# Self-Tuning Controller Based On Process Fuzzy Model

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*Abstract:* - A self-tuning controller based on the Takagi-Sugeno fuzzy model of the process is proposed. The fuzzy model is represented as a linear time-varying model and the pole-placement design procedure modified for the time-varying systems is applied to obtain the desired closed-loop poles. The stability is proven for a class of SISO systems. Because of its simplicity, the proposed method is suitable for the on-line controller design in the adaptive control systems. As an example, the proposed method is applied in adaptive control of a laboratory liquid level rig. The obtained results are much better than the results obtained with the self-tuning controller based on linear process model. *IMACS/IEEE CSCC'99* Proceedings, Pages: 2161-2166

*Key-Words:* - Fuzzy control, fuzzy model, adaptive control, pole-placement design

## 1 Introduction

Fuzzy logic was originally introduced in process control as a means of representing the qualitative and uncertain knowledge about the dynamic behaviour of the system. The representation of qualitative knowledge by fuzzy sets proposed by Zadeh [1-3], and the application of the fuzzy set theory in control proposed by Mamdani [4] have proved to be very successful in dealing with nonlinearities, parameter uncertainties and cases when it is difficult to obtain a useful mathematical model.

A commonly known disadvantage of fuzzy control is the lack of analytical tools. It has proved to be extremely difficult to develop a general stability analysis theory and a design procedure for fuzzy systems. Tanaka and Sugeno [5] introduced the method of analysis based on the fuzzy model in which sets of fuzzy rules are used to imply suitable local linear state space models from which local controllers can be determined. The stability of the overall system is determined by the Lyapunov stability analysis. The stability conditions require that for all the local linear models a common positive-definite matrix **P** should be found to satisfy the Lyapunov equation, and this is a very difficult problem.

Cao et al. [6] suggested a way to avoid determining the **P** matrix. Their method is based on the linear uncertain system theory. In this method the stability analysis of a fuzzy control system is converted to the stability analysis of linear time-varying subsystems. In [7] the necessary and sufficient condition for the stabilisation of the MIMO fuzzy control system are given and the procedure for obtaining a stabilising feedback control law is proposed based on the decomposition principle by which the design of a fuzzy discrete-time control system is decomposed into the design of the "extreme" subsystems.

The methods proposed in [5-7] can be used in analysis and design for a wide class of complex control system. These methods can be very useful for off-line controller design, but they are not suitable for use as on-line controller design procedures in adaptive control because of their complexity.

In this paper a simple controller design method for SISO systems is proposed. It is a version of the pole-placement method [8] modified for time-varying systems and it is suitable for use in adaptive control. The method is based on the Takagi-Sugeno input-output fuzzy model [9], but it uses an approach that differs from the methods proposed by Tanaka and Sugeno [5] and Cao et al. [6-7]. Instead of designing the local control laws for local linear models and then checking the stability of the overall system, in this method the overall fuzzy input-output model is considered as a linear time-varying model and a linear time-varying controller is designed that provides the closed-loop system behaviour that matches the desired dynamics. The stability is proven for a class of control systems.

This paper is organised as follows: Section 2 shows the outline of the Takagi-Sugeno input-output fuzzy model and describes how it can be represented as a linear time-varying model. Section 3 gives a method of pole placement design for time-varying systems. Section 4 gives the controller design procedure for a class of systems for which the stability is proven. Section 5 shows the application of the proposed method in adaptive control of the laboratory liquid level rig.

## 2 Dynamic Fuzzy Model

Assume that the fuzzy model, proposed by Takagi and Sugeno [9-11], is used for description of the dynamic behaviour of the process. The fuzzy model is composed of local linear models using following inference rules:

$$\mathrm{R}^i\text{: } \mathbf{IF}\ \left[x_1(k) \text{ is } F_1^i\right] \mathbf{AND}\ \ldots\ \mathbf{AND}\ \left[x_{\mathrm{nx}}(k) \text{ is } F_{\mathrm{nx}}^i\right]$$

$$\mathbf{THEN}\ \ y^i(k+d+1) = -\sum_{j=1}^{\mathrm{na}} a_j^i y(k-j+d+1) + + \sum_{j=1}^{\mathrm{nb}} b_j^i u(k-j+1) + c^i, \quad i = 1, \ldots, \mathrm{nr}, \tag{1}$$

where $\mathrm{R}^i$ denotes $i$th inference rule; $x_j$ is $j$th variable of the premises; $F_j^i$ denotes the fuzzy set defined on the universe of discourse of the variable $x_j$, used in $i$th inference rule; $y^i$ output of the $i$th local model; $y$ output of the model; $u$ input of the model; $a_j^i$, $b_j^i$, $c^i$ parameters of the $i$th local model (consequence parameters) and $d$ the process dead time.

The premises of the discussed model can have more than one variable. The local models are of a SISO type. The premise variables $x_j$ can be the values of the output $y$, the input $u$ in the past time instants, or some other signals.

Given the values of premise variables $x_j(k)$, the final output of the fuzzy process model is inferred by taking the weighed average of the local model outputs $y^i$:

$$y(k+d+1) = \sum_{i=1}^{\mathrm{nr}} v^i(k) \left\{ -\sum_{j=1}^{\mathrm{na}} a_j^i y(k-j+d+1) + + \sum_{j=1}^{\mathrm{nb}} b_j^i u(k-j+1) + c^i \right\}, \tag{2}$$

where

$$v^i(k) = \frac{\prod_{j=1}^{\mathrm{nx}} m_j^i\left[x_j(k)\right]}{\sum_{l=1}^{\mathrm{nr}} \prod_{j=1}^{\mathrm{nx}} m_j^l\left[x_j(k)\right]}, \tag{3}$$

$m_j^i$ - a membership function of the fuzzy set $F_j^i$.

Using the shift operator $q^{-1}$ the model (2) can be expressed in the transfer-function form:

$$\mathbf{A}(q^{-1},k)y(k+d+1) = \mathbf{B}(q^{-1},k)u(k) + c(k), \tag{4}$$

where

$$\begin{aligned}
\mathbf{A}(q^{-1},k) &= 1 + \sum_{j=1}^{\mathrm{na}} a_j(k) q^{-j}, \\
\mathbf{B}(q^{-1},k) &= \sum_{j=1}^{\mathrm{nb}} b_j(k) q^{-j+1}, \\
a_j(k) &\mathrel{\hat=} \sum_{i=1}^{\mathrm{nr}} a_j^i v^i(k), \\
b_j(k) &\mathrel{\hat=} \sum_{i=1}^{\mathrm{nr}} b_j^i v^i(k), \\
c(k) &\mathrel{\hat=} \sum_{i=1}^{\mathrm{nr}} c^i v^i(k).
\end{aligned} \tag{5}$$

The equation (4) is the representation of the fuzzy model (1) in the form of a linear time-varying model. The consequence parameters $a_j^i$, $b_j^i$, $c^i$ can be obtained by the least squares method as described in [9].

Non-measurable disturbance and unmodeled dynamics of the process can be represented by introducing the disturbance signal $z$ superposed to the output $y$:

$$\begin{aligned}
y(k) &= y_0(k) + z(k), \\
\mathbf{A}(q^{-1},k)y_0(k+d+1) &= \mathbf{B}(q^{-1},k)u(k) + c(k),
\end{aligned} \tag{6}$$

where $y_0$ is the output without disturbance. Combining the equations (6) yields the following model:

$$\mathbf{A}(q^{-1},k)y(k+d+1) = \mathbf{B}(q^{-1},k)u(k) + c(k) + + \mathbf{A}(q^{-1},k)z(k+d+1) \tag{7}$$

## 3 Controller Design

It can be shown that the system with desired closed-loop poles can be obtained using the following control law:

$$\mathbf{R}(q^{-1},k)u(k) = -\mathbf{S}(q^{-1},k)y(k) + \mathbf{T}(q^{-1},k)u_r(k) - - \mathbf{P}(q^{-1},k)c(k) \tag{8}$$

where

$$\mathbf{S}(q^{-1},k)=\sum_{j=0}^{\text{ns}} s_j(k)q^{-j} ,$$

$$\mathbf{T}(q^{-1},k)=\sum_{j=0}^{\text{nt}} t_j(k)q^{-j} ,$$

$$\mathbf{P}(q^{-1},k)=\sum_{j=0}^{\text{np}} p_j(k)q^{-j} .$$

**Definition 1:** Let **W** denote the set of all polynomials in the shift operator $q^{-1}$ with time-varying coefficients, and let:

$$\mathbf{G}(q^{-1},k)=\sum_{j=0}^{\text{ng}} g_j(k)q^{-j} ,$$

$$\mathbf{H}(q^{-1},k)=\sum_{j=0}^{\text{nh}} h_j(k)q^{-j}$$

be the elements of the set **W**. Operation $\circ:\mathbf{W}\times\mathbf{W}\rightarrow\mathbf{W}$ is defined by:

$$\mathbf{G}(q^{-1},k)\circ\mathbf{H}(q^{-1},k)=\sum_{i=0}^{\text{ng}}\sum_{j=0}^{\text{nh}} g_i(k)h_j(k-i)q^{-(i+j)} .$$

The **B** polynomial can be factorised as:

$$\mathbf{B}(q^{-1},k)=b_1(k)\mathbf{B}'(q^{-1},k) ,$$

where the $\mathbf{B}'$ polynomial is monic. If the **R** polynomial is such that it can be denoted by:

$$\mathbf{R}(q^{-1},k)=\mathbf{R}'(q^{-1},k)\circ\mathbf{B}'(q^{-1},k) , \tag{9}$$

where

$$\mathbf{R}'(q^{-1},k)=1+\sum_{j=1}^{\text{nd}} r_j'(k)q^{-j} ,$$

then the following equation holds:

$$\mathbf{D}(q^{-1},k)\circ\mathbf{B}(q^{-1},k)=b_1(k)\left[\mathbf{R}'(q^{-1},k)\circ \circ\mathbf{B}'(q^{-1},k)\right], \tag{10}$$

where

$$d_i(k)=r_i'(k)\frac{b_1(k)}{b_1(k-i)} . \tag{11}$$

Using (7), (8) and (10) the closed-loop system can be denoted by:

$$\begin{aligned}&\left\{\left[\mathbf{D}(q^{-1},k)\circ\mathbf{A}(q^{-1},k)\right]q^{d}+b_1(k)\mathbf{S}(q^{-1},k)q^{-1}\right\}y(k+1)=\\&=b_1(k)\mathbf{T}(q^{-1},k)u_r(k)+\left[\mathbf{D}(q^{-1},k)-b_1(k)\mathbf{P}(q^{-1},k)\right]c(k)\\&+\left[\mathbf{D}(q^{-1},k)\circ\mathbf{A}(q^{-1},k)\right]q^{d}z\,(k+1).\end{aligned} \tag{12}$$

Let the coefficients of polynomials **D**, **S**, **T** and **P** be chosen such that the following equations hold:

$$\begin{aligned}&\left[\mathbf{D}(q^{-1},k)\circ\mathbf{A}(q^{-1},k)\right]q^{d}+b_1(k)\mathbf{S}(q^{-1},k)q^{-1}=\\&=\mathbf{A}_M(q^{-1})\mathbf{A}_O(q^{-1}) ,\end{aligned} \tag{13}$$

$$b_1(k)\mathbf{T}(q^{-1},k)=\mathbf{B}_M(q^{-1})\mathbf{A}_O(q^{-1}) , \tag{14}$$

$$\mathbf{D}(q^{-1},k)-b_1(k)\mathbf{P}(q^{-1},k)=0 , \tag{15}$$

where

$$\mathbf{A}_M(q^{-1})=1+\sum_{j=1}^{\text{nm}} a_j(k)q^{-j} ,$$

$$\mathbf{B}_M(q^{-1})=\sum_{j=1}^{\text{nm}} b_j(k)q^{-j+1} .$$

Then the closed-loop system is obtained that can be described by the following equation:

$$\mathbf{A}_M(q^{-1})y(k+d+1)=\mathbf{B}_M(q^{-1})u_r(k)+y\,(k+d+1) ,$$

where

$$y\,(k+d+1)=\left[\mathbf{D}(q^{-1},k)\circ\mathbf{A}(q^{-1},k)\right]z\,(k+d+1)$$

is the filtered disturbance signal. By solving the equations (13-15) in each sampling period, and calculating the control signal using control law (8), the linear time-invariant closed-loop system with desired poles can be obtained.

### 3.1 Stability Analysis

Although the control law based on the proposed method provides the stability of the closed-loop system, the coefficients of the **R** polynomial in (8) can be time-varying. The stability of such a controller is in general hard to prove. A sufficient condition for the stability of the controller is that the coefficients of the **R** polynomial should be time-invariant and that all its roots should be inside the unit circle of the complex z-plane.

Consider a discrete-time process described by the linear time-varying model (7), with the following characteristics:

1. $d$ = 0 (process is without dead time);
2. All coefficients of the $\mathbf{B}'$ polynomial are time-invariant;
3. All roots of the $\mathbf{B}'$ polynomial are inside the unit circle. (16)

It can be proven that if the process satisfies the conditions (16), there is a control law given by equation (8) such that **R** is the polynomial with time-invariant coefficients and all roots inside the unit circle, that provides a linear time-invariant closed-loop system with the desired poles.

If $d = 0$ then equation (13) can be denoted by:

$$\mathbf{L}(q^{-1},k) + b_1(k)\mathbf{S}(q^{-1},k)q^{-1} = \mathbf{A}_Z(q^{-1}) \qquad (17)$$

where

$$\mathbf{L}(q^{-1},k) = \mathbf{D}(q^{-1},k) \circ \mathbf{A}(q^{-1},k) \qquad (18)$$

is a polynomial with time-varying coefficients $\lambda_i(k)$, and

$$\mathbf{A}_Z(q^{-1}) = \mathbf{A}_M(q^{-1})\mathbf{A}_O(q^{-1}) \qquad (19)$$

is a polynomial with the coefficients $g_i$. If the order of the polynomial **L** is nl = na + nd, where na and nd are the orders of the polynomials **A** and **D** respectively, and the order of the polynomial $\mathbf{A}_z$ is $\mathrm{nz} \geq \mathrm{nl}$, then the solution of the equation (17) is

$$s_i(k) = \frac{g_{i+1} - l_{i+1}(k)}{b_1(k)}, \quad 0 \leq i < \mathrm{nl};$$
$$s_i(k) = \frac{g_{i+1}}{b_1(k)}, \quad \mathrm{nl} \leq i < \mathrm{nz}. \qquad (20)$$

It is obvious that the equation (17) is satisfied for arbitrary coefficients $l_i$. Thus the coefficients of the **D** polynomial can also be arbitrary. The relation between the coefficients of the polynomials **D** and $\mathbf{R}'$ is given by equation (11), so the coefficients of the polynomial $\mathbf{R}'$ can have arbitrary values. The polynomial **R** is given by (9). If the coefficients of the polynomial $\mathbf{B}'$ are time-invariant then, choosing the polynomial $\mathbf{R}'$ with constant coefficients, the polynomial **R** with the time-invariant coefficients can be obtained.

### 3.2. Controller design procedure

If the conditions (16) are satisfied, the following procedure can be used for the controller design:

*Step 1:* Determine the coefficients $a_i(k)$, $b_i(k)$ and $c(k)$ using membership functions obtained by the fuzzyfication procedure and formulas (3) and (5).

*Step 2:* Choose the polynomials $\mathbf{A}_M$, $\mathbf{B}_M$ and $\mathbf{A}_o$.

*Step 3:* Choose the $\mathbf{R}'$ polynomial and form the **D** polynomial using (11).

*Step 4:* Form the **L** polynomial using (18) and the $\mathbf{A}_z$ polynomial using (19).

*Step 5:* Calculate the coefficients of the **S** polynomial using (20).

*Step 6:* Calculate the coefficients of the polynomials **R**, **T** and **P** using the equations (9), (14) and (15) respectively.

For the above design procedure the values of the membership functions have to be known. If one of the membership functions depends on the current control signal value $u(k)$, the procedure is much more complicated. Instead of the numeric values of coefficients $a_i(k)$, $b_i(k)$ and $c(k)$, the expressions $a_j[u(k),k]$, $b_j[u(k),k]$ and $c[u(k),k]$ have to be used in all steps of the design procedure. The control signal is obtained by solving the following equation

$$\begin{aligned} u(k) = -\sum_{i=1}^{\mathrm{nr}} \Bigg\{ & \sum_{j=1}^{\mathrm{nd+nb}} r_j^i[u(k),k]u(k-j) - \\ & - \sum_{j=1}^{\mathrm{ns}} s_j^i[u(k),k]y(k-j+1) + \\ & + \sum_{j=1}^{\mathrm{nt}} t_j^i[u(k),k]u_r(k-j+1) - \\ & - \sum_{j=1}^{\mathrm{nd}} p_j^i[u(k),k]c(k-j+1) \Bigg\}, \end{aligned}$$

where $r_j^i[u(k),k]$, $s_j^i[u(k),k]$, $t_j^i[u(k),k]$ and $p_j^i[u(k),k]$ are the expressions for the coefficients of the polynomials **R**, **S**, **T** and **P**.

## 4 Example: A Laboratory Liquid Level Rig

The proposed method is applied in adaptive control of a laboratory liquid level rig shown in Fig. 1.

A pump with a squirrel cage induction motor controlled by a frequency converter provides the water flow. The control signal $u$ is the frequency converter reference. The operation range of the motor is from 10 to 45 Hz. The valve opening $z$ is considered as a measurable disturbance, ranging from 55 to 100%.

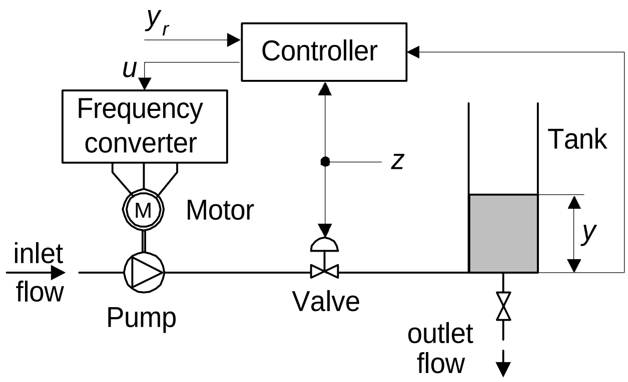


Fig. 1. Laboratory liquid level rig

The objective is to control the liquid level $y$ in the tank in such a way that the response to the step of the level reference signal $y_r$ has an overshoot not greater than 5% of the level reference change. The level reference range is from 600 to 650 mm.

The performance of the self-tuning controller based on Takagi-Sugeno fuzzy process model is compared to the performance of the self-tuning controller based on linear process model.

The linear model of the process is given by the following equation:

$$y(k+1) = a_1 y(k) + a_2 y(k-1) + b_1 u(k) + cz(k)\,. \qquad (21)$$

The effect of the measurable disturbance signal is modelled with the last term of the equation (21). The performance of the self-tuning controller with linear process model is shown in Fig. 2. As can be seen, the effect of the disturbance to the closed-loop response of the system is significant. This is the consequence of the fact that the changes of the disturbance signal cause the changes of the process parameters. Because the changes of the process parameters are large and frequent the parameter estimation algorithm cannot follows them, resulting in inaccurate values of the model parameters.

The fuzzy model is given by the following rules:

$R^1$: **IF** $\left[z(k)\text{ is }F^1\right]$ **THEN**

$$y^1(k+1) = a_1^1 y(k) + a_2^1 y(k-1) + b_1^1 u(k) + c^1$$

$R^2$: **IF** $\left[z(k)\text{ is }F^2\right]$ **THEN**

$$y^2(k+1) = a_1^2 y(k) + a_2^2 y(k-1) + b_1^2 u(k) + c^2$$

$R^3$: **IF** $\left[z(k)\text{ is }F^3\right]$ **THEN**

$$y^3(k+1) = a_1^3 y(k) + a_2^3 y(k-1) + b_1^3 u(k) + c^3\,.$$

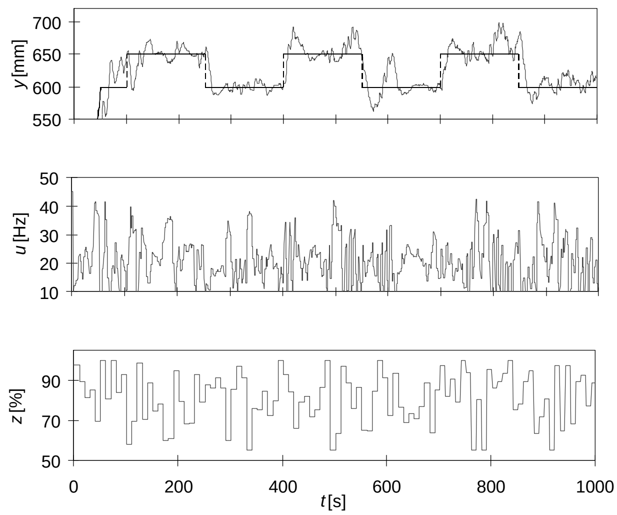


Fig. 2. The performance of the self-tuning controller with linear process model.

The membership functions of the premise fuzzy sets are shown in Fig. 3.

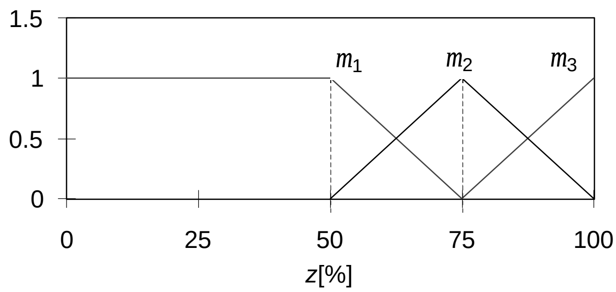


Fig. 3. The membership functions of the premise fuzzy sets.

Thus, the fuzzy process model consists of three local linear models combined using fuzzy logic. Every local model corresponds to an operating area characterised by the value of the measurable disturbance signal $z$. The main advantage over the linear model is that the parameters don't have to be modified as the operating area changes.

The performance of the self-tuning controller with fuzzy process model is shown in Fig. 4. System behaviour is obviously much better than the system behaviour obtained with the self-tuning controller based on linear model. The compensation of the disturbance is much better and the responses to the step changes of the reference signal $y_r$ are without overshoots for any values of the valve opening in the considered range. Additionally, it can be seen that the control signal $u$ is less active when the self-tuning controller with fuzzy model is used (see Fig. 4) than when the self-tuning controller with linear model is used (see Fig. 2). This feature is of great importance in the real-world applications.

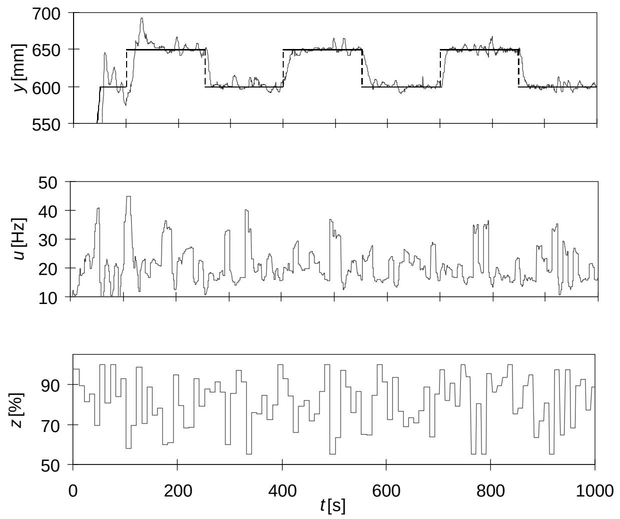


Fig. 4. The performance of the self-tuning controller with fuzzy process model.

## 5 Conclusion

A self-tuning controller based on the Takagi-Sugeno fuzzy model of the process is proposed. The fuzzy model is represented as a linear time-varying model and the pole-placement design procedure modified for the time-varying systems is applied to obtain the desired closed-loop poles. The stability is proven for a class of SISO systems. Because of its simplicity, the proposed method is suitable for the on-line controller design in the adaptive control systems.

The proposed self-tuning controller is compared with the self-tuning controller based on linear process model. Both controllers are experimentally tested on a laboratory liquid level rig. System behaviour obtained with the proposed controller is much better than the system behaviour obtained with the self-tuning controller based on linear process model.


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