Limit Cycles in Wave Digital Filters One-Port versus Two-Port Approach.

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Abstract: - In this paper we present the results obtained from the comparative study of limit cycles in fixed-point implementation of different ladder wave digital filters. Part of them were derived using the one-port approach and the rest of the structures employing the two-port approach. Various Butterworth, Chebyshev and Elliptic filters have been analyzed in detail and the information obtained can be seen summarized in different tables and figures. In order to evaluate the real impact of these oscillations in practical situations, maximum amplitude and period of limit cycles have been obtained. Although hidden oscillations don't appear at the output of the filter, the remaining energy at the inner storage registers can degrade the good performance of the filter, so this kind of cycles has also been analyzed.

*Key-Words:*limit cycles,ladder wave digital filters,fixed point,one-port approach,two-port approach.Proc.pp..2071-2074

1 Introduction

One reason to transform analog networks of low losses into their corresponding digital networks according to the wave digital filter theory is their low sensitivity to variations in component values [1]-[2]. So, in order to obtain a ladder wave digital filter we need the correspondent LC analog filter. This relationship between analog and digital domains is the principal reason for the many interesting properties of wave digital filters.

It is known that finite wordlength conditions can result in parasitic oscillations at the output of a digital filter working with zero input signal. These oscillations can degrade the filter behavior to such an extent that for certain applications it could not be used. It is therefore desirable to be able to predict if a digital filter is free or not from limit cycles, and it is also very useful to know the maximum amplitude and period of the oscillations that can appear when the filter is not limit cycle free.

2 Background

Different theoretical studies about limit cycles in wave digital filters can be found but not exhaustive practical works that value the supposed good performance of these filters in relation to the parasitic oscillations.

Although it has been shown that it is possible to eliminate these cycles when certain constrains are imposed on quantization at the outputs of the adaptors, this suppression is achieved at the expense of some additional complexity, and, in general, when no restrictions are imposed limit cycles may occur. So, in this work, once filters have been designed, no specific means are employed to eliminate oscillations, and under these conditions we perform the exhaustive search of limit cycles.

2.2 One-port approach

To derive a wave digital filter from a reference analog network we need to translate elements and sources to the digital domain and, in order to complete the equivalence, we must also simulate the interconnections (topological constrains). The lumped components, applying the bilinear transformation, are converted in a simple digital building block with one input and one output. The interconnection to obtain the complete filter is achieved using special n-ports circuits called adaptors (series or parallel). This technique is known as the one-port approach [1] and was developed by Fettweis. In figure 1 a wave digital filter derived from a 3rd order elliptic LC structure by means of the one-port approach is shown.

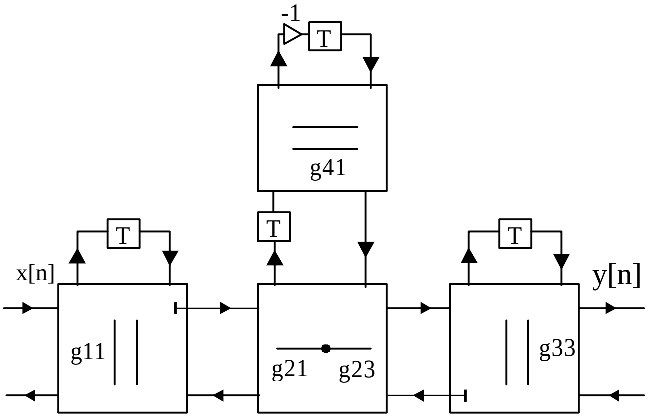

Figure 1: One-port approach

2.2 Two-port approach

The two-port approach was developed by Lawson and Constantinides [3]. In this case, each inductor and capacitor in the ladder reference circuit is considered as a two-port network described by the ABCD parameters. Applying the bilinear transformation we obtain the corresponding digital building block and finally we can connect these blocks directly without using adaptors. So, we transform each analog two-port (series inductor, series-tuned circuit, parallel capacitor, parallel-tuned circuit ...) into digital form.

Figure 2 shows the overall digital structure corresponding to the same 3rd order elliptic filter, the design was carried out using the two-port approach from the source-end.

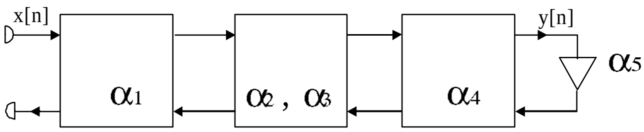

Figure 2: Two-port approach.

2.3 Exhaustive search of limit cycles

In this work have employed the general method described in [4] for searching the presence or not of limit cycles in fixed-point digital filters. This procedure is very broad and useful for any digital filter structures. In order to obtain the bounds for both maximum amplitude and period in a parasitic oscillation, the system of linear constant-coefficient equations that characterises the filter is studied and the way the quantification errors spread from one equation to another is analysed. Figure 3 corresponds to the flowchart of the exhaustive search algorithm. When we are testing an initial state vector x(0), the apparition of a limit cycle usually requires several recursions, so the algorithm runs faster if we estimate such a number of iterations (Ttrans) in order to detect the oscillation as soon as possible.

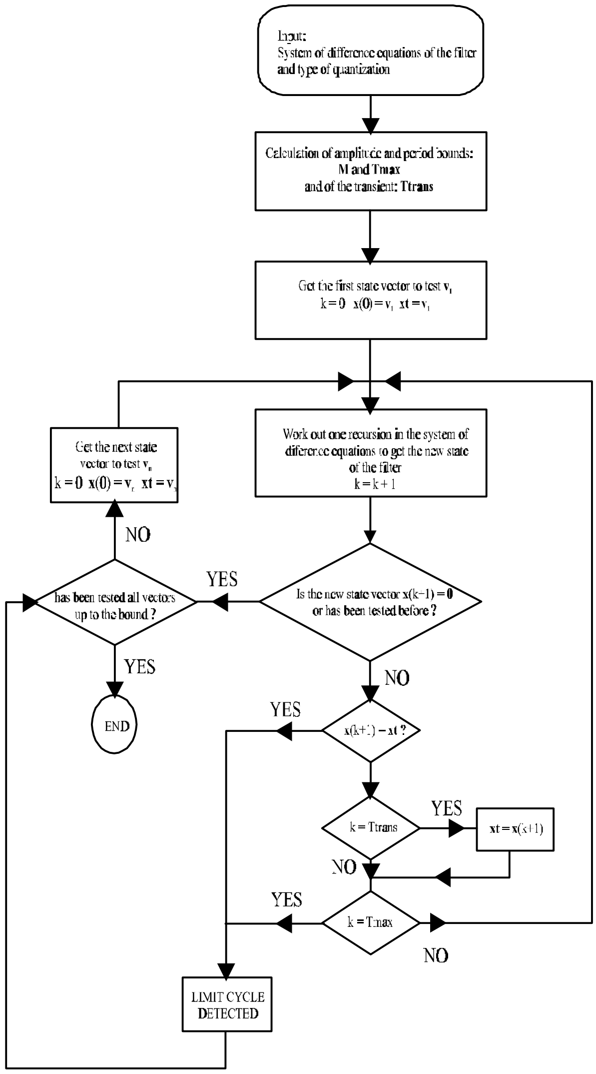

Figure 3: Flowchart of the algorithm.

3 Results

The following tables correspond to the analysis of six different low pass filters implemented with fixed-point arithmetic based on 16 bits. The normalised specification employed for all of them is as follows: passband edge frequency fp=0.3, stopband edge frequency fs=0.6, passband ripple αp=0.5 dB and minimum stopband attenuation αs=20 dB. Butterworth filters obtained are of 5^{th} order, Chebyshev of 4^{th} order and Elliptic of 3^{rd} order. The maximum values of the output, state and period of the limit cycles detected during the exhaustive search can be seen summarised in different tables, and for reasons of simplicity all the values are expressed as integer multiples of the quantization step. The three-dimensional graphics represents the states reached in the different limit cycles that have been detected during the analysis of two 3rd order Butterworth filters. Each axis represents the value stored in the corresponding inner register.

Aproach	Type	Output	State	Period
One-port Approach		1	$\begin{bmatrix}1\\4\\2\\4\\1\end{bmatrix}$	6
	Chebyshev	3	$\begin{bmatrix}2\\9\\3\\5\end{bmatrix}$	16
	Elliptic	4	$\begin{bmatrix}8748\\32504\\8748\\32502\end{bmatrix}$	26
Approach	Butterworth	2	$\begin{bmatrix}1\\4\\2\\4\\1\end{bmatrix}$	10
		1	$\begin{bmatrix}2\\7\\1\\1\end{bmatrix}$	28
		10	$\begin{bmatrix}9\\6\\6\\9\end{bmatrix}$	4

Table 1: Rounding results

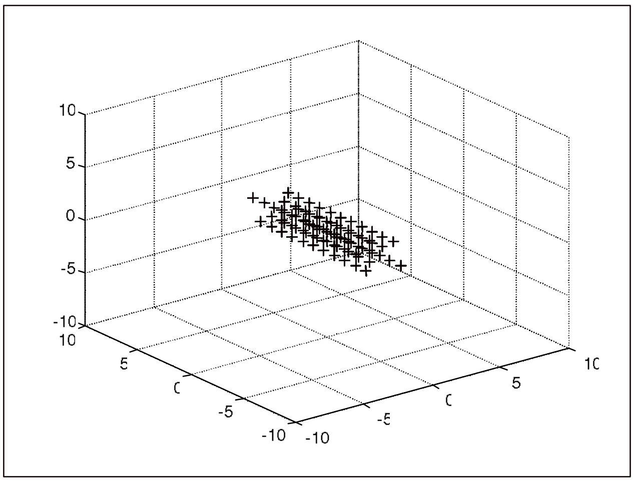

Figure 4: States reached in the limit cycles detected working with rounding and the one-port approach.

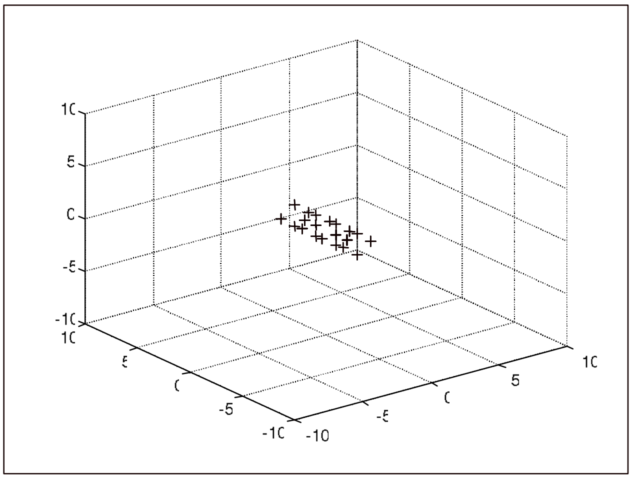

Figure 5: States reached in the limit cycles detected working with rounding and the two-port approach.

For the filter designed with the one-port approach have been detected 87 different possible states while in the filter corresponding to the two-port approach only appear 20.

Aproach	Type	Output	State	Period
One-port Approach		3	$\begin{bmatrix}2\\3\\2\\3\\2\end{bmatrix}$	7
	Chebyshev	2	$\begin{bmatrix}1\\5\\1\\2\end{bmatrix}$	6
	Elliptic	4	$\begin{bmatrix}8748\\32510\\8746\\32506\end{bmatrix}$	262
Approach	Butterworth	3	$\begin{bmatrix}3\\4\\2\\3\\2\end{bmatrix}$	3
		16	$\begin{bmatrix}5\\10\\6\\11\end{bmatrix}$	29
		5	$\begin{bmatrix}4\\2\\2\\4\end{bmatrix}$	4

Table 2: Two's complement truncation results.

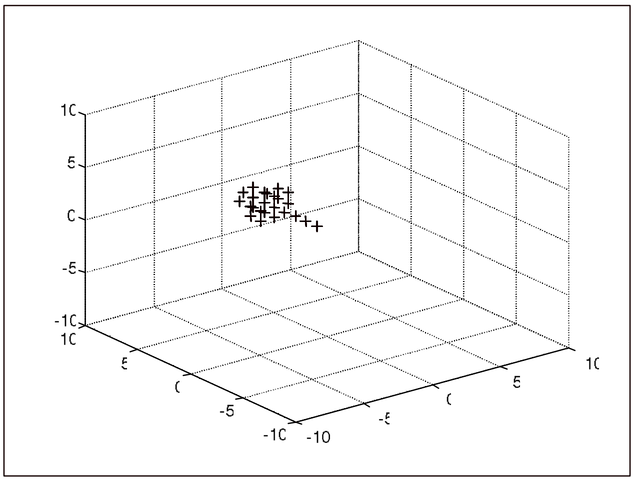

Figure 6: States reached in the limit cycles detected working with two’s complement truncation and the one-port approach.

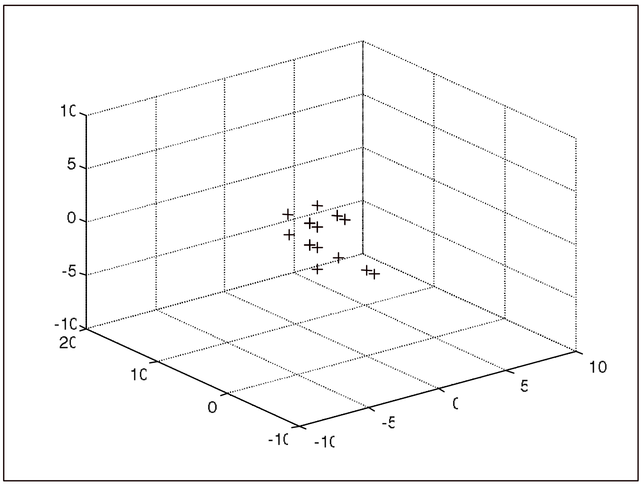

Figure 7: States reached in the limit cycles detected working with two’s complement truncation and the two-port approach.

4 Conclusion

Just as we expected, the best performance has been obtained working with signed magnitude truncation, although hidden oscillations have been detected studying the one-port approach, the values reached at the inner registers are very small. The results achieved working under rounding and two’s complement truncation are quite different from one filter to another, but they tend to suggest that Butterworth filters present better stability. Considering that the maximum amplitudes obtained are expressed as integers multiples of the quantization step, we can assert that limit cycles found in the filters analysed are very small and so, the importance of parasitic oscillations working with the usual wordlengths (16 bits or more) is very slight and frequently may not be disturbing at all.

Aproach	Type	Output	State	Period
Approach	Butterworth	0	$\begin{bmatrix}0\\2\\0\\2\\0\end{bmatrix}$	1
		0	$\begin{bmatrix}0\\2\\0\\1\end{bmatrix}$	1
	Elliptic	0	$\begin{bmatrix}0\\3\\0\\3\end{bmatrix}$	2
Two-port Approach		0	$\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$	0
	Chebyshev	0	$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$	0
	Elliptic	0	$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$	0

Table 2: Signed magnitude truncation results

References:

[1] Fettweis A.: *"Wave Digital Filters: Theory and Practice"*. Proc. IEEE, vol. 74, , Feb. 1986 pp 270-327.

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