Robust nonlinear control for ship steering

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Abstract: - In the paper a velocity-based linearisation approach for a design of a gain-scheduling controller for ship steering is presented. While ship steering and gain-scheduling are well known problems with well-developed solutions, the described method differs from previous approaches to gain-scheduled controller design in that the closed-loop system with nonlinear controller retains properties of the linearised system with linear controller designed in one of the design stages. Stochastic robustness analysis with Monte Carlo simulations is applied to confirm robustness performance of the nonlinear control which was the objective for control design.

Key-Words: gain scheduling, robust nonlinear control, ship steering problem. *CSCC'99 Proceedings*, Pages:2971-2976

1 Introduction

The analysis and design of nonlinear dynamic systems is relatively difficult. On the other hand, techniques for the analysis and design of linear time-invariant systems are better established, even although systems with genuinely linear time-invariant dynamics in reality do not exist. It is, therefore, attractive to adopt the design approach where a nonlinear system is decomposed into the design of a family of linear time-invariant systems. This type of strategy forms the basis of one of the most widely, and successfully, applied techniques for the design of nonlinear controllers; namely, gain-scheduling.

However, in the conventional, and most common, gain-scheduling design approach, each linear controller is typically associated with a specific equilibrium operating point of the plant and is designed to ensure that, locally to the equilibrium operating point, the performance requirements are met.

In [4, 5], a framework is proposed for the analysis and design of gain-scheduled and nonlinear systems which associates a family of velocity-based linearisations with a nonlinear system. Each operating point of the nonlinear system, including operating points far from equilibrium, has an associated member of the velocity-based linearisation family which describes the dynamic characteristics in the vicinity of that operating point. In contrast to the conventional series expansion linearisation about an equilibrium operating point, the velocity-based linearisation family indicates the plant dynamics not only in the vicinity of a single equilibrium operating point but also during transitions between equilibrium operating points and when operating far from equilibrium.

In ship steering control much use is made of proportional plus integral plus derivative (PID) control methods. However, ship dynamics change significantly with forward speed, with depth of water and with load. The optimal settings of a conventional controller of this type are thus likely to change with changes of operating condition.

Adaptive controllers have been considered for ship steering control (e.g. [8, 1]), but, as in many other types of safety-critical application, there is some concern about potential instabilities and other problems associated with adaptive system behaviour. Nonlinear control strategies (other than adaptive control) thus appear to offer some advantages over other forms of control for ship steering and the main objective of the work described in this paper has been to carry out a study to investigate the potential of controllers based on the velocity-based linearisation families for this application.

The paper is organised as follows. Description of the ship steering problem is given in Section 2. Section 3 provides an outline of the nonlinear control system design and section 4 demonstrates the robustnes properties through stochastic robustness analysis. Conclusions are stated in Section 5.

2 Ship steering problem

Automatic systems for the steering of ships have been in existence for over seventy years. Interest in the problems of ship steering control has increased during the past two decades because of potential cost benefits arising from fuel savings when sophisticated control algorithms are applied.

The ship model is an extended version of Nomoto's first-order model [6] which has been the basis of other ship steering studies (e.g. [2]). For the case of a vessel with a symmetrical hull the mathematical model takes the form

$$\frac{T}{K}\ddot{\psi} + \frac{n_1}{K}\dot{\psi} + \frac{n_3}{K}\dot{\psi}^3 = u \tag{1}$$

where ψ is the yaw angle and u is rudder angle. The parameters are T which is a time constant, K which is a gain constant and n_1 and n_3 which are two constants known as Norrbin coefficients. For a ship of length 45 m at a forward speed U_0 of 5 m/s a suitable set of parameters [2] is as follows:

$$K = 0.5\ \mathrm{s}^{-1}, T = 31\ \mathrm{s}, n_1 = 1.0, n_3 = 0.4\ \mathrm{s}^2$$

Nonlinear effects due to limitations on the permissible maximum deflection of the rudder (u_{max}) and the

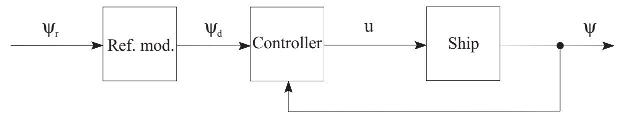

Figure 1: Block diagram of model reference control system

maximum angular rate ($\dot{u}_{max}$) must also be considered. Appropriate values are $\pm$ 35 degrees for the maximum rudder deflection and $\pm$ 7 degrees/s for the maximum rudder rate.

Two steering modes are involved in the specification of ship steering control system performance. These are the *course keeping* and *course changing* performance.

For the course changing mode there are well defined criteria which can be expressed in terms of a specific form of desired step response in the time domain. The optimisation of course-keeping control characteristics is more complex since the requirements in confined waters and in open sea conditions are different.

Particular consideration has been given to course-changing control problems during this investigation. A model reference approach has been used to define the desired response characteristics for a course-changing control system, as shown in Figure 1

The dynamics of the reference model should be matched to the dynamics of the ship regardless of the magnitude of the demanded change of reference yaw angle. A reference model which is too sluggish cannot produce an optimal performance since the ship cannot reach the required heading in the minimum time. On the other hand we should not use a reference model which is too fast compared with the ship response characteristics because this may cause rudder actuator saturation and performance degradation.

One appropriate reference model proposed by [8] is described by a second order transfer function

$$\frac{\psi_d(s)}{\psi_r(s)} = \frac{K_m}{T_m s^2 + s + K_m} \tag{2}$$

where the time constant T_m must be chosen to be smaller than the dominant time constant of the ship model. A ratio of 2 or 3 to 1 between T_m and the dominant time constant of the vessel has been suggested [8]. The damping factor for this second-order model is given by

$$Z = \frac{1}{2\sqrt{K_m T_m}} \tag{3}$$

To take account of the nonlinearities associated with the rudder, van Amerongen suggested introducing two additional factors in the reference model so that the gain factor K_m is replaced by a factor $\frac{K_m f}{1+sTa}$ where f and T_a are calculated in real time and depend on the demanded rudder deflection, the maximum deflection and the maximum angular rate [8].

This gives a reference model characterised by the equation

$$\begin{aligned}\dddot{\psi}_d + \left(\frac{1}{T_a} + \frac{1}{T_m}\right)\ddot{\psi}_d &+ \frac{1}{T_a T_m}\dot{\psi}_d + \frac{K_m f}{T_a T_m}\psi_d \\ &= \frac{K_m f}{T_a T_m}\psi_r \end{aligned} \tag{4}$$

In this investigation a slightly different approach has been taken to the generation of a reference model although the structure of van Amerogen's third order model has been retained. The reference model of equation (4) may be rewritten in the form

$$\dddot{\psi}_d + A\ddot{\psi}_d + B\dot{\psi}_d + C\psi_d = C\psi_r \tag{5}$$

Then if one real root of the characteristic equation (say r) is known the other two roots can be found. It can be shown that for this to be true the parameters of the reference model must be related in such a way that

$$C = r(B - rA + r^2) \tag{6}$$

and for critical damping in the remaining poles it is necessary that

$$B = \frac{1}{4}(A^2 + 2Ar - 3r^2) \tag{7}$$

For values of B less than the critical value of equation (7) the reference model has overdamped characteristics and, correspondingly, for larger values of B it is underdamped.

Although van Amerongen suggested that f and T_a should depend on the rudder angular deflection and angular rate, a constant value of f of unity was chosen in the current work and T_a was also taken to be a constant. Since the time constant of the ship (T) is known to be dependent upon the forward speed it is clearly appropriate to use a time constant T_m in the reference model which depends upon T. This was achieved by using a relationship

$$T_m = \frac{T}{\gamma} \tag{8}$$

where the factor γ depends on the operating condition. The problem then becomes one of finding appropriate values for the factor γ and for the time constants T_a and T_m.

An empirical approach was adopted in order to find suitable values for these parameters of the reference model. The ship was simulated with a feedback linearisation controller designed for a forward speed of 5 m/s. Tests were performed on this simulated system for step changes of reference ranging from 5 degrees to 60 degrees for a number of different values of T_a and T_m. The aim was to find combinations of reference model parameter values which, together with the controller, would produce responses which were as fast as possible without causing the rudder to reach limiting conditions in terms of its deflection or angular rate. Satisfactory results were found involving a value of T_a of 3 and values of γ which were dependent on the size of the reference step. Similarly a relationship was found between the size of the reference step and the value of the real root r in the characteristic equation of the model.

3 Control design

As stated in the Introduction, nonlinear control has already been applied as a solution for the ship steering problem. In contrast to the previous approaches the velocity-based linearisation analysis offers several advantages. The family of velocity-based linearisations can be pieced together to approximate the solution to a nonlinear system so stability as well as the transient behaviour of the nonlinear system can be investigated. Consequently, the velocity-based linearisation theory has considerable potential for supporting the design and analysis of gain-scheduled controllers.

Given the direct relationship between the velocity-based form of the nonlinear systems and their associated velocity-based linearisation families and the strong correspondence in their dynamic behaviour, the velocity-based linearisation families constitute a much more appropriate framework for the analysis and design of gain-scheduled controllers than conventional approaches.

The foregoing analysis suggests the following design procedure [5].

1. Determine the velocity-based linearisation family associated with the nonlinear plant dynamics.

2. Based on the plant velocity-based linearisation family, determine the required controller velocity-based linearisation family such that the resulting closed-loop family achieves the performance requirements. Since each member of the plant family is linear, conventional *linear* design methods can be utilised to design each corresponding member of the controller family.

3. Realise a nonlinear controller corresponding to the family of linear controllers designed at step 2. The velocity-based form of the controller can be obtained directly from the family of linear controllers by simply permitting the point of linearisation to vary with the operating point.

The reader is referred to [3, 4] and [5] for further explanations and proofs concerning the framework which has been adopted.

This design procedure maintains the continuity with linear design methods which is an important feature of the conventional gain-scheduling approach. However, in contrast to the conventional gain-scheduling approach, the resulting nonlinear controller is valid throughout the operating envelope of the plant, not just in the vicinity of the equilibrium operating points. This extension is a direct consequence of employing the velocity-based linearisation framework rather than the conventional series expansion linearisation about an equilibrium operating point. The main advantage of the approach is that the analysis investigation of the velocity-based linearised system is not confined to the operating point but is global. Therefore the analysis results of system properties like stability and robustness are valid throughout the operating envelope of the plant [5].

The described approach is applied for ship steering as follows. Consider the second order plant describing ship dynamics based on (1) with parameters scaled for different forward speeds [2]

$$\ddot{\psi} + \frac{n_1}{T_0}\left[\frac{U_0}{U}\right]^3 \dot{\psi} + \frac{n_3}{T_0}\left[\frac{U_0}{U}\right]^5 \dot{\psi}^3 = \frac{K}{T_0}\left[\frac{U_0}{U}\right]^2 u \tag{9}$$

which can be written as

$$\ddot{\psi} + a\dot{\psi} + b\dot{\psi}^3 = cu \tag{10}$$

where

$$a = \frac{n_1}{T_0}\left[\frac{U_0}{U}\right]^3$$

$$b = \frac{n_3}{T_0}\left[\frac{U_0}{U}\right]^5$$

$$c = \frac{K}{T_0}\left[\frac{U_0}{U}\right]^2$$

The requirement is to design a controller for ship's yaw angle ψ such that the closed-loop system satisfies robustness demands due to the variations in speed and parameters and provides a closed-system bandwidth sufficient for satisfactory tracking properties.

At an equilibrium point (ψ_0, u_0)

$$u_0 = 0 \tag{11}$$

Hence, the series expansion linearisation of (9) relative to the equilibrium operating point (ψ_0, u_0), is

$$\delta\ddot{\psi} + (a + 3b\dot{\psi}_0^2)\delta\dot{\psi} = c\delta u \tag{12}$$

$$\delta\psi = \psi - \psi_0, \delta u = u - u_0$$

Based on the conventional series expansion linearisation at an equilibrium operating point, an appropriate local controller is the controller

$$\dot{u} + 500u = K_P\dot{e} + K_P e(a + 3b\dot{\psi}_0^2) \tag{13}$$

$$\delta e = e - e_0, u = \delta u + u_0$$

with

$$e = \psi_d - \psi.$$

The transfer function of the controller (13) is $\frac{K_P}{s+500}(s + a + b3\dot{\psi}_0^2)$. The appropriate bandwidth and phase margin figures are achieved for the closed-loop system through appropriate selection of the parameter K_P.

The dynamics, (12), are the same at every equilibrium operating point and so the controller, (13), may be employed at every equilibrium operating point.

Nevertheless, the linear controller does not achieve the required performance. In order to incorporate information about the plant dynamics at non-equilibrium operating points into the controller design, reformulate the nonlinear plant, (10), by differentiating, as

$$\ddot{w} + (a + 3b\dot{\psi}^2)\dot{w} = c\dot{u} \qquad (14)$$

The velocity-based linearisation family at the general operating point (ψ_1, u_1) associated with the nonlinear plant, (10), consist of the frozen forms of (14) obtained when ψ is constant,

$$\ddot{\hat{w}} + (a + 3b\dot{\psi}_1^2)\dot{\hat{w}} = c\dot{u} \qquad (15)$$

The required velocity-based linearisation family of the controller is determined by using linear methods to design a local controller for each of the members of the plant velocity-based linearisation family. Employing the same structure as previously, consider the linear controller family

$$\ddot{u} + 500\dot{u} = K_P\ddot{e} + K_P(a + 3b\dot{\psi}^2)\dot{e} \qquad (16)$$

At equilibrium operating points, the members of the linear controller family correspond to the controller dynamics (13), determined previously. However, at non-equilibrium points, where ψ is not constant, the parameters of the controller are now different from their equilibrium values and are designed to compensate for the variation in the dynamics of the members of the plant velocity-based linearisation family. The response of the closed-loop system throughout the whole operating region for selected $K_P = 10000$ (bandwidth is app. 2.5 rad/s, phase margin is app. $90°$) is depicted in Figure 2. It can be seen that the performance requirements (tracking of the reference model signal) are met for the full operating range.

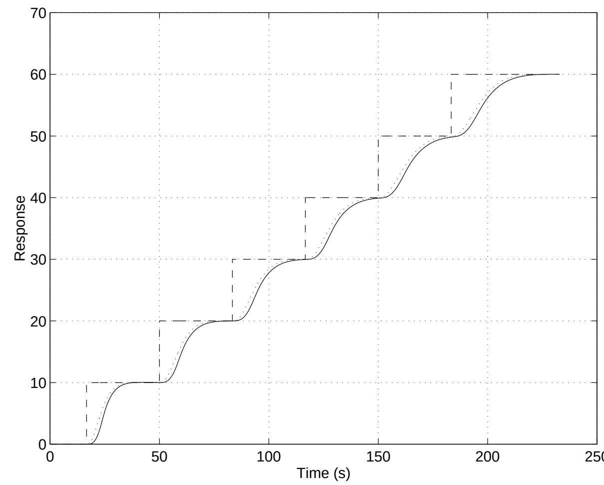

Figure 2: Response (full line) to reference model response (dotted line) obtained with step commands (dashed line) of magnitudes ranging from 10 to 60 degrees with nonlinear controller

4 Stochastic robustness analysis of closed-loop system

Control systems robustness is defined as the ability to maintain satisfactory stability or performance characteristics in the presence of all conceivable system parameter variations. There exist a wide range of methods for robustness analysis of linear systems [9]. In the area of nonlinear control the robustness analysis is not so well developed. One possible approach to nonlinear systems robustness analysis is Stochastic Robustness Analysis [7]. This method determines the *stochastic robustness* of a system by calculation of time responses with Monte Carlo methods. Time responses provide the most clear-cut means of evaluating performance. When time responses are computed, stochastic performance robustness can be portrayed as a distribution of possible trajectories around a nominal or desired trajectory. Envelopes can be defined around a nominal trajectory based on stated performance criteria, and the probability of exceeding the envelope becomes the scalar, binomial performance robustness metric [7].

Responses of the designed ship control system evaluated by Monte Carlo analysis are given in figure 3. Parameters K_0 [0.25,0.75], T_0 [15,45] and speed U [5,20]m/s have been changing in 600 simulations. It can be seen from Figure 3 that the chosen control strategy satisfactorily deals with the uncertainties in the parameter values while changes in the operating conditions are covered by the nonlinear nature of the controller.

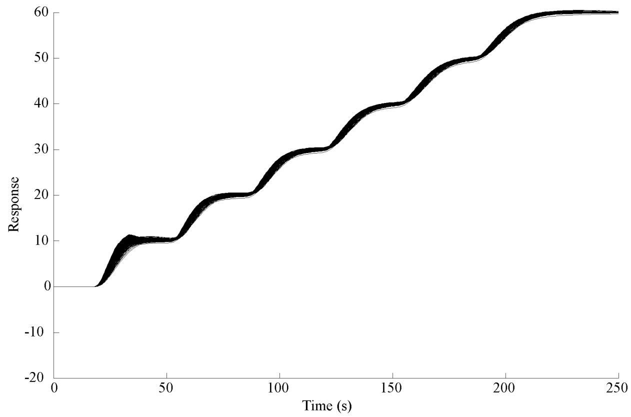

Figure 3: Stochastic robustness evaluation of the closed-loop system to reference model response with variations in parameters and speed

5 Conclusion

In the paper a robust nonlinear controller design for ship steering has been presented. The method used is based on gain-scheduled controller design with velocity linearisation. This approach effectively links nonlinear systems design with knowledge used for conventional linear systems design.

A nonlinear controller was designed via velocity-linearised nonlinear system description. Robustness had been achieved during the linear phase of design and was preserved when the nonlinear form of controller was applied. The nonlinear controller succesfully performed its task regardless of process parameter variations, as shown by stochastic robustness analysis methods.

The advantage of the proposed approach is that it provides a single controller, of moderate complexity, which is valid for a wide range of operating conditions and is robust to parameter variations.

ACKNOWLEDGEMENT

The authors would like to thank prof. John O'Reilly from University of Glasgow for his usefull comments regarding this paper.

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