RAPID RESTORATION OF DISTORTED SIGNALS AND IMAGES BY INVERSE FILTERING METHOD*

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Abstract An express-reconstruction of distorted signals and images based on a fast filtering algorithm for inverting linear convolution by sectioning method combined with effective real-valued split radix fast Fourier transform (FFT) algorithms is proposed.

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The reconstruction of digital signals (images) can be reduced to the search of a filter with a finite impulse response (FIR-filter) inverse to the one that caused the distortion. It is a typical inverse problem and belongs to the class of ill-posed problems [1] in which it is necessary to determine the original signal (image) through the known distorted one [1,2]. The main sources of signal (image) distortion to which digital reconstruction is applied are blurring due to motion (of the object and videocamera) or atmospheric turbulence, and defocusing of the optics [1,2]. Inverse filtering of a more general kind, when there is noise, is mainly considered in the basic studies [2-4]. There are several different iterative and non-iterative methods for solving the inverse filtering problem (with or without noise) [1-4]. However, most iterative methods (the method of steepest descent, Galerkin's method, Fienup's method and iterative inversion of convolutions under constraints [1,4]) have the serious drawback that it is difficult to predict in advance how many approximations will be needed to obtain a solution with pre-assigned accuracy.

For the algorithm for inverting a linear convolution (LC) by sectioning proposed in [5], it is assumed that the impulse response of the distorting effect is specified exactly, so that an absolutely accurate solution of the inverse filtering problem can be obtained. Unfortunately, this algorithm is not effective because fast calculation algorithms of the type of FFT are not used in it.

Our purpose here is to synthesize a fast algorithm for inverting a LC based on the sectioning method combined with the most efficient real-valued FFT algorithms [6-10], which will reduce the computational complexity of solving digital signal(image) processing problem of large dimension. A known overlap-add method is taken as a basis of this algorithm. This method is used for the direct problem - filtering. We will consider the digital filtering of signals by inversion of an LC of the form :

$$y_m = \sum_{n=0}^{m} h_{m-n} x_n \, , m = 0,1,\ldots,N+M-2, \qquad (1)$$

where x_n is the incoming one-dimensional signal, y_m is the distorted signal and h_n is the impulse response which describes a linear FIR-filter. Since the direct method of inverting cyclic convolution (CC) matrix (circulant) seldom gives positive result (there are zeros in Fourier spectrum of impulse characteristics), the relationship of circulant and triangular Toeplitz matrices, which always have inverse matrices, is investigated. According to the proposed approach, triangular Toeplitz matrices of LxL size are complemented to the LC matrices of (2L-1)xL size. Then the LC matrices are transformed to the square matrices of CC of 2Lx2L size by complementing them with zeros.

The algorithms based on calculating triangular Toeplitz matrices by means of twice the dimensions can be written as follows [10-12]:

(1) for i= 1 we compute a 2L-point CC of the form

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$$x_l^{(1)} = \sum_{m=0}^{2L-1} \tilde{h}^{(-1)}_{((l-m))} \tilde{y}_m^{(1)}, l = 0,1,\ldots,2L-1, \tag{2}$$

where $\{\tilde{h}_m^{(-1)}\} = \{h_0^{(-1)},\ldots,h_{L-1}^{(-1)},0,\ldots,0\}$, $\{\tilde{y}_m^{(1)}\} = \{y_0^{(1)},\ldots,y_{L-1}^{(1)},0,\ldots,0\}$, are 2L-point sequences, $((l-m)) = (l-m) \bmod 2L;$

(2) for i ≥ 2 we form the (N-1)-point sequence $\{\bar{x}_l^{(i)}\} = \{x_{M+l}^{(i-1)}\}, l = 0,1,\ldots,N-2$ and compute the CC of the form

$$\bar{y}_m^{(i)} = \sum_{l=0}^{2N-1} \tilde{h}_{((m-l))} \tilde{\bar{x}}_l^{(i)}, \; l = 0,1,\ldots,2N-1, \tag{3}$$

where $\{\tilde{h}_n\} = \{h_0,\ldots,h_{N-1},\ldots,0,\ldots,0\}$, $\{\tilde{\bar{x}}_l^{(i)}\} = \{\bar{x}_0^{(i)},\ldots,\bar{x}_{N-2}^{(i)},0,\ldots,0\}$ are 2N-point sequences, $((m-l)) = (m-l) \bmod 2N;$

(3) for i ≥ 2 we form the L-point sequence $\{y_m^{(i)}\} = \{\bar{y}_0^{(i)},\ldots,\bar{y}_{N-2}^{(i)}, y_{N-1}^{(i)},\ldots,y_{L-1}^{(i)}\}$ and then
compute the 2L-point CC :

$$x_l^{(i)} = \sum_{m=0}^{2L-1} \tilde{h}^{(-1)}_{((l-m))} \tilde{y}_m^{(i)}, l = 0,1,\ldots,2L-1, \tag{4}$$

where $\{\tilde{h}_m^{(-1)}\} = \{h_0^{(-1)},\ldots,h_{L-1}^{(-1)},0,\ldots,0\}$
and $\{\tilde{y}_m^{(i)}\} = \{\bar{y}(_0^{(i)},\ldots,\bar{y}_{N-2}^{(i)},y_{N-1}^{(i)},\ldots,y_{L-1}^{(i)},0,\ldots,0\}$ are 2L-point sequences, $((l-m)) = (l-m) \bmod 2L;$

The computational complexity of algorithm (2)-(4) is given by expressions (5) :

$$M(R) = O((6\log_2 L + 12/L - 5)R), \tag{5a}$$
$$A(R) = O((18\log_2 L + 20/L - 9)R), \tag{5b}$$

which show a gain over the initial algorithm of [5], the computational complexity of which is characterized by relations (6) :

$$M(R) = O(1.25(L + 2/5)R), \tag{6a}$$
$$A(R) = O(1.25(L - 2 + 8/(5L))R). \tag{6b}$$

Thus, despite the view held by the author [5], the cost of solving inverse filtering problems using inversion of an LC by sectioning [10-12] is reduced by using FFT algorithms (in this case, the real-valued split-radix FFT (RFFT-SR) algorithm [7-10], one of the best). As a result we comes to the matrix of CC two times larger in size then the initial Toeplitz one, but it can be calculated on the basis of effective fast algorithms.

The proposed fast inverse convolution algorithm for reconstructing distorted signals by sectioning the inverse convolution and using the RFFT-SR algorithm was programmed for IBM. A computer experiment was performed in which distorted sequences of P=1024-4096 readings were reconstructed, using impulse responses of various lengths (N=65, 129, 257, 513 elements). When impulse response is quite long (N ≥257), the signal is reconstructed 1.6 times faster on average by the proposed algorithm than with the approach based on the algorithm of [5]. The advantage is especially pronounced when the sequence is fairly long, of length (P≥2048).

The fast algorithm elaborated was also used for solving distorted (blurred and defocused) image restoration problems. Let y_{m_1,m_2} is a digital photographic image that has been distorted by blurring or defocusing of optics (or blurring in two directions). Assuming that impulse response of the distorting filter is separable: $h_{n_1,n_2} = h_{n_1} h_{n_2}$,for a digital reconstruction of the distorted image, we apply the proposed above algorithm to y_{m_1,m_2} , first by rows, and then by columns:

$$x_{n_1,n_2} = \sum_{m_2=0}^{n_2}\left(\sum_{m_1=0}^{n_1} y_{m_1,m_2} h^{(-1)}_{n_1-m_1}\right) h^{(-1)}_{n_2-m_2}, \; m_1 = 0,1,\ldots,L_1-1,\; m_2 = 0,1,\ldots,L_2-1,$$

where $h_n^{(-1)}$ is impulse response of the inverse FIR-filter. Then, according to (2)-(4), digital reconstruction of the distorted images, first along the rows and then along the columns, reduces to computing the following CC:

$$x^{(i_1,i_2)}_{l_1,l_2} = \sum_{m_2=0}^{2L_2-1}\left(\sum_{m_1=0}^{2L_1-1} \tilde{h}^{(-1)}_{((l_1-m_1))}\tilde{y}^{(i_1,i_2)}_{m_1,m_2}\right)\tilde{h}^{(-1)}_{((l_2-m_2))}, l_1 = 0,\ldots,2L_1-1, l_2 = 0,\ldots,2L_2-1,$$

where

$$\{\tilde{h}^{(-1)}_{m_1}\} = \{h^{(-1)}_0,\ldots,h^{(-1)}_{L_1-1},0,\ldots,0\}, \{\tilde{h}^{(-1)}_{m_2}\} = \{h^{(-1)}_0,\ldots,h^{(-1)}_{L_2-1},0,\ldots,0\}$$

and

$$\{\tilde{y}^{(i_1,i_2)}_{m_1,m_2}\} = \{\bar{y}^{(i_1,i_2)}_{0,0},\ldots,\bar{y}^{(i_1,i_2)}_{N_1-2,N_2-2},\ldots,y^{(i_1,i_2)}_{N_1-1,N_2-1},\ldots,y^{(i_1,i_2)}_{L_1-1,L_2-1},0,\ldots,0\},$$

$$\{\tilde{h}_{n_1}\} = \{h_0,\ldots,h_{N_1-1},0,\ldots,0\}, \quad \{\tilde{h}_{n_2}\} = \{h_0,\ldots,h_{N_2-1},0,\ldots,0\},$$

$$\{\tilde{\bar{x}}^{(i_1,i_2)}_{l_1,l_2}\} = \{\bar{x}^{(i_1,i_2)}_{0,0},\ldots,\bar{x}^{(i_1,i_2)}_{N_1-2,N_2-2},0,\ldots,0\}, \quad \{\bar{x}^{(i_1,i_2)}_{l_1,l_2}\} = \{x^{(i_1-1,i_2-1)}_{M_1+l_1,M_2+l_2}\},$$

$$l_1 = 0,1,\ldots,N_1-2, \quad l_2 = 0,1,\ldots,N_2-2, \quad i_1 = 1,2,\ldots,S_1, \quad i_2 = 1,2,\ldots,S_2,$$

$$S_1 = \frac{R_1 - N_1 + 1}{L_1 - N_1 + 1}, \quad S_2 = \frac{R_2 - N_2 + 1}{L_2 - N_2 + 1}.$$

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