Dynamics and Synchronization of a Second-Order Nonlinear and Nonautonomous Electric Circuit

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Abstract: We study the chaotic dynamics of a second-order nonlinear and non-autonomous electric circuit. The circuit has only one nonlinear element, a nonlinear resistor with a piecewise-linear v-i characteristic of N-type. We also study chaos synchronization of two identical circuits by one-way coupling.

Key-Words: nonlinear circuits, chaos, bifurcation, antimonotonicity, synchronization
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1. Introduction

The study of nonlinear electric circuits is a convenient yet powerful experimental and analytical tool in studying chaotic behavior in nonlinear dynamics. The study of a series of simple nonlinear and non-autonomous second-order electric circuits revealed all the routes to chaos [1-9].

Recently, J. G. Lacy [10] presented a simple nonautonomous second-order electric circuit (Fig.1), with a nonlinear resistor, whose v-i characteristic is piecewise linear of N-type (Fig.2).J. G. Lacy studied the chaotic behavior of the circuit for a single value of the frequency f

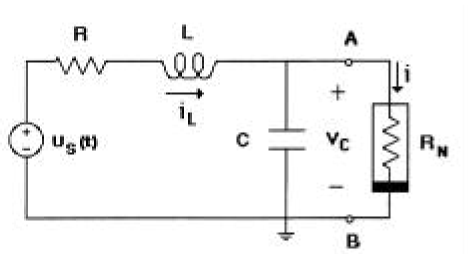


Fig.1 The electrical circuit under consideration.

(f = 5.0kHz) of the sinusoidal voltage source, using its amplitude V_S as the bifurcation parameter.

In the present paper, we have studied, by experiment and computer simulation, the dynamics of Lacy's circuit over a wide range of frequencies, using a nonlinear resistor, whose picewise linear v-i characteristic has different break points, $\pm B_p$. We have also studied the chaos synchronization of two identical circuits by one-way coupling.

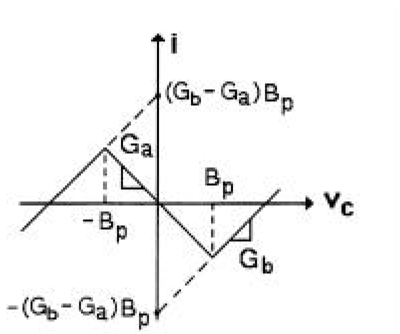


Fig.2 i-v characteristic of the nonlinear resistor N_R of our circuit

2. Experimental Apparatus and Differential Equations

The parameter values of the circuit of Fig.1 are L = 32.9mH, R = 700Ù, C = 62.9nF, B_p = 1.787V, G_a = -2.2mS, G_b = 1.0mS, while V_S and f_S are the control parameters. The realization of the nonlinear resistor is shown in Fig.3, where $R_1 = R_3$ = 1.0kÙ and R_2 = 2.2kÙ. The two back-to-back zener diodes (E_Z = 6V) set the break points $\pm B_p$.

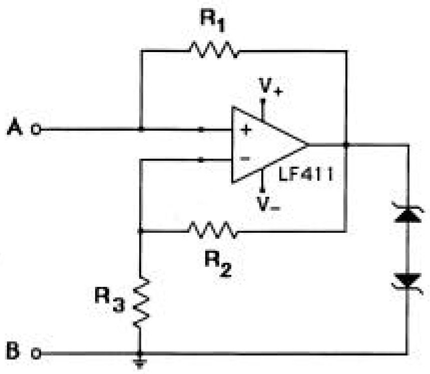


Fig.3 Laboratory realization of the nonlinear resistor

The differential equations of the circuit of Fig.1 are the following

$$C(dv_C/dt) = i_L - g(v_C) \quad (1)$$

$$L(di_L/dt) = - Ri_L - v_C + V_S\sin(2ðft) \quad (2)$$

where

$$g(v_C)= G_b v_C + 0.5(G_a – G_b)\{|v_C +B_p|-|v_C – B_p|\} \quad (3)$$

is the equation of the v-i characteristic of Fig.2.

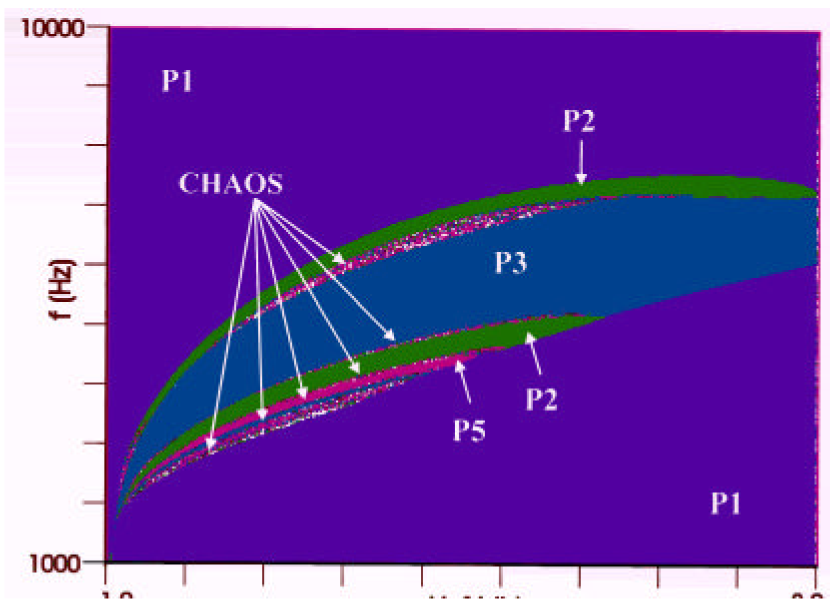


Fig.4 Parametric diagram, f vs. V_S , for our system

3. The Dynamics of the System

We have studied the dynamics of the system for different fixed values of the frequency f, as the voltage V_S was increased. The superposition of the bifurcation diagrams give the parametric diagram, f vs. V_S of Fig.4. In this diagram, in the upper part, we can clearly observe the period-doubling route to chaos. In Figs.5 and 6, the bifurcation diagrams V_C vs. V_S are shown, for f = 4.5kHz and 5.0kHz respectively.

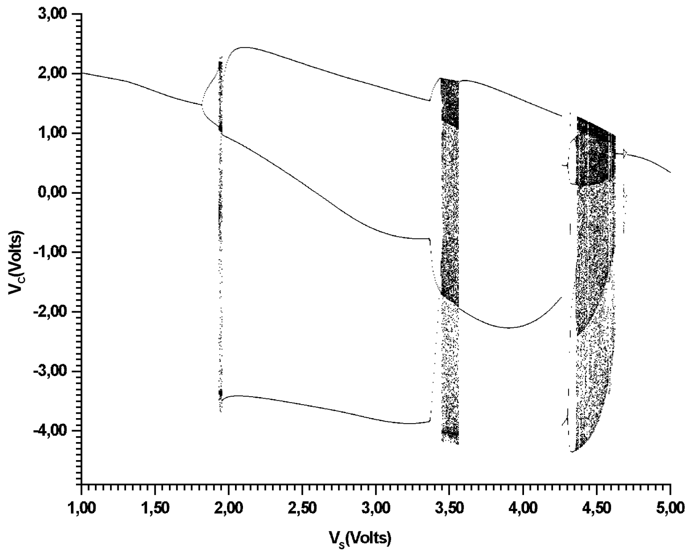


Fig.5 Bifurcation diagram, V_C vs. V_S, for f = 4.5kHz .

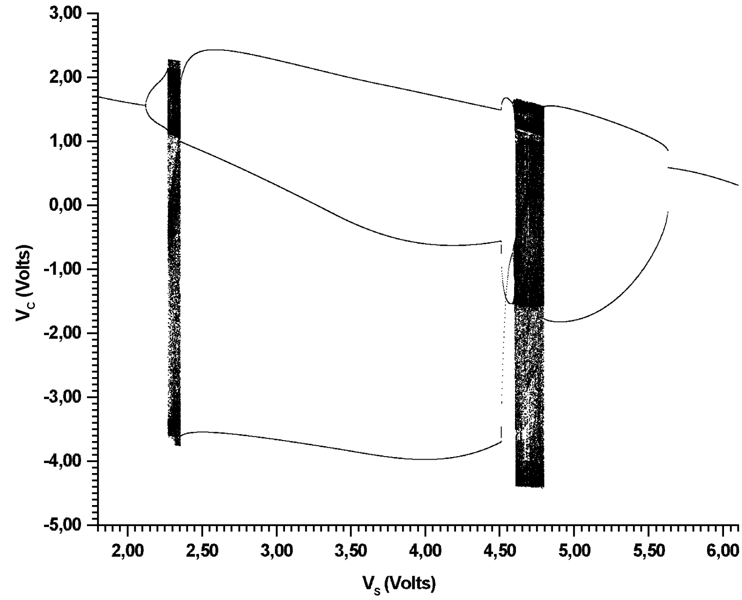


Fig.6 Bifurcation diagram, V_C vs. V_S, for f_S = 5.0kHz

Cascades of period-doubling bifur-cations have long been recognized to be one of the common routes to chaos, as exemplified by the one-dimensional logistic map x_{n+1} = ëx_n(1-x_n). As the parameter ë in the logistic map is increased, it is known that periodic orbits are only created but never destroyed [11]. Unlike the monotone bifurcation behavior of the logistic map, Dawson et al. [12] showed, that in many common nonlinear dynamical systems periodic orbits must be both created and destroyed infinitely often, as a parameter is varied. They named this concurrent creation and annihilation of periodic orbits **antimonotonicity**.

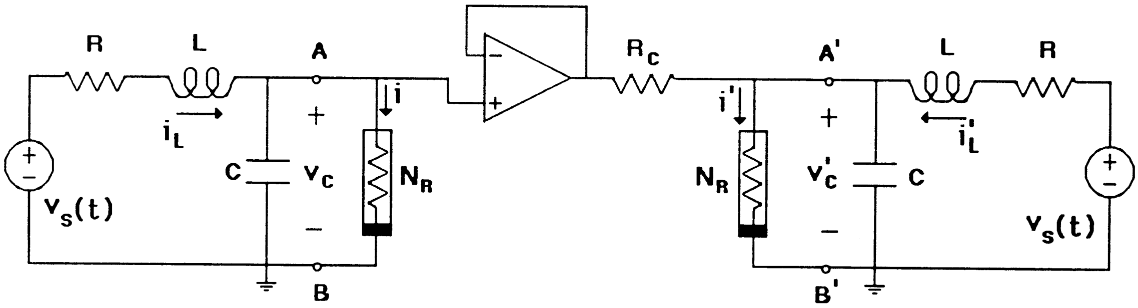
Reversals of period-doubling cascades have been observed in various nonlinear physical systems both numerically and experimentally. In one of the first studies of this phenomenon [13], the occurrence of such reverse sequences was connected to the dynamics of a cubic 1-D map. As examples of numerical simulations, we cite the van der Pol equation [14], Duffing's oscillator [15], bad-cavity laser equation [16], the 1-D Chua map [17], and a RC-ladder chaos generator [18]. Experimental manifestations of antimonotonicity have been observed on the Belousov-Zhabotinsky chemical reaction [19], the driven R, L, p-n junction nonlinear circuit [20-22], a driven third-order nonlinear electical circuit [23], and an autonomous third-order nonlinear electrical circuit with a nonlinear resistor, with an asymmetric v-i characteristic [24]. In our circuit, for 4600 Hz < f_S < 5050 Hz, we can observe period doubling reversals, while for the rest of the frequency values the system ends to a period-1 limit cycle by boundary crisis [25].

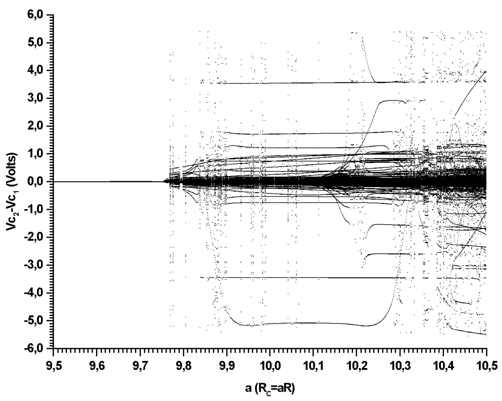
4. Synchronization of Chaos

It has been shown, that it is possible to construct a set of chaotic systems, so that their common signals will have identical or synchronized behavior [26-28]. Generally, there are two methods of chaos synchronization available in the literature. According to the first method due to Pecora and Carroll [26], a stable subsystem of a chaotic system could be synchronized with the separate initial chaotic system under certain suitable conditions. The second method to achieve chaos synchronization among two identical nonlinear systems is due to the effect of one-way coupling [29-31], without requiring to construct any stable subsystem.

By one-way coupling we mean, that the behavior of one (response) system is depended on the behavior of another identical (drive) system, but the second one is not influenced by the behavior of the first. In addition, the response system can have a different set of initial conditions, other than that of the drive system. As time progresses, the two identical chaotic systems can achieve a perfect synchronization among their state variables and maintain it, depending upon the one-way coupling strength.

The schematic circuit realization of the two identical circuits with one-way coupling is shown in Fig.7. The two circuits are coupled by a linear resistor R_C and a buffer. The buffer acts as a signal driving element, which isolates the drive system variables, being affected by the response system variables, thereby providing one-way coupling.





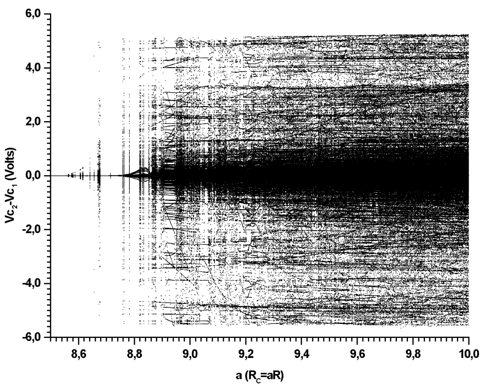


Fig.7 The schematic circuit realization of the two identical circuits of Fig.1 with one-way coupling.

Fig.8. The bifurcation diagram ($v_{C2} - v_{C1}$) vs. a for f= 5.0kHz and V_S = 2.28V.

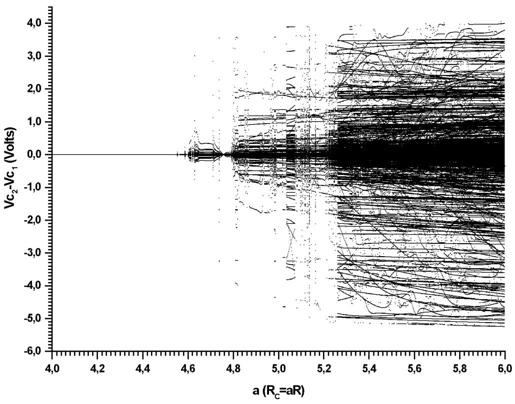


Fig.9. The bifurcation diagram ($v_{C2} - v_{C1}$) vs. a for f= 5.0kHz and V_S = 4.70V.

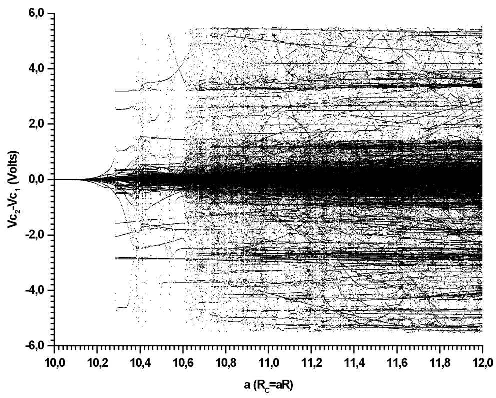
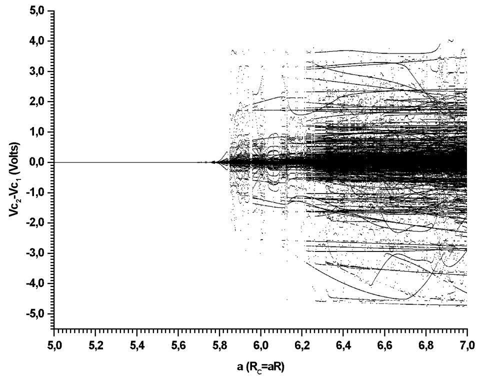


Fig.10. The bifurcation diagram ($v_{C2} - v_{C1}$) vs. a for f= 4.5kHz and V_S = 1.95V.

Fig.11. The bifurcation diagram ($v_{C2} - v_{C1}$) vs. a for f= 4.5kHz and V_S = 3.50V.

Fig.12. The bifurcation diagram ($v_{C2} - v_{C1}$) vs. a for f= 4.5kHz and V_S = 4.40V.

We have studied the chaos synchronization of the coupled circuits for different chaotic regimes, using a = R_C/R as the control parameter. In Figs. 8-12, we have plotted the bifurcation diagrams ($v'_C - v_C$) = ($v_{C2} - v_{C1}$) vs. a for the different chaotic regimes of the bifurcation diagrams of Figs.5 & 6 using the same initial conditions. We observe, that the onset of synchronization is different in each



diagram, meaning that is depended on the amplitude and the frequency of the driven signal. The onset of synchronization is also depended on the initial conditions.

5. Conclusion

In this paper we have studied the chaotic dynamics of a second-order non-linear circuit driven by a sinusoidally voltage source. We observed forward and reverse period-doubling bifurcations, i.e. the phenomenon of antimonotonicity.

We have also studied the synchronization of two identical circuits by one-way coupling in different chaotic regimes. The onset of synchronization is depended on the amplitude and frequency of the driving signal, on the initial conditions, and on the coupling parameter a.

According to Carroll and Pecora [32], periodically forced synchronized chaotic circuits are much more noise-resistant than autonomous synchronized chaotic circuits, so our circuit could be a good candidate for secure communications.

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