A New Model for A Complex Computer Network Reliability Design and Its Solution Using Trust Region Method

1st. Baith Mohamed(1), M. El-Alem(2), Belal Ayyoub(3)

Abstract

A model that maximizes the reliability and minimizes the cost for a complex computer network reliability design is introduced. The model takes in to its consideration, for the first time to the best of our knowledge, the criticality of network links. The model problem is formed as a nonlinear optimization problem. The model problem is solved using an efficient algorithm that uses trust-region active-set approach to ensure convergence from any starting point. Our numerical experiments show that our approach is promising especially for complex computer networks.

Introduction

The ultimate objective of this paper is to give design engineers procedures to enhance their ability to design network for which reliability is an important consideration. Ideally, one would like to generate network design algorithms that take as input the characteristics of network components as well as network criteria, and produce as output an optimal network design. This is known as network synthesis, and it is very difficult to achieve. Instead, we consider a network that is already designed then try to improve this design by maximizing the link reliability, which will maximize the overall network reliability

In the most theoretical reliability problems the two basic methods of improving the reliability of systems are improving the reliability of each component or adding redundant components. Of course, the second method is more expensive than the first.

Our paper considers the first method, in which, addition to being cheaper, can be converted when needed to the second method. We present a new model, which considers the measure of criticality for each link. The model involves only probability of success to the links as a variable.

Our aim is to obtain the optimal system design with the following constraints:

1. basic cost reliability data to links. We use a linear-cost-reliability relation for each component [2].
2. criticality of links. The designer should take this in to account before building a reliable network because of its importance in saving money and arranging the priority of maximizing link reliability.

Notation

In this section we define all parameters used in our new model.

Rn	: Reliability of network.
pi	: Probability of data to pass through link i successfully or, the reliability of link i.
f	: Certain flow (packet units).
qi	: Probability of failure of link i.
Qn	: Probability of failure to network.
n	: Total number of links.
i	: Link number.
ICRi	: Index of criticality measures [3].
ISTi	: Index of structure measures [3].
Ct	: Total cost of links.

1) Arab Academy for Science and Technology, College of Engineering and Technology, Computer Engineering Department Alexandria, Egypt,
2) Department of Mathematics, Faculty of Sciences, Alexandria University, Alexandria Egypt,
3) Arab Academy for Science and Technology, College of Engineering and Technology Electronics and Communication Department Alexandria Egypt,
(This author would like to thank Prof. M.Saeb for his useful suggestion).

Ci	: Cost of link (Production, design, and maintenance costs).
Cc	: The capital money or the budget of improvement.
ai:	Initial value to accept level of critical link in maintenance and repair
$p_{(i)min}$	: Minimum accepted threshold of link i reliability.

Mathematical model

In this section, we present the assumptions under which the formulation of our new model is presented.

A. Assumptions.

1. There are many different methods used to derive the expression of total reliability of complex network, which are derived in a certain flow of data-gram. We state our network expressions according to the methods of paper [3], [4] and [5].
2. We used a cost-reliability curve to derive an equation to express each cost link according to its reliability and then the total network cost will be additive in term of cost at constitute link. See figure (1).

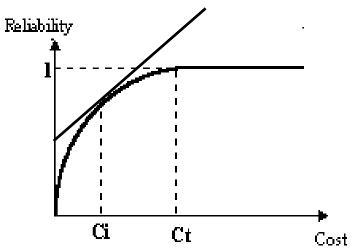


Figure (1): Cost-reliability curve

3: We calculate the ICRi for each link from its structural measure [3], which is given by,

ICRi = (ISTi ∗ qi /Qn),

Where;

ISTi = (dR_n / dpi).

-Every ICRi must be lower than initial value a_i this value is a minimum accepted level of criticality measure to every link.

B. Formulation of the problem.

The objective function in general, has the form : Maximize,

Rn= f (p_1, p_2, p_3,…, p_n).

This function is to be maximized subject to the following constraints.

1. ICRi ≤ ai. i=1,2,…,n.

These constraints guarantee that the solution is in an acceptable level of maintenance and repair value.

$$2\sum_{i=1}^{n} (p_i - p(i)_{\min}) /(1- p(i)_{\min} \le C_C$$

$$i.e \quad \sum_{i=1}^{n} p_i * C_t \le C_C$$

This ensures that the total cost of links is less than or equal to the budget.

3. $p_{(i)min}$ ≤ pi ≤ 1, i=1,2,…n.

This set of constrains permits only positive link cost.

Method of solution

The algorithm used was suggested by El-Alem[6]. It uses an active-set technique with the trust-region approach to ensure global convergence from any starting point. The algorithm is iterative. At each iteration, the set of active constants is identified using some indicator matrices (see [6]). On this set, the trial step is computed using a projected Hessian technique in the tradition of numerous work on equality constrained optimization. See for example, Dennis, El-Alem, and Maciel[8]. At each iteration, two model trust-region sub-problems are solved to obtain a trial step. The two trust-region sub-problems are similar to the trust-region sub-problem for the unconstrained optimization [7]. The first component of the trial step decreases the amount of infeasibility. The second component is computed on the null space of the first one. It goes towards optimality. The starting point can be infeasible. The solution obtained by this algorithm is feasible and satisfies the optimality conditions (see, Fiacco and McCormik[9]). Namely, the gradient of the Lagrangian function is zero and the solution satisfies the constraints. More details about this algorithm can be found in [6].

The algorithm described above was programmed on Matlab version 4.2a with machine epsilon about (10^{-16}).

In the next section, we present our numerical experience with the network reliability model descried above.

Experimental Results

_In the following examples, we use famous networks cases configurations like a Bridge, Delta, and Arpa network.

Case study I:

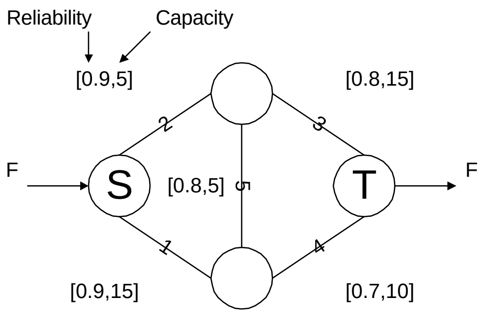


Figure (2): Bridge network example

To find the polynomial for a complex, network we must know that it is always given at a certain amount of flow to be transmitted from source (s) to sink (t), see Figure (2), (i.e. The polynomial will differ from a flow to another.) The Objective function to be maximized has the form**:**

$Rn(10)=1-(q_1+q_4.q_5.p_1+\ q_3.q_4.p_1.p_5 + q_2.q_4.p_1.p_5.p_3)$,

Maximizing the above function is equivalent to minimizing the following function [3].

$Qn =(q_1+q_4.q_5.p_1+q_3.q_4.p_1.p_5+ q_2.q_4.p_1.p_5.p_3)$.

Where,

$Rn= 1 - Qn$, and $q_i =1-p_i$.

This function will be minimized under the following constraints.

1.The ICRi constrain.

$ICRi \le 0.3$. . i=1,2,…,5.

We choose common value for ai.

2. We use the values in the Figure (2) as initial values for links reliabilities to improve the network:

$p_{(1)min}=0.9$, $p_{(2)min}=0.9$, $p_{(3)min}=0.8$, $p_{(4)min}=0.7$, $p_{(5)min}=0.8$.

3. We use the cost-reliability curve as seen in Figure (3),

$$Ci=\sum_{i=1}^{5} (pi-p(i)min)/(1-p_{(i)min}) \le 3.9995$$

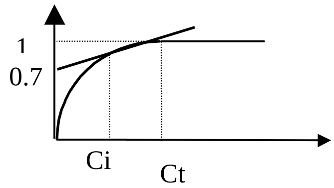


Figure (3): Cost-reliability curve

We choose the cost-reliability curve to permit distribution of cost depending on ranking of links according to their criticality.

The model was built in such a way that it reduces the failure of the most critical links. This is done by increasing the reliability of the most critical links, which tend to maximizes the over all reliability, what is our target. We summarized our results in the following table (1) and table (2).

	Computed value	**Rank**
p_1	0.9999	1
p_2	0.9985	3
p_3	0.9125	4
p_4	0.9999	2
p_5	0.8902	5
Rn	0.9999	

Table (1): links and network reliabilities

	Value in units
C_1	0.9999
C_2	0.98
C_3	0.5625
C_4	0.9999
C_5	0.451
C_t	3.9989

Table (2): The links and network costs

Case study II:

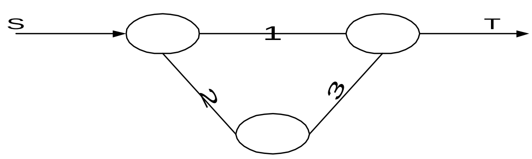


Figure (4) Delta network

Using the same procedures as in example (1), we obtain the following optimization problem for Delta network given in Figure (4).

Max. $Rn = p_1 + p_2.p_3 - p_1.p_2.p_3.$

Subject to

$ICRi \leq 0.3$

$$\sum_{i=1}^{3} C_i. * p_i \leq 5.4$$

$p_i \geq 0.7. \quad i=1,2,3.$

The following two table's (3) and (4) summarized the results.

	Computed value	Rank
p_1	0.9999	1
p_2	0.700	3
p_3	0.9378	2
Rn	0.9999	

Table (3): Links and network probabilities:

	Cost values
C_1	0.9999
C_2	0.7000
C_3	0.9378
C_t	2.6377

Table (4): Links and network costs

Case study III:

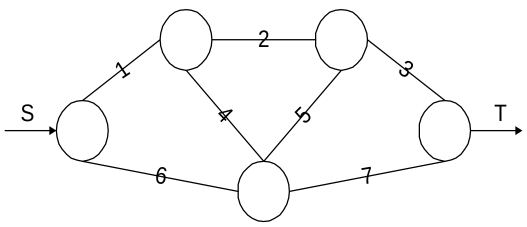


Figure (5): An Arpa network

For Arpa network given in Figure (5), the optimization problem has the form.

$Rn = p_1.p_4.p_6.p_7 + p_1.p_2.p_3.p_6.p_7 - p_1.p_2.p_3.p_4.p_6.p_7 + p_1.p_2.p_5.p_6.p_7 - p_1.p_2.p_4.p_5.p_6.p_7 - p_1.p_2.p_3.p_5.p_6.p_7 + p_1.p_2.p_3.p_4.p_5.p_6.p_7$

Subgect to.

$ICR\ i \leq 0.5$

$$\sum_{i=1}^{7} Ci.\ (p_i) \leq 6.6$$

$p_i \geq 0.5 \quad i=1,2,\ldots,n.$

The following two tables (5) and (6) show the results

	Computed value	Rank
p_1	0.9999	1
p_2	0.500	7
p_3	0.7964	6
p_4	0.9999	4
p_5	0.9847	5
p_6	0.9999	2
p_7	0.9999	3
Rn	0.9999	

Table (5): Links and network probabilities

Ci	Cost values
C_1	0.9999
C_2	0.5000
C_3	0.7964
C_4	0.9999
C_5	0.9847
C_6	0.9999
C_7	0.9999
C_t	6.283 units

Table (6): Links and network costs

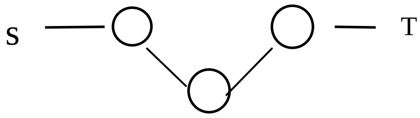
Important Comments

About case study I:

	ai≤ 0.01	ai≤ 0.1	ai≤ 0.3	ai ≤0.9
pi	0 ≤ pi ≤1	0 ≤ pi ≤1	0 ≤ pi ≤1	0 ≤ pi ≤1
p1	0.9999	0.9999	0.9999	0.9999
p2	0.8100	08937	0.9962	0
p3	0.0800	0.0506	0.0018	0
p4	0.9999	0.9999	0.9999	0.9999
p5	0.0913	0.0491	0.0018	0
Rn	0.9999	0.9999	0.9999	0.9999
capit-al	3.00	3.00	3.00	2.00
Ct	2.9927	2.9934	2.9998	2.00

Table(7):Varying of ai and its effect on Rn and Ct.

-In order to understood the depending of the total reliability and cost on ai, problem 1 was resolved for different values of ai table (7) summarizes the results. As we can see easily see in table (7), decreasing the value of ai will cause a decrease in Ct and an increase in Rn. The most critical link has the priority to be improved according to a certain capital of money Cc. In addition, we can optimize the configuration of the network as shown in column 5 of table (7). (When ai ≤0.9, and the capital is 2.00). The new configuration for a network will be as shown in Figure (6).



Figure(6):Optimum configuration

The ranking of links obtained by our method consider with those obtained in [3], see table (8).

link No.	ICRi ranking in [3]	Our (Pi) ranking
1	1	1
2	5	5
3	3	3
4	2	2
5	4	4

Table (8): Ranking of links according to its criticality [3].

About case study II:

-We summarize our experimental results in a table (9) below:

You can find the change of the cost and Rn while we increase the value of ai.

	ai ≤ 0.01	ai ≤ 0.1	ai ≤ 0.3
p1	0.9999	0.9999	0.9999
p2	0.7000	0.7000	0.7000
p3	0.9025	0.9212	0.9378
Rn	0.9999	0.9999	0.9999
Ct	2.20504	2.2424	2.27569

Table (9): Changing ai for Delta networks (This values taken at: Capital =5.4units, and 0.7≤Pi ≤1).

-As we see in table (9), decreasing ai give us lower cost and better link reliabilities cost.

- In addition to that we let a_1=0.9, a_2 and a_3=0.5, (less reliable links), the C_t became 2.1799. (which is lower).

About case study III:

	ai ≤ 0.1	ai ≤ 0.5	ai ≤ 0.8
p_1	0.9999	0.9999	09999
p_2	0.5000	0.5000	0.5491
p_3	0.9890	0.7964	0.5192
p_4	0.9999	0.9999	0.9999
p_5	0.9939	0.9847	05000
p_6	0.9999	0.9999	0.9999
p_7	0.9999	0.9999	0.9999

pi	0.5 ≤ pi ≤1	0.5 ≤ pi ≤1	0..≤ pi ≤1
Capital	6.6	6.6	5.6
R_n	0.9999	0.9999	0.9999
C_t	6.483	6.461	5.586

Table (10): Changing ai and its effect on Rn, Pi and Ct

- From the results in table (10), decreasing the value of ai will decrease Ct, until ai reach to a certain value (0.1), the Ct will increase, this lead us that ai have a certain level to apply our approach.

General comments

-To under stand the effect of removing the ICR constraints from the model, we resolved problem

The results (when removing ICRi constraints) are:

p_1=0.9999,p_2=0.8949,p_3=0.8949.

C_t=2.7898 (at: **0.7≤Pi ≤1).**

Comparing with table (9), you can see the difference. Notice that C_t=2.7898 is bigger that Ct= obtained in table (8).

-We compare our results of case study, against these different method, (see [2] and [11]). The results listed below. See table (11).

	SA results	I-NESA	[11]	Our results
p1	0.93566	0.93747	0.93924	0.9999
p2	0.93674	0.93291	0.93454	0.9999
p3	0.79299	0.78485	0.77154	0.9235
p4	0.93873	0.93641	0.93938	0.9999
p5	0.92816	0.93342	0.92844	0.8929
Rn	0.99001	0.9900	0.99004	0.9999
Ct	5.01997	5.01993	5.02001	4.816

Table (11) : A comparison between our results and global optimal solution all problems have the same qualifications.

- As seen in the above table the total cost obtained by method was the smallest, this indicates the affectedness of our approach.

- Measure of ICRi is an efficient constraint for network design and operation. Network optimization, inspection, and generation of maintenance schedules can be based on the links, rank. In case of emergencies, the knowledge of network links relative criticality helps making rational designs regarding the allocation of material, labor and time for repairing different failed links.

-The results show that it is better to improve the less reliable link (and with high critical), than to improve the high reliable link (with most critical).

-In our new model which include the optimal level of links reliability and can be converted to a number of redundancies when in each of stages simultaneously we can distinguish which is better is to improve the individual link reliability or to adding link redundancy ; this what was the Tillman hopped to do this in his 'future work' paper in [10].

-it should be noted that we can apply our model in any communication network.

- As seen from the results, the trust approach is effective, and can handle this kind of problem very easily. The next step of generalizing our model is to consider a network where each of its node is network by itself. The mathematical model of such problem is a multilevel optimization problem.

-Extending our trust-region algorithm to handle such problem is a research topic that deserves to be investigated.

Conclusion

To solve complex network design problem:

1. we must formulate a model, that is correctly represent the real problem.
2. to the best of maximization of total reliability and minimization of the total coast of a network take in to consideration the link ranking according to its criticality, then arrange the most critical links gradually; then try to resolve the model by decreasing the critical links until reach to the biggest value of Rn with minimum paid cost.
3. Trust-region method is an efficient technique to solve complex computer design problems (especially those of multi constraints).

References

[1] F. Moskowits and J.B. Mclean, "Some reliability aspects of system design" IRE Tran. Rel. Quality Control, vol. RQC-8, pp. 7-35, Sept. 1956.

[2] V. Ravi, B.S.N. Murty, P.J. Reddy "Non equilibrium Simulated Annealing-Algorithm Applied to reliability Optimization of Complex System". IEEE, Trans. on reliability, vol46, No. 2, June 1997.

[3] Magdy saeb, Roland Schinzinger"On Measures of Network Reliability and Critical Components", IEEE, Trans. on Reliability 1988.

[4] W.H Debany, Jr., P.K. Varshney,C.R.P Hartmann "Network Reliability Evaluation Using Probability Expressions", IEEE, Trans. on reliability, Vol. R-35, No. 2, 1986june.

[5] K.K. Aggarwal, Y.C. Chora, J.S. Bajwa" Capacity Consideration in Reliability of Communication Systems

[6] M. El-Alem "A new Active-set Trust Region Algorithm for general nonlinear programming1999, (submitted for pub-lication).

[7] J.E Dennis, JR. Robert B. Scnabel. Numerical methods for unconstrained optimization and nonlinear equations, 1983.

[8] A.S Elbakry, R. A. Tapia, Y .Zhang, and T. Tsuchiya. On the formulation and theory of the Newton Interior- Point Method for Nonlinear Programming. Journal of Optimization Theory and Applications, vol.89(3), pp. 507-542, 1996.

[9]A.V. Fiacco and G.P. McCormick. Nonlinear Programming: Sequential Un-constrained Minimization Techniques, john Willey and Sons, New Yourk, 1968.

[10] Frank A. Tillman, Ching-Lai Hwang, Way Kuo. "Optimization Techniques for System Reliability with Redundancy- A Review"..IEEE, Tans. on reliability, Vol. R-26, No. 3, August 1977.

[11] C.Mohan, K. Shanker 'Reliability optimization of complex system using random search techniques', microelectronics & Reliability, vol.28, 1988, pp513-518.