design time T that is equal to x_0. This solution is not optimal because it has been obtained with arbitrary value of the vector Ψ_0 that is not correct. However, this solution is the first approximation to the optimal solution.

To minimize the total computer design time T it is necessary to improve the initial approximation Ψ_0 . This problem can be solved by different methods. First of all it is possible to use some gradient method with the calculation of the T function's gradient $\nabla T = \left(\frac{\partial T}{\partial y_{10}}, \frac{\partial T}{\partial y_{20}}, \ldots, \frac{\partial T}{\partial y_{N0}} \right)$ and movement along anti-gradient. Other way is the solution of the equations system $\frac{\partial T}{\partial y_{i0}} = 0$; $i = 1, 2, \ldots, N$ by Newton's method. In that case it is necessary to calculate the matrix of the second derivatives but the number of iterations can be reduced significantly.

4 Examples

Two simple circuits have been investigated to demonstrate the optimal control theory approach.

4.1 Example 1

In Fig.1 there is a nonlinear circuit that has 4 independent variables (K=4) as admittance: y_1, y_2, y_3, y_4 and 3 dependent variables (M=3) as nodal voltages: V_1, V_2, V_3 in the nodes 1, 2, 3.

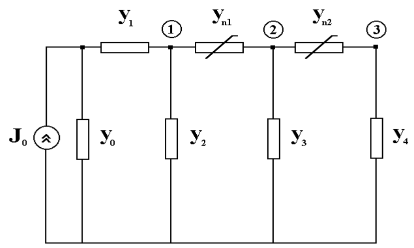

Fig. 1 Circuit topology for K=4 and M=3.

We suppose that other elements are defined and each nonlinear admittance has dependency by the law $y_{ni} = a_{ni} + b_{ni} \cdot V_i$, where $i = 1$ or 2, $b_{n1} = b_{n2} = 0.001$. The results of the analysis of a complete set of the design strategies that were obtained by general design idea are given in Table 1.

Table 1. A complete set of the design strategies by general design idea for example 1.

N	Vector of the control functions U (u_1,u_2,u_3)	Total design time (sec)	The iterations number
1	0 0 0	16.53	3514
2	0 0 1	48.66	28170
3	0 1 0	22.25	13295
4	0 1 1	57.78	35895
5	1 0 0	9.28	5354
6	1 0 1	56.41	30618
7	1 1 0	25.70	14512
8	1 1 1	46.91	38191

The first line of this table corresponds to the traditional design strategy. All control functions u_j (j = 1, 2, 3) are equal to 0. We have two traditional parts of the design process in this case. One part is the system of three equations for three nodal voltages and the other is the optimization procedure. The gradient method is used for all examples as the optimization procedure. The total computer design time in this case is equal to 16.53 sec. It is interesting that the modified traditional strategy (u_j = 1; j = 1, 2, 3) has the total computer design time for this example almost 3 times more than traditional strategy. It can be explain by very small values of the non linearity parameters b_{n1}, b_{n2} .

More interestingly is that among the rest of the strategies exists the strategy number 5 that has minimum of the design time for all strategies that can be defined by means of general strategy idea. This design time is equal to 9.28 sec and is almost 2 times less than for traditional strategy. However this strategy is not optimal. It is necessary to find the optimal strategy by means of some optimization procedure or by maximum principle.

Data of the optimum and some quasi-optimum strategies are given in Table 2.

Table 2. Optimum and quasi optimum strategies for example 1.

N	Vector of the control functions U (u_1 u_2 u_3)	Total design time (sec)	The switching points	The iterations number
1	(0 0 0); (1 0 0)	8.34	3	4828
2	(1 1 1); (1 0 0)	9.23	1	5352
3	(1 0 0); (0 0 0)	5.54	3055	3116
4	(0 1 0); (1 0 0)	8.62	40	4985
5	(0 0 1); (1 0 0)	8.24	30	4792
6	(0 1 1); (1 0 0)	8.68	5	5025
7	(0 0 1); (1 0 0); (0 0 0)	4.56	30; 2500	2559
8	(0 0 1); (1 0 0); (1 1 1)	4.45	30; 2420	2646

All given strategies have computer design time less than the best strategy number 5 from Table 1. The strategy number 8 is optimum one and has the minimum computer design time that is equal to 4.45 sec. This strategy has the gain almost four times with respect to the traditional design strategy.

The behavior of the control functions u_1, u_2, u_3 during the total design process is shown in Fig. 2.

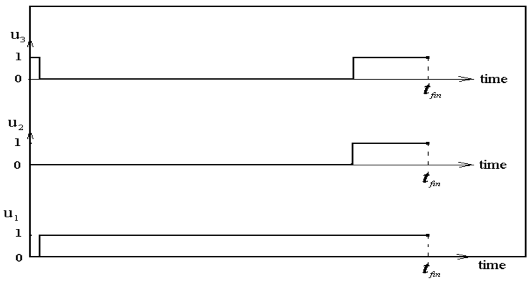

Fig. 2 Optimum dependency of the control functions for example 1.

These time dependencies have no any definite law and can be obtained by means of a special optimization procedure with the maximum principle application.

4.2 Example 2

In Fig. 3 there is a nonlinear circuit that has 6 independent variables as admittance: $y_1, y_2, y_3, y_4, y_5, y_6$ and 5 dependent variables as nodal voltages: V_1, V_2, V_3, V_4, V_5 in the nodes 1, 2, 3, 4, 5.

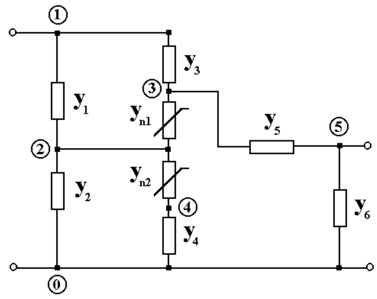

Fig. 3 Circuit topology for K=6 and M=5.

The results of the analysis of a complete set of the design strategies that were obtained by general design idea are given in Table 3.

Table 3. A complete set of the design strategies by general design idea for example 2.

N	Vector of the control functions U (u_1 u_2 u_3 u_4 u_5)	Total design time (sec)	The iterations number
1	00000	168.40	2071
2	00001	350.86	6141
3	00010	138.80	1926
4	00011	278.63	5615
5	00100	154.50	2103
6	00101	305.33	6081
7	00110	93.76	1925
8	00111	184.49	5741
9	01000	155.11	2151
10	01001	310.05	6388
11	01010	91.23	1896
12	01011	195.04	6194
13	01100	106.11	2159
14	01101	206.96	6439
15	01110	21.09	1621
16	01111	62.34	6082
17	10000	118.26	2085
18	10001	238.77	6137
19	10010	84.09	1736
20	10011	176.69	5492
21	10100	103.54	2107
22	10101	199.27	6197
23	10110	49.05	1561
24	10111	113.70	5475
25	11000	106.28	2199
26	11001	213.22	6765
27	11010	44.49	1417
28	11011	150.82	7268
29	11100	70.42	2200
30	11101	141.05	6857
31	11110	18.89	1859
32	11111	30.10	8297

There are 32 different strategies in this case. The first line of the table corresponds to the traditional design strategy. The last line corresponds to the modified traditional strategy. For this example the modified traditional strategy has gain in computer time more than 5 times with respect to traditional strategy, but it is not the most effective. Among all these strategies there are two strategies, number 15 and number 31 which have the total design time less than modified traditional strategy. The strategy number 31 has design time that equals to 18.89 sec, but as for the example 1 this strategy is not optimal either. Optimum trajectory can be fined by special optimization procedure.

Data of the optimum and some quasi optimum strategies are given in Table 4.

Table 4. Optimum and quasi optimum strategies for example 2.

N	Vector of the control functions U ($u_1u_2u_3u_4u_5$)	Total design time (sec)	The switching points	The iterations number
1	(01001); (11110)	10.71	1	1052
2	(11010); (01110)	9.56	100	596
3	(11011); (11110)	8.12	46	757
4	(01011); (11110)	7.91	20	744
5	(00110); (11110)	7.86	1	768
6	(11110); (01110)	7.80	38	610
7	(01010); (11110)	7.75	12	716
8	(10011); (11110)	7.69	2	755
9	(11001); (11110)	7.58	4	737
10	(01110); (11110)	7.57	79	725
11	(11111); (11110)	7.14	121	781
12	(11010); (11111)	4.39	61	740
13	(11110); (11111)	2.86	11	770
14	(11010); (11110); (11111)	3.07	3; 50	746
15	(01111); (11110); (11111)	2.96	6; 11	789
16	(11110); (11111); (00000)	2.85	11; 748	749
17	(11110); (11111); (00000); (11111)	2.47	15; 424; 425	639

All strategies of this table have computer design time less than the best strategy number 31 from Table 3. The strategy number 17 is optimum one and has the minimum computer design time that is equal to 2.47 sec. This strategy has the time gain 68 times with respect to the traditional design strategy and 12 times with respect to the modified traditional strategy.

The behavior of the control functions u_1, u_2, u_3, u_4, u_5 during the total design process is shown in Fig. 4.

Fig. 4 Optimum dependency of the control functions for example 2.

These time dependencies have no any definite law and were obtained by a special optimization procedure too.

5 Conclusions

The optimum design algorithm depends on the number and the order of the equations that are excepted from the main system. The problem of the optimum algorithm construction can be solved more adequately on the base of the optimum control theory application. Maximum principle serves in that case as the base for all control functions determination. In that case we have as the result the optimal trajectory X and optimal dependency of the control functions u_j. These optimal control functions can be used for the minimization of the computer design time for some systems that have similar topology. In that case it is possible to reduce the total computer time for a large system design.

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