Recursive Least-Squares Algorithm for Blind ISI Cancelation in Multiuser Systems

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Abstract:- This paper proposes a new blind ISI cancelation method that involves the use of two RLS-based adaptive filters. By minimizing the distance between the output of the filter and a reference signal for both filters, the output of one filter turns out to be exactly equal to the ISI component of the output of the other filter. The difference between the output of these filters is an ISI-free estimate of the signal. Computer simulation has been conducted to illustrate the performance of this new method.

Keywords: Blind Equalization, Blind ISI Cancelation, Multiuser Systems

1. INTRODUCTION

In the recent years, theoretical investigations have shown that the capacity of a multipath wireless channel can be enormously increased as long as there are sufficient amount of multipath scattering. A multi-user *space division multiple access*(SDMA) system, which essentially separates users based on differences in their locations, can be viewed as one potential application of these investigations. However, The received signal of such systems tends to suffer from *intersymbol interference*(ISI) and *co-channel interference*(CCI). Traditionally, the cancelation of ISI and CCI rely on the periodic transmission of training sequences which are known at the receivers. This strategy, however, results in a significant reduction of the effective communication rate because of the fast fading of radio channels. "Blind" technologies that do not need the transmission of training sequences have therefore attracted a lot of research attentions recently[1, 2, 3, 4, 5, 6]. It is now well established that blind ISI cancelation can be achieved by exploiting channel diversity, whereas blind CCI cancelation or blind source separation, can be achieved possibly only by resorting to higher-order statistics if no additional information is available[3, 14, 15]. Many blind ISI cancelation methods based on channel diversity have been developed in the past decade. Most of them can be viewed as either stochastic methods[1, 2, 4, 5] or deterministic one[6, 7, 8]. Tong *et al*[1] were the first to realize the signal cyclo-stationarity in the identification of non-minimum phase FIR channels for single-user systems. Subspace decomposition is generally the most important tool in the implementation of these methods. The methods based on deterministic model can provide an acceptable solution within a short input sequence. These methods however require exact knowledge of the channel order which is never met in practice. In view of this shortcoming, several methods[9, 10] have been proposed based on matrix decomposition which are unsuited for adaptive implementation. Some of them are computationally intensive too. Gesbert *et al*[13] developed an on-line adaptive blind equalizer by mutually referenced filters. This equalizer is conceptually similar to that reported in [11] where the coefficients of the equalization filter bank is updated such that the output vector-sequence forms a *Hankel* matrix. Although this on-line equalizer was claimed to be more robust in the presence of channel order mismatch and exhibit global convergence, it needs to update too many filters which can be computationally expensive.

In this paper, we propose a blind ISI cancelation method that is not affected by channel order mismatch when an over-estimated channel order is available. Its implementation can be realized by the recursive least squares(RLS) algorithm. In Section 2, a criterion for ISI-free estimate is introduced. By minimizing this criterion, a solution to ISI cancelation is obtained. We propose our method in Section 3, and the RLS algorithm is presented in Section 4. Some simulation results are listed in Section 5 and the conclusions are given in the last section.

2. A CRITERION FOR ISI-FREE ESTIMATES

Consider a multi-user system with u user inputs and N outputs derived from multiple antennas and/or oversampling where $N > u$. Denote the symbol rate by $1/T$ and the system input at time index n as $\mathbf{x}[n] = [\alpha_1(n), \alpha_2(n), \cdots, \alpha_u(n)]^t$, where $\alpha_j(n)$ is the n-th symbol of the j-th user and the superscript ^{t} represents *transpose*. We further assume that each dynamic channel can be modeled by a FIR system with order m_i for $i = 1, 2, \cdots, uN$ and all orders of channels are not larger than m. The sampled output signal is then an $N \times 1$ vector that can be represented by

$$\mathbf{y}[n] = \sum_{k=0}^{m} \mathbf{H}[k]\mathbf{x}[n-k] + \mathbf{w}[n], \tag{1}$$

where $\mathbf{w}[n]$ is noise and $\mathbf{H}[k]$ is the k-th tap coefficient matrix of channel impulse response with size $N \times u$. When one and only one $\mathbf{H}[k]$ is nonzero and each row of this nonzero $\mathbf{H}[k]$ has only one nonzero element, the received signal $\mathbf{y}[n]$ can be viewed as $\mathbf{x}[n-k]$ up to certain constants for each of their components. ISI cancelation is thus developed to keep one nonzero $\mathbf{H}[k]$ for $k = 0, 1, \cdots, m$, and force the remainders to be zero.

For received signals with a high signal-to-noise ratio(SNR), $\mathbf{w}[n]$ can be neglected. For the ease of algorithm derivation, we assume zero noise $\mathbf{w}[n] = 0$ at first. Let

$$\begin{aligned}\mathbf{s}[n] &= [\mathbf{x}[n]^t, \mathbf{x}[n-1]^t, \cdots, \mathbf{x}[n-L-m+1]^t]^t, \\ \mathbf{o}[n] &= [\mathbf{y}[n]^t, \mathbf{y}[n-1]^t, \cdots, \mathbf{y}[n-L+1]^t]^t, \end{aligned} \tag{2}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}[0] & \cdots & \mathbf{H}[m] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}[0] & \cdots & \mathbf{H}[m] & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}[0] & \cdots & \mathbf{H}[m] \end{bmatrix},$$

we then have

$$\mathbf{o}[n] = \mathbf{A}\mathbf{s}[n], \tag{3}$$

where L is an integer such that $LN > (m+L)u$. Let $M = m + L$, then $\mathbf{A}$ is a $L \times M$ block Toeplitz matrix. For further discussion, we assume that the components of input sequence $\mathbf{x}[n]$ are white and uncorrelated to each other, and $\mathbf{A}$ is full-column rank.

Let $\mathbf{B}$ be a $uM \times LN$ matrix that generates an output as

$$\mathbf{e}[n] = \mathbf{B}\mathbf{o}[n] = \mathbf{B}\mathbf{A}\mathbf{s}[n]. \tag{4}$$

Denote the i-th block row of $\mathbf{B}$ by $\mathbf{B}_i$, $i = 1, 2, \cdots, M$, then $\mathbf{B}_i$ is a $u \times LN$ matrix. Let

$$\mathbf{z}_{ij}[n] = \mathbf{e}_i[n+j-i] - \mathbf{e}_j[n],\ j < i.$$

We define a criterion

$$\mathbf{\Phi}(\mathbf{B}_i, \mathbf{B}_j) = E\{\mathbf{z}_{ij}[n]\mathbf{z}_{ij}[n]^\dagger\}, \tag{5}$$

where † stands for *conjugate transpose* operator and $\mathbf{e}_i[n] = \mathbf{B}_i\mathbf{o}[n]$, $\mathbf{e}_j[n] = \mathbf{B}_j\mathbf{o}[n]$. Obviously, $\mathbf{\Phi}(\mathbf{B}_i, \mathbf{B}_j)$ is a $u \times u$ matrix and is of non-negative definite for any pair of i, j. We define that the minimum of $\mathbf{\Phi}(\mathbf{B}_i, \mathbf{B}_j)$ is the matrix whose eigenvalues all arrive at their minimum points. Let $\mathbf{B}_i\mathbf{A} = [\mathbf{D}_{i1}, \mathbf{D}_{i2}, \cdots, \mathbf{D}_{iM}]$, $i = 1, 2, \cdots, M$ and

$$\mathbf{R}_{xx}(0) = E\{\mathbf{x}[n]\mathbf{x}[n]^\dagger\},$$

then

$$\begin{aligned} &\mathbf{\Phi}(\mathbf{B}_i, \mathbf{B}_j) \\ = &\sum_{k=1}^{M+j-i} (\mathbf{D}_{i,k} - \mathbf{D}_{j,k-j+i})\mathbf{R}_{xx}(0)(\mathbf{D}_{i,k} - \mathbf{D}_{j,k-j+i})^\dagger \\ + &\sum_{k=1}^{i-j} \mathbf{D}_{j,k}\mathbf{R}_{xx}(0)\mathbf{D}_{j,k}^\dagger + \sum_{k=M+j-i+1}^{M} \mathbf{D}_{i,k}\mathbf{R}_{xx}(0)\mathbf{D}_{i,k}^\dagger. \end{aligned}$$

Since $\mathbf{R}_{xx}(0)$ is positive definite, minimizing $\mathbf{\Phi}(\mathbf{B}_i, \mathbf{B}_j)$ with respect to $\mathbf{B}_i$, subject to a constant $\mathbf{B}_j$ gives

$$\begin{cases} \mathbf{D}_{i,k} = \mathbf{D}_{j,k-j+i}, & \text{for } k = 1, 2, \cdots, M+j-i. \\ \mathbf{D}_{i,k} = \mathbf{0}, & \text{for } k = M+j-i+1, \cdots, M. \end{cases} \tag{6}$$

This property of criterion (5) will play an important role in our algorithm development. Notice that if the channel order m is known then $M = L + m$ is known. In that case, letting $i = M$ and $j = 1$, we have

$$\begin{cases} \mathbf{D}_{M,k} = \mathbf{0}, & \text{for } k = 2, 3, \cdots, M, \\ \mathbf{D}_{M,M} = \mathbf{D}_{1,1} \end{cases} \tag{7}$$

which results in an ISI-free output[11]:

$$\mathbf{e}_M[n] = \mathbf{D}_{M,M}\mathbf{x}[n - M + 1], \tag{8}$$

where $\mathbf{D}_{M,M} = \mathbf{D}_{1,1} \neq \mathbf{0}$, or in other words, $\mathbf{B}_1$ is assumed to be a vector that is not orthogonal to the first block column of $\mathbf{A}$. However, in practice, two facts render the estimate (8) useless: smaller leading coefficient $\mathbf{H}[0]$ and additive noise at the receiver. Because of the special structure of $\mathbf{A}$, a smaller $\mathbf{H}[0]$ will lead to a smaller $\mathbf{D}_{1,1}$, and thus a poor SNR estimate of $\mathbf{x}[n]$. When the received signal is corrupted by noise, the channel order cannot be accurately estimated. The ISI-free estimate (8), however, requires an exact channel order. In order to overcome these shortcomings, we propose a new ISI cancelation scheme as is described below.

3. OUR PROPOSED METHOD

Note that the estimate obtained by $\mathbf{B}_i$ can be represented by

$$\mathbf{e}_i[n] = \mathbf{B}_i\mathbf{o}[n] = \sum_{k=1}^{M} \mathbf{D}_{i,k}\mathbf{x}[n - k + 1].$$

If $\mathbf{B}_i$ minimizes the criterion (5), then from (6) the last $i - j$ components of $[\mathbf{D}_{i1}, \mathbf{D}_{i2}, \cdots, \mathbf{D}_{iM}]$ must be zero. The estimate can thus be re-written as

$$\mathbf{e}_i[n] = \sum_{k=1}^{M+j-i} \mathbf{D}_{j,k-j+i}\mathbf{x}[n - k + 1].$$

In other words, the span of ISI in $\mathbf{e}_i[n]$ is shrunk from M to $M + j - i$. Keeping $\mathbf{B}_j$ constant and minimizing $\mathbf{\Phi}(\mathbf{B}_{i+1}, \mathbf{B}_j)$, similarly, gives

$$\mathbf{e}_{i+1}[n] = \sum_{k=1}^{M+j-i-1} \mathbf{D}_{j,k-j+i+1}\mathbf{x}[n - k + 1].$$

Therefore, let

$$\hat{\mathbf{x}}[n] = \mathbf{e}_i[n] - \mathbf{e}_{i+1}[n - 1]. \tag{9}$$

Then

$$\begin{aligned} \hat{\mathbf{x}}[n] &= \sum_{k=1}^{M+j-i} \mathbf{D}_{j,k-j+i}\mathbf{x}[n - k + 1] \\ &\quad - \sum_{k=2}^{M+j-i} \mathbf{D}_{j,k-j+i}\mathbf{x}[n - k + 1] \\ &= \mathbf{D}_{j,i-j+1}\mathbf{x}[n] \end{aligned}$$

which is a scaled ISI-free estimate of $\mathbf{x}[n]$ without any time-delay when $\mathbf{D}_{j,i-j+1} \neq \mathbf{0}$. Now, let us consider the case where $\mathbf{w}[n] \neq \mathbf{0}$. The ISI-free estimate is

$$\hat{\mathbf{x}}[n] = \mathbf{D}_{j,i-j+1}\mathbf{x}[n] + \mathbf{B}_i\mathbf{n}[n] - \mathbf{B}_{i+1}\mathbf{n}[n - 1],$$

where $\mathbf{n}[n]$ is the noise component defined by

$$\mathbf{n}[n] = [\mathbf{w}[n]^t, \mathbf{w}[n - 1]^t, \cdots, \mathbf{w}[n - L + 1]^t]^t.$$

Since the values of $\mathbf{B}_{i+1}$ and $\mathbf{B}_i$ are related to $\mathbf{A}$, they are the solutions of criterion (5) and cannot be controlled. The SNR improvement of $\hat{\mathbf{x}}[n]$ is thus determined by the value of $\mathbf{D}_{j,i-j+1}$, or the selection of $\mathbf{B}_j$.

Observe that the middle elements of channel impulse coefficients $\mathbf{H}[k]$, $k = 0, 1, \cdots, m$ in (1) are generally larger than its leading and last few elements. One possible solution of selecting $\mathbf{B}_j$ is to use the received signal directly as the reference, and let $\mathbf{D}_{j,i-j+1}$ be a middle element of the channel impulse response. To do so, let $\mathbf{B}_j = \mathbf{B}_1 = [\mathbf{I}_u, \mathbf{0}, \mathbf{0}, \cdots, \mathbf{0}]$ where $\mathbf{I}_u$ is the identity with dimension u. Denote $\mathbf{H}_u[k]$ as the matrix consisting of the first u rows of $\mathbf{H}[k]$ for $k = 0, 1, \cdots, m$, and $\mathbf{w}_u[n]$ as the first u elements of $\mathbf{w}[n]$ for $n = 1, 2, \cdots, p$. Then,

$$\mathbf{e}_1[n] = \mathbf{B}_1\mathbf{o}[n] = \sum_{k=0}^{m} \mathbf{H}_u[k]\mathbf{x}[n - k] + \mathbf{w}_u[n],$$

and

$$\hat{\mathbf{x}}[n] = \mathbf{H}_u[i-1]\mathbf{x}[n] + \mathbf{B}_i\mathbf{n}[n] - \mathbf{B}_{i+1}\mathbf{n}[n-1], \tag{10}$$

for $2 \leq i \leq M$. When $\mathbf{H}_u[i - 1] \neq \mathbf{0}$, $\hat{\mathbf{x}}[n]$ is a scaled ISI-free estimate of the original input signals. In practice, an estimate of $\mathbf{x}[n]$ with good SNR is highlight needed. For single-user systems ($u = 1$), $\mathbf{H}_u[i - 1]$ degenerates into a constant. Therefore, the larger the absolute value of

$\mathbf{H}_u[i-1]$, the better the estimate $\hat{\mathbf{x}}[n]$. For multi-user systems, $\mathbf{H}_u[i-1]$ is a $u \times u$ mixing matrix. The criterion of a good integer i becomes complicated. Generally speaking, if the determinant of $\mathbf{H}_u[i-1]$ is far away from zero, then it can be viewed as a good candidate. However, from the overall system standpoint, some other candidates should be valued. Since $\hat{\mathbf{x}}[n]$ is used as the input of a source separation algorithm for CCI cancelation, ideally, if $\mathbf{H}_u[i-1]$ is the matrix whose rows contain the unique nonzero element, it is possibly another good candidate. In such case, some components of $\hat{\mathbf{x}}[n]$ come from only one user input. Consequently, by letting index i be different integers, several ISI-free estimates can be obtained with (10). It is likely that with these estimates the CCI cancelation procedure can be simplified. This is perhaps another contribution of this paper.

4. RECURSIVE LEAST-SQUARE ALGORITHM

Based on the above discussion, an ISI-free estimate of the original input can be obtained by (10). The key point is to minimize the criteria related to both $\mathbf{B}_i$ and $\mathbf{B}_{i+1}$ respectively, with the channel output as their references. Now, let us consider the solution of $\mathbf{B}_i$ first. Let $\mathbf{B}_1 = [\mathbf{I}_u, \mathbf{0}, \cdots, \mathbf{0}]$ and

$$\begin{aligned}\mathbf{\Phi}(\mathbf{B}_i, \mathbf{B}_1) &= \mathbf{B}_i \mathbf{R}_{oo}(0)\mathbf{B}_i^\dagger + \mathbf{B}_1 \mathbf{R}_{oo}(0)\mathbf{B}_1^\dagger \\ &- \mathbf{B}_i \mathbf{R}_{oo}(i-1)\mathbf{B}_1^\dagger - \mathbf{B}_1 \mathbf{R}_{oo}(i-1)^\dagger \mathbf{B}_i^\dagger,\end{aligned}$$

where $\mathbf{R}_{oo}(0) = E\{\mathbf{o}[n]\mathbf{o}[n]^\dagger\}$ and $\mathbf{R}_{oo}(i-1) = E\{\mathbf{o}[n+1-i]\mathbf{o}[n]^\dagger\}$. At the minimum point of $\mathbf{\Phi}(\mathbf{B}_i)$, $\mathbf{B}_i$ is the solution to the *Wiener-Hopf* equation and is given by

$$\mathbf{B}_i = \mathbf{B}_1 \mathbf{R}_{oo}(i-1)^\dagger \mathbf{R}_{oo}^{\#}(0), \tag{11}$$

where superscript $\#$ stands for *pseudo-inverse* operator. Correlation matrices $\mathbf{R}_{oo}(0)$, $\mathbf{R}_{oo}(i-1)$ and $\mathbf{R}_{oo}(i)$ can be approximated by the received sample sequence $\mathbf{o}[n]$. By substituting i with $i+1$, $\mathbf{B}_{i+1}$ is obtained by (11). Consequently, a batch algorithm of ISI cancelation can be easily designed. The ISI-free estimate is computed as

$$\hat{\mathbf{x}}[n] = \mathbf{B}_i \mathbf{o}[n] - \mathbf{B}_{i+1}\mathbf{o}[n-1].$$

Although this batch algorithm is less sensitive to the error of channel order estimate in comparison with other existing algorithms, it still involves a pseudo-inverse process that is time-consuming. In order to reduce the computation, the *Recursive Least-Squares*(RLS) algorithm is applied for the computation of $\mathbf{R}_{oo}(0)$ at the arrival of each received sample. The algorithm can be illustrated in Figure 1. In this block diagram, $\hat{\mathbf{x}}$[n+1-i] is the ISI-free estimate of the baseband signal. Two procedures are involved in this algorithm for the adaptation of $\mathbf{B}_i$ and $\mathbf{B}_{i+1}$. The reference signal $\mathbf{e}_1$[n] consists of the first u components of the received signal $\mathbf{y}$[n], and is used to generate the error signals for the adaptation of $\mathbf{B}_i$ and $\mathbf{B}_{i+1}$. Note that the criterion in (5) does in fact represent the energy of the error signal. $\mathbf{B}_i$ and $\mathbf{B}_{i+1}$ are then updated iteratively so that the energies of both error signals reach their minimum points.

Let $0 < \lambda < 1$. λ is referred to as the *forgetting factor*. Denote $\mathbf{R}_{oo}^{(n)}(0)$ as the values of $\mathbf{R}_{oo}(0)$ at time index n. Then,

$$\mathbf{R}_{oo}^{(n)}(0) = \lambda \mathbf{R}_{oo}^{(n-1)}(0) + \mathbf{o}[n]\mathbf{o}[n]^\dagger$$

where $\mathbf{R}_{oo}^{(0)}(0) = \mathbf{0}$ and $n > 0$. According to the *matrix inversion lemma*(pp.565, [12]), the inverse of $R_{oo}(0)$ can be updated by

$$\mathbf{R}_{oo}^{(n)\#}(0) = \lambda^{-1}(1 - k^{(n)}\mathbf{o}[n]^\dagger)\mathbf{R}_{oo}^{(n-1)\#}(0)$$

where $\mathbf{R}_{oo}^{(0)\#}(0) = \delta^{-1}\mathbf{I}$, δ is a small positive constant and $\mathbf{I}$ is the identity with the dimension equal to that of $\mathbf{R}_{oo}(0)$, and

$$k^{(n)} = \frac{\lambda^{-1}\mathbf{R}_{oo}^{(n-1)\#}(0)\mathbf{o}[n]}{1 + \lambda^{-1}\mathbf{o}[n]^\dagger \mathbf{R}_{oo}^{(n-1)\#}(0)\mathbf{o}[n]}.$$

The procedure involved in our proposed algorithm is now summarized below.

1. Suppose that the channel order is over-estimated as m, then L is chosen as the smallest integer greater than $mu/(N-u)$. Let i be a certain positive integer less than m, δ be a small positive constant and $0 < \lambda < 1$, and $M = m + L$. $\mathbf{R}_{oo}^{(0)\#}(0) = \delta^{-1}\mathbf{I}$, $\mathbf{R}_{oo}^{(0)}(i-1) = \mathbf{0}$, $\mathbf{R}_{oo}^{(0)}(i) = \mathbf{0}$.
2. For $n = 1, 2, \cdots$,
   - collect the received sample $\mathbf{y}[n]$ and construct $\mathbf{o}[n]$ by (2) where, if $n < 1$ in $\mathbf{y}[n]$ then let $\mathbf{y}[n] = \mathbf{0}$.

– update

$$\mathbf{R}_{oo}^{(n)}(i-1) = \lambda \mathbf{R}_{oo}^{(n-1)}(i-1) + \mathbf{o}[n]\mathbf{o}[n+1-i]^{\dagger},$$

$$\mathbf{R}_{oo}^{(n)}(i) = \lambda \mathbf{R}_{oo}^{(n-1)}(i) + \mathbf{o}[n]\mathbf{o}[n-i]^{\dagger},$$

and

$$k^{(n)} = \frac{\lambda^{-1}\mathbf{R}_{oo}^{(n-1)\#}(0)\mathbf{o}[n]}{1+\lambda^{-1}\mathbf{o}[n]^{\dagger}\mathbf{R}_{oo}^{(n-1)\#}(0)\mathbf{o}[n]},$$

$$\mathbf{R}_{oo}^{(n)\#}(0) = \lambda^{-1}(1-k^{(n)}\mathbf{o}[n]^{\dagger})\mathbf{R}_{oo}^{(n-1)\#}(0)$$

– compute

$$\mathbf{B}_i = \mathbf{B}_1\mathbf{R}_{oo}(i-1)^{(n)\dagger}\mathbf{R}_{oo}^{(n)\#}(0),$$

$$\mathbf{B}_{i+1} = \mathbf{B}_1\mathbf{R}_{oo}(i)^{(n)\dagger}\mathbf{R}_{oo}^{(n)\#}(0),$$

– estimate the ISI-free output

$$\hat{\mathbf{x}}[n] = \mathbf{B}_i\mathbf{o}[n] - \mathbf{B}_{i+1}\mathbf{o}[n-1].$$

3. End.

The algorithm can be divided into three parts for each new received signal sample: determining three correlation matrices, computing $\mathbf{B}_i$ and $\mathbf{B}_{i+1}$, and making an estimate of the original signal $\mathbf{x}[n]$. The estimate $\hat{\mathbf{x}}[n]$ has a fixed number of symbol time delays $i-1$. Its SNR is strongly dependent on the selection of the integer i. As mentioned above, for single-user communication systems, it is better to select a value of i so that the (i)-th coefficient of the channel impulse response is far away from zero. However, because of the "blind" limitation, no knowledge about the channel impulse response is available prior to the setting of the value of i. As the absolute values (determinant) of the center coefficients are generally large, one possible way is to let i be an integer in $[2, m-1]$.

5. SIMULATION EXAMPLES

In this section, the ISI cancelation algorithm is evaluated by a single-user wireless communication system. The source data is shaped by a pulse shaped filter before being transmitted. The pulse shaped filter has a raised-cosine impulse response function $p(t)$ with roll-off factor $\beta = 0.1$. Let the channel be a two-ray multi-path whose impulse response is expressed by

$$h(t) = p(t) - 0.4(1+j)p(t-1.3T_s), \qquad (12)$$

where T_s is the baud period and $j = \sqrt{-1}$. The raised-cosine pulse $p(t)$ is truncated to $4T_s$. The channel order is then $m = 4$. The estimate of channel order is set to be 5 in our simulations in order for demonstrating the robustness of our algorithm to channel order error. Under SNR=30dB, 1000 source symbols of a single user input drawn from uniform 4 QAM are transmitted. The received signal is oversampled by a factor of 2. The channel impulse response is shown in Figure 2 where the coefficients with odd index consist of sub-channel one and those with even index consist of sub-channel two. Let $i = 2$. Figure 3 is an eyes diagram after equalization at SNR=30dB. Figure 4 shows the impulse coefficients of two "composite channels" obtained by $\mathbf{B}_2\mathbf{A}$ and $\mathbf{B}_3\mathbf{A}$. It is obvious to see that the first coefficient of (a) and (b) in Figure 4 is almost equal to the second coefficient of sub-channel one in Figure 2, *i.e.* the third (2nd odd) in both real and imaginary parts respectively; the second of (a) and (b), and the first of (c) and (d) in Figure 4 is equal to each other respectively, and is equal to the fifth coefficient in Figure 2, and so on. Figure 5 presents the ISI situation of $\hat{\mathbf{x}}(n)$. The ISI of $\hat{\mathbf{x}}(n)$ comes from the difference of the two composite channel coefficients represented by "circle" and "star", and is indeed very small.

6. CONCLUSIONS

In this paper, by taking the advantage of channel diversity in linear multi-user systems, we proposed a new blind RLS-type ISI cancelation algorithm. The algorithm updates two filter parameters by using received signal as reference so that the output of one filter is exactly equal to the ISI part of the other. The difference between these two filter outputs is therefore an ISI-free estimate of the input. This algorithm works well in the presence of channel order mismatch and, is simple and easy to implement by RLS structure.

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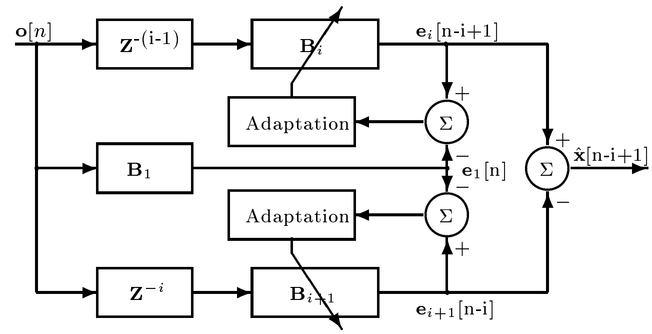

Figure 1. Block Diagram of The ISI Cancelation Algorithm

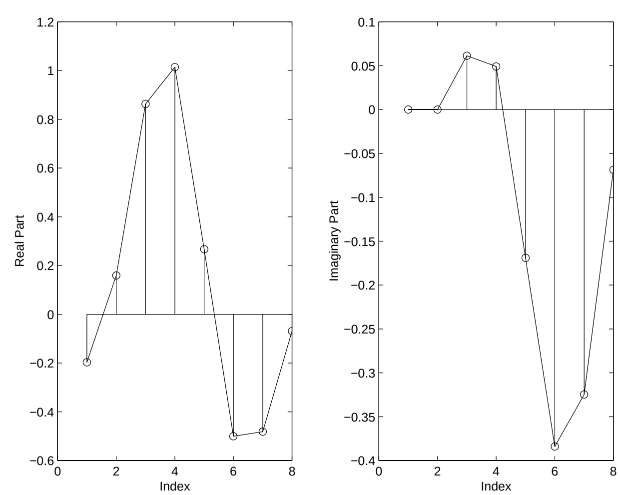

Figure 2. Real and imaginary parts of the communication channel $h(t)$.

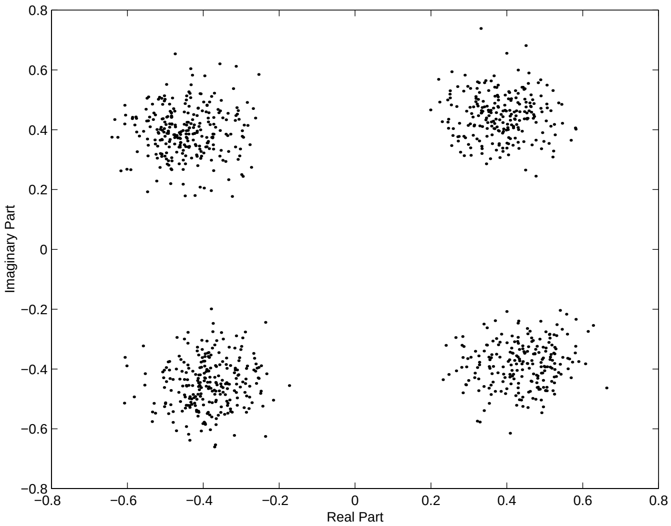

Figure 3. Eyes diagram after equalization under SNR=30dB.

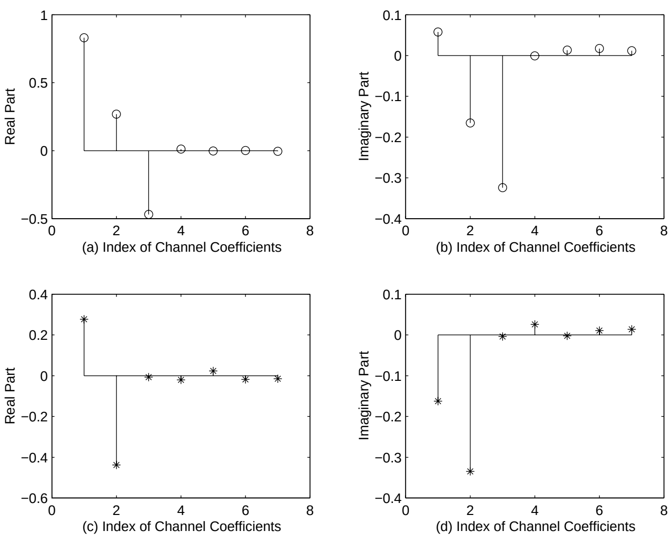

Figure 4. Impulse Coefficients of "Composite Channels": (a) and (b) consist of the composite channel whose output is $e_i(n)$; whereas (c) and (d) consist of the one whose output is $e_{i+1}(n)$

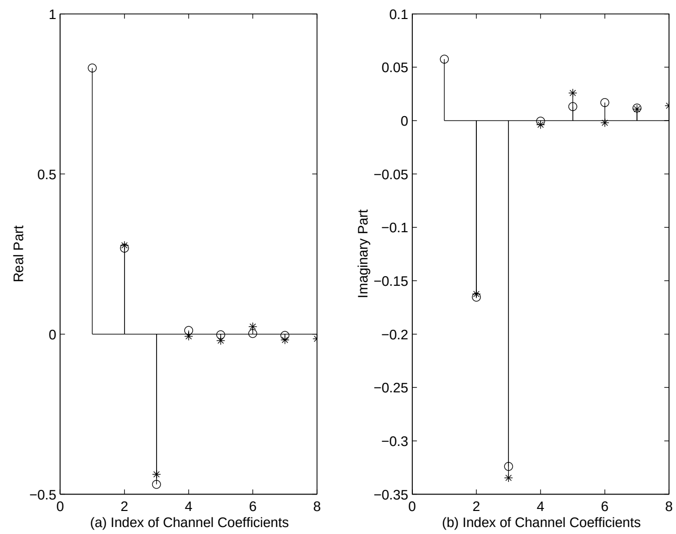

Figure 5. Impulse Coefficients of "Composite Channel" for $\hat{x}(n)$: the coefficient is the distance between "circle" and "star" at each index.