A new edge detector based on the extended Russ operator and its application to mammogram segmentation

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Abstract: This paper introduces a novel edge detector based on a new operator—the extended Russ operator—first introduced by J. C. Russ, based on the work of H. E. Hurst. One of the components of the extended Russ operator has been found to be edge sensitive. It has been used to good effect to pre-process mammograms before they are submitted to a conventional Canny edge detector. The resulting images provide an unbroken skin line and good detail in the nipple region, sufficient to facilitate further segmentation with ease. Although computationally demanding, it has the scope to be developed into an edge detector in its own right, once the theory behind its effectiveness is investigated further.

Keywords: edge detection, mammograms, segmentation, Hurst exponent, Russ operator

1 Introduction

The edge detector introduced in this paper has its origins in the work of H. E. Hurst, a civil engineer who studied the river Nile and aimed to regulate its flow by designing reservoirs to avoid droughts and floods. His analysis of the yearly outflow of water along the river system, as a time series, led to a method of data analysis [1, 2] now known as the *method of Hurst*. Mandelbrot and co-workers [3, 4] explained this method in terms of fractional Brownian motion (fBm) and renamed it *rescaled range (R/S) analysis*. J. C. Russ [5] extended this method to texture analysis of two-dimensional intensity images, although his calculations did not exactly mirror those of Hurst [6, chapter 9].

We have modified Russ's method by releasing the data analysis from its fractal underpinnings and interpreting the resulting derived parameters purely as features extracted from the data. Experiments then showed that one of the new parameters functioned as an edge detector that gave clean, continuous edges when tested on mammograms. The resulting images when presented to a Canny edge detector gave far better binary edge images than when the originals were used as inputs. This new edge detector needs further investigation, especially into its theory of operation and domain of applicability.

2 The original Russ operator

The original Russ operator was called by him the "local Hurst operator" and defined as follows:

1. Define an octagonal mask of a given "radius" r as shown in Figure 1. The different pixels are then at different distances from the centre pixel as given in Table 1.

2. For each set of pixels at the same Euclidean distance ρ from the centre pixel, find the maximum and the minimum *pixel values*. Their difference is the *range*

3. Plot $\log R$ against $\log \rho$ and fit a straight line to the data to minimize the square of the error.

4. The *slope* of the plot, m, is a measure of "local roughness (in the sense of the Hurst coefficient)" [5, p 250] at the centre pixel. It

		7	6	7		
	5	4	3	4	5	
7	4	2	1	2	4	7
6	3	1	0	1	3	6
7	4	2	1	2	4	7
	5	4	3	4	5	
		7	6	7		

Figure 1: Octagonal mask of "radius" $r = 3$ and diameter 7. Pixels at the same Euclidean distance ρ_k from the central pixel, numbered 0 above, are labelled with the same index. See Table 1 for the relevant distances.

could be used to plot a transformed image that could later be segmented on the basis of the m values.

5. The square of the coefficient of correlation, η^2, is a measure of the goodness of fit of the straight line and should be close to 1 for images exhibiting fractal characteristics.

We refer to the above operator as the *Russ operator*, denoted by $\mathcal{R}$. It uses an octagonal mask of radius r, centred on a pixel, $\mathbf{p} = (a, b)$ in an image. It maps that pixel $\mathbf{p}$ in the image to a specific value of m, which we denote by $m_{\mathbf{p}}$:

$$\mathcal{R}(r, \mathbf{p}) = m_{\mathbf{p}} \qquad (1)$$

This value, $m_{\mathbf{p}}$, represents the gradient of the straight line fitted to the plot of $\log R$ versus $\log \rho$ as shown, for example, in Figure 2.

3 The extended Russ operator

In his original formulation, Russ focused principally on the gradient m which he used as a measure of local roughness. However, the line fitting step 3 in section 2 above actually yields *three* parameters: the gradient m, the y-axis intercept c and the coefficient of correlation η. From our experiments with the Russ operator on mammograms, we have found

Pixel Label k	No. of Pixels	Euclidean Distance ρ_k
0	1	0
1	4	1
2	4	$\sqrt{2}$
3	4	2
4	8	$\sqrt{5}$
5	4	$2\sqrt{2}$
6	4	3
7	8	$\sqrt{10}$

Table 1: The Euclidean distances ρ_k corresponding to the indices k shown on the octagonal mask of radius $r = 3$ in Figure 1.

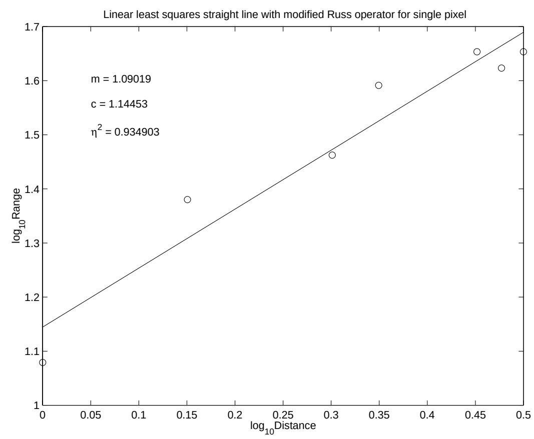

Figure 2: Graph of straight line fitted to the seven points arising from the mask of radius 3. The values refer to a pixel at co-ordinates (230, 350) at 400 μm per pixel for MIAS mammogram image mdb028rl. The vales of (m, c, η^2) are $(1.09, 1.14, 0.935)$.

that both c and η^2 embody useful information. We have therefore chosen to extend the Russ operator and parametrize the resulting *extended Russ operator*, $\mathcal{E}$, so:

$$\mathcal{E}(r, \mathbf{p}) = (m, c, \eta^2)^T_{\mathbf{p}} \qquad (2)$$

4 The new edge detector: results with mammograms

When the extended Russ operator was applied to mammograms it was found that the parameter c associated with each pixel is edge-sensitive. An image displaying the c-values at each pixel, scaled appropriately, emphasizes the edges in the original image. *Thus the c component of the extended Russ operator can function as an edge detector.*

Figure 3 shows the results of experiments with two mammograms, mdb028rl and mdb003ll, from the MIAS [7] database; the original images are shown in parts (a) and (e) of the figure respectively.

The edge separating the breast from the background on a mammogram varies in strength across the image. Thus, even an optimal and robust conventional edge detector like the Canny edge detector, claimed to be "less likely than the others to be"fooled" by noise, and more likely to detect true weak edges" [8], does not give an unbroken, unambiguous skin line when applied directly to mammograms, as shown in the binary edge images in Figures 3(b) and (f). The images in (c) and (g) show the edge-enhanced c-component images resulting from the application of the extended Russ operator on the original images. Note especially the clearly visible skin line and the detail in the nipple regions on both images. When these latter, edge-enhanced images are presented in place of the original mammograms to the same Canny edge detector, the results are shown in Figures 3(d) and (h) respectively. Although the edge detail within the breast region is so excessive as to be quite useless, the skin line and nipple regions are clearly outlined and can be used as inputs to an algorithm for segmenting the mammogram into the breast and background.

5 Conclusions

Our results lead us to conclude that the c-component of the extended Russ operator functions well as an edge pre-processor that emphasizes anatomical edges of varying strength before an image is submitted to a conventional edge detector. This improves the continuity and detectability of the edges and enables a useful binary image to be produced by the edge detector, suitable for image segmentation. It is our belief that with further development, the c-component may used as an edge detector in its own right. Before that can take place, however, it is necessary to understand the theoretical basis for this empirically discovered edge detector.

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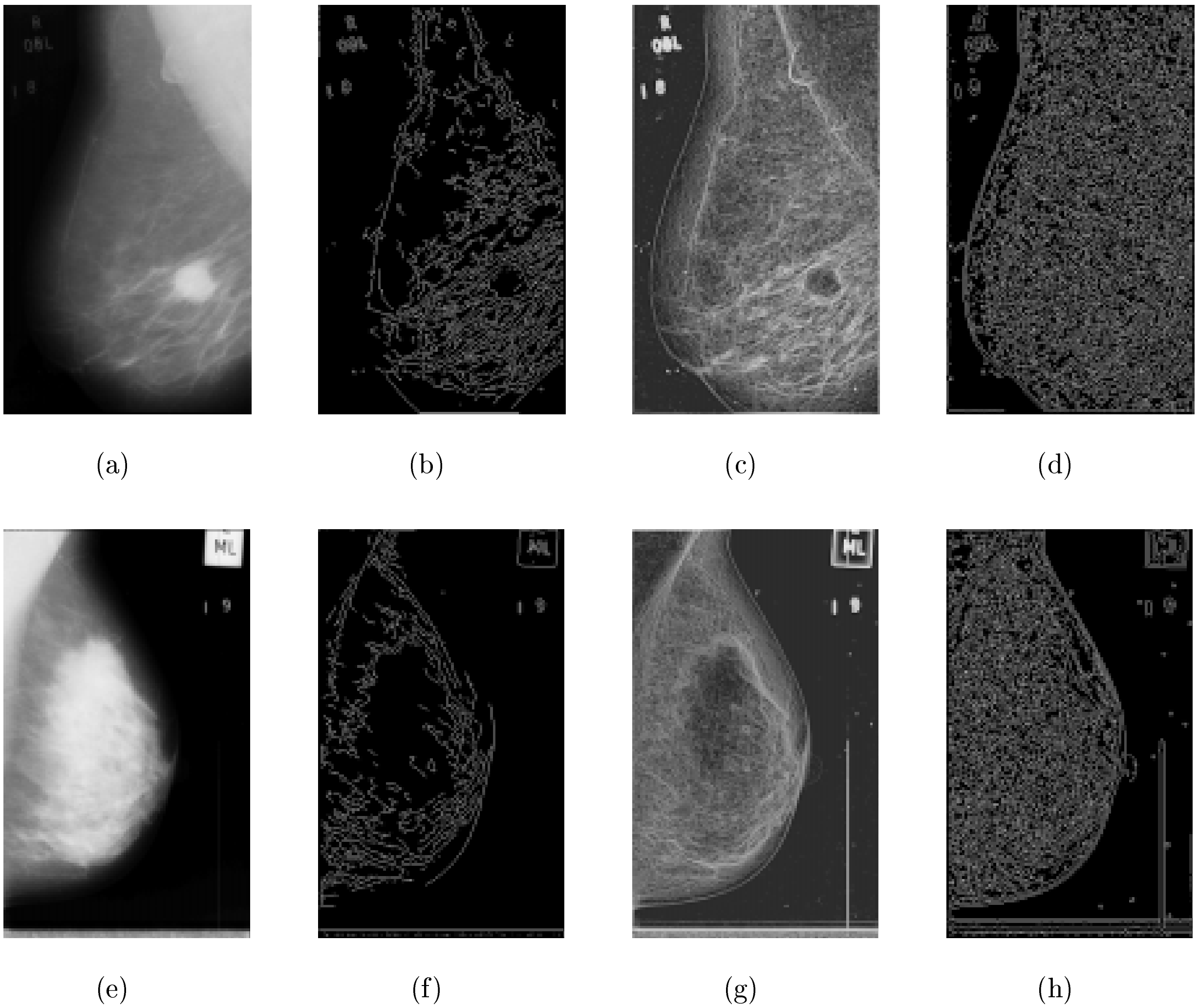

Figure 3: Edge image results with mammograms: (a), (e): original mammograms mdb028rl and mdb003ll from MIAS database; (b), (f): edge images from the Canny edge detector applied to original mammograms; (g), (h): edge enhanced images obtained by plotting the c-component of the modified Russ operator with a mask of radius 3; (d), (h): binary edge images when the Canny edge detector is applied to the edge-enhanced images in (c) and (g) respectively.