Sigma Delta and Sliding Mode Control of Pulse Width Modulation Audio Power Amplifiers

J. FERNANDO SILVA
CAUTL, IST, DEEC, Electrical Machines and Power Electronics (24)
Instituto superior Técnico, Universidade Técnica de Lisboa
Av. Rovisco Pais 1, 1049-001 Lisboa
PORTUGAL

Abstract: - Robust sliding mode controllers for high fidelity PWM audio power amplifiers are designed and compared with Sigma Delta first and second order modulators. The obtained 3^{th} order Bessel polynomial sliding mode controller, presents faster dynamics, no overshoot, better tracking performance, no steady-state error, and flatter passband response. The proposed sliding mode approach provides a new, non linear theoretical frame for solving the control problem of PWM audio power amplifiers.

Key-Words: - Sigma Delta, Sliding mode, PWM audio power amplifiers. *CSCC'99 Proceedings,* Pages:3271-3277

1 Introduction

The increasing miniaturisation of today audio and video home electronics, together with the rising requirements of home video, multimedia sound and home theatre, can only be accomplished by reducing the overall power consumption of the audio apparatus. This enables the use of smaller and lightweighter power supplies, but requires more efficient audio power amplifiers. Furthermore, loudspeakers with lower size, weight and cost together with higher linearity response, are required. However, wide and flat speaker response imply a lower efficiency, hence more power must be delivered by the audio power amplifier. To reduce volume, weight and cost, or to lengthen battery life in portable systems, the audio amplifier efficiency must be increased from the 15-20% to roughly 90%. Class AB linear amplifiers can be astonishing [1], but have efficiencies as low as 15-20% with speech and music signals [2]. Improvements on the supply scheme of AB amplifiers [3], [4], class D amplifiers with active AB filters [5], are steps increasing the efficiency. Still, PWM switching power amplifiers [6], [7], present higher efficiencies, thus enabling the reduction of the power supply cost and compensating the efficiency loss of loudspeakers. Moreover, PWM amplifiers can provide a complete digital solution for audio power processing.

For high fidelity systems, PWM audio amplifiers must present flat passbands of, at least, 16Hz-20kHz (±0.5dB), distortions less than 0.1% at the rated output power, fast dynamic response, and signal to noise ratios above 90dB. This requires fast power semiconductors (usually MOSFET transistors), capable of switching at frequencies near 500kHz and modern fast non linear control methods, such as sliding mode control [8], [9], to synthesise the PWM circuits that can drive the power semiconductors to switch "Just In Time". This switching, as in the JIT management paradigm, provides the precise and timely control actions needed to accomplish the mentioned requirements and to eliminate the non-linearities in the LC filter and in the loudspeakers.

Oppositely, linear power converter control methods based on state space average models, and linear regulators, must change the modulation index slowly, to ensure stability, thus loosing response speed. Moreover, some PWM schemes, being programmed off-line (or generated on-line, using modulators with a limited number of patterns), can not take into account the real time perturbations in the power supply voltages, or the frequency dependent filter and loudspeakers impedance. Consequently, the linear feedback control of the filter output voltage of the PWM audio amplifier becomes slow, difficult and load sensitive.

Replacing the open-loop PWM modulator, by Sigma Delta modulators [10], is the easiest and effective way to deal with the non-linear behaviour of the PWM audio amplifier. Still, the output filter, used to attenuate the high switching frequency, will introduce non-linear effects and delays depending on the input signal frequency. Since the power converter topologies used as PWM amplifiers are variable structure systems, the integral application of sliding mode control theory to the PWM amplifier will bring definite advantages as the system order is reduced [11], and the non-linear effects, together with the frequency dependent delays are cancelled out and power supply rejection ratio is increased. Furthermore, sliding mode control is particularly interesting due to its known characteristics of robustness (to parameter uncertainty, supply perturbations and noise) and appropriateness to the on-off behaviour of power switches.

This paper presents a new paradigm for the control of PWM power amplifiers. Using sliding mode control, and state space models, the variables to be measured and controlled are determined, and the PWM amplifier presents shorter response times, not obtainable with linear controllers.

After the modelling of the PWM audio amplifier in section 2, the paper considers first and second order Sigma Delta controllers in section 3, and the sliding mode controller in section 4, showing test results in both sections. The performances of the controllers are also compared. The results show that the performance of the designed sliding mode controller exhibits no steady-state error, no overshoot, no frequency dependent delay or losses and presents the fastest possible dynamic behaviour.

2 Modelling the PWM audio amplifier

A PWM audio power amplifier, able to provide over 80W to 8 Ω loads (V_{dd}=50V), can be obtained using an half bridge power inverter, coupled to an output filter for high frequency attenuation (fig. 1). A low sensitivity doubly terminated passive ladder (double LC) low pass filter, using 4^{th} order Chebyshev approximation polynomials was selected, given its ability to meet, while minimising the number of inductors, the following requirements: passband edge frequency 21kHz, passband ripple 0.5dB, stopband edge frequency 300kHz and 90dB minimum attenuation in the stopband (L_1 = 79.4μH; L_2 = 84.9μH; C_1 = 1.66μF; C_2 = 822nF; R_2 = 8Ω; r_1 = 0.474 Ω).

Neglecting switch delays, on state semiconductor voltage drops, snubber networks, and supposing small dead times, the switching strategy must avoid power supply internal shorts between the two half bridge switches. Being the switches always in complementary states, their state can be represented by the time dependent variable γ, defined as:

$$\gamma = \begin{cases} 1 \longrightarrow \text{if Q1 is ON and Q2 is OFF} \\ -1 \longrightarrow \text{if Q1 is OFF and Q2 is ON} \end{cases} \tag{1}$$

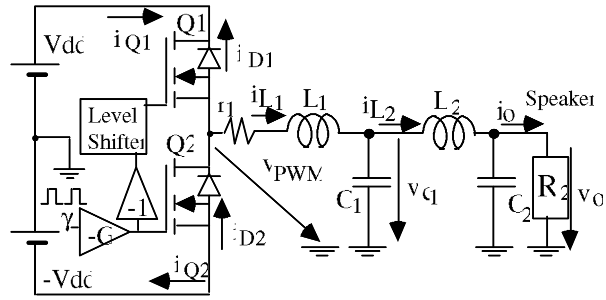


Fig. 1. PWM audio amplifier with 4th order Chebyshev low pass output filter and loudspeaker load.

Therefore, the half bridge output voltage, v_{PWM}, is:

$$v_{PWM} = \gamma\, V_{dd} \tag{2}$$

Considering the state variables and circuit components represented in fig. 1 and dealing with the loudspeaker load as a perturbation represented by the current i_o (this ensures robustness against the frequency dependent impedance of the speaker), the state-space model of the PWM audio amplifier is (3).

$$\begin{bmatrix} \frac{di_{L_1}}{dt} \\ \frac{dv_{C_1}}{dt} \\ \frac{di_{L_2}}{dt} \\ \frac{dv_o}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-r_1}{L_1} & \frac{-1}{L_1} & 0 & 0 \\ \frac{1}{C_1} & 0 & \frac{-1}{C_1} & 0 \\ 0 & \frac{1}{L_2} & 0 & \frac{-1}{L_2} \\ 0 & 0 & \frac{1}{C_2} & 0 \end{bmatrix} \begin{bmatrix} i_{L_1} \\ v_{C_1} \\ i_{L_2} \\ v_o \end{bmatrix} + \begin{bmatrix} \frac{\gamma}{L_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{-1}{C_2} \end{bmatrix} \begin{bmatrix} V_{dd} \\ i_o \end{bmatrix} \tag{3}$$

This model will be used to define the output voltage v_o controllers.

3 Sigma Delta Controlled PWM Audio Amplifier

To control the switching audio amplifier, obtaining the PWM control signal γ, the v_{PWM} voltage (fig. 1) can be assumed as the audio output voltage, since the 4^{th} order Chebyshev low pass filter can be considered an ideal filter removing only the high frequency content of the v_{PWM} voltage.

3.1 First order Sigma Delta Controller

Considering the previous assumption, a first order sigma delta modulator [10] can be used to control the v_{PWM} voltage. Sliding mode control theory can establish this modulator, if the error e_1 between the audio signal input v_i and the sampled output v_{PWM} voltage, αv_{PWM}, is considered:

$$e_1 = v_i - \alpha\, v_{PWM} = v_i - \alpha\, \gamma\, V_{dd} \qquad (4)$$

This is a system with a strong relative degree of zero, as the v_{PWM} voltage is not a state variable. It is, thus, necessary to increase the system order, writing:

$$\frac{d}{dt}\left[\int e_1\, dt\right] = [\, v_i - \alpha\, \gamma\, V_{dd}\,] \qquad (5)$$

According to the theory of sliding mode control [8,9], the sliding surface $S(e_1,t)$, for the new system (5), and the commutation strategy for the time dependent γ, considering a comparator with histerysis ε, are respectively (6) and (7).

$$S(e_1,t) = \beta \int e_1\, dt = \varepsilon \to 0 \qquad (6)$$

$$\gamma(t_k) = \mathrm{SGN}\,\{S(e_1,t_k) + \varepsilon\, \mathrm{SGN}\,[S(e_1,t_{k-1})]\} \qquad (7)$$

The β parameter must be calculated to ensure the switching period $1/f_{PWM}$. To impose the switching frequency f_{PWM}, since $\beta \int_0^{1/f_{PWM}} (\alpha\, V_{dd} - v_{imax})dt = 2\varepsilon$, the parameter β must be:

$$\beta = \frac{2\,\varepsilon\, f_{PWM}}{(\alpha\, V_{dd} - v_{imax})} \qquad (8)$$

Practical implementation of this control strategy can be done using an integrator with gain β (β 1500), and a comparator with hysterisis ε (ε 5mV), fig. 2. Such arrangement is well know as the Sigma Delta modulator [10].

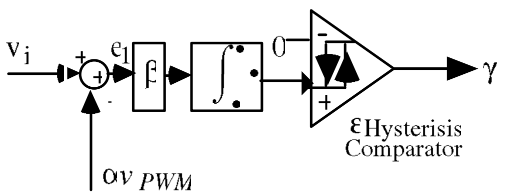


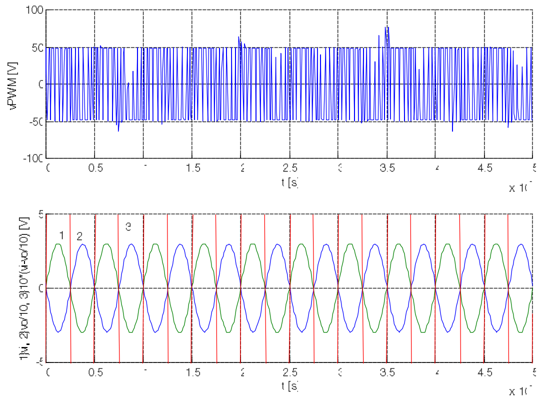
Fig. 2. First order Sigma Delta modulator for the PWM audio amplifier.

As the hysteresis comparator has a very high gain (theoretically infinity), with a high enough switching frequency, $S(e_1,t)$ must be almost zero. Assuming also that v_i is nearly constant over the switching period $1/f_{PWM}$, equation (6) gives:

$$0 \quad \beta\left[\int_0^{1/f_{PWM}} v_i\, dt - \int_0^{1/f_{PWM}} \alpha v_{PWM}\, dt\right] \qquad (9)$$

This means that the Sigma Delta controlled amplifier has a gain of $1/\alpha$, as the mean value of the output voltage $\overline{v_{PWM}}$ in each switching cycle, is:

$$\overline{v_{PWM}} = \frac{v_i}{\alpha} \qquad (10)$$



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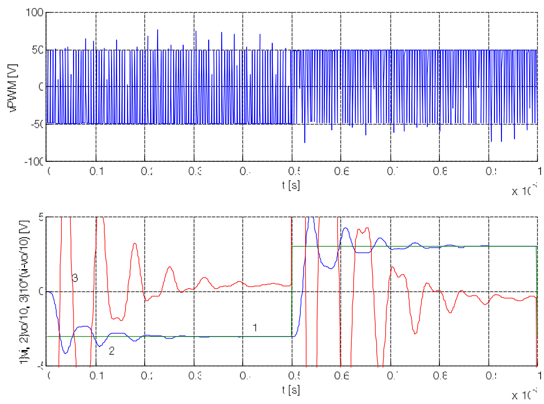

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Fig. 3. First order Sigma Delta audio amplifier performance; a) v_{PWM}, v_i, and $v_o/10$ voltages for a 20kHz sine input, at 55W output power; b) v_{PWM}, v_i, $v_o/10$, and $10^*(v_i - v_o/10)$ voltages for a 1kHz square wave input, at 100W output power.

Figure 3 a) shows the v_{PWM}, v_i, and $v_o/10$ waveforms for a 20kHz sine input. The overall behaviour is the expected, as the practical filter and loudspeaker are not ideal, but notice the 0.5dB loss and phase delay of the speaker voltage v_o, mainly due to the output filter and speaker inductance. In figure 3 b), the v_{PWM}, v_i, $v_o/10$, and the error $10^*(v_i - v_o/10)$ for a 1kHz square input are shown. Note the oscillations and steady state error of the speaker voltage v_o, due to the filter dynamics and double termination.

3.2 Second order Sigma Delta Controller

A second order Sigma Delta modulator is a good compromise between circuit complexity and signal to quantization noise ratio. As the switching frequency of the two power MOSFET (fig. 1) cannot be further increased, the second order structure named "Cascaded integrators with feedback"(fig. 4) was selected, and designed to eliminate the step response overshoot found on fig. 3b).

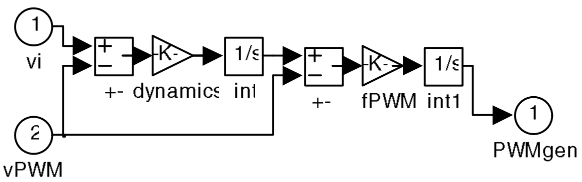
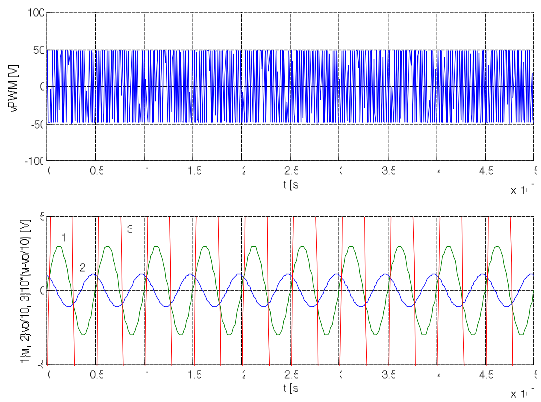


Fig. 4. Second order Sigma Delta modulator.



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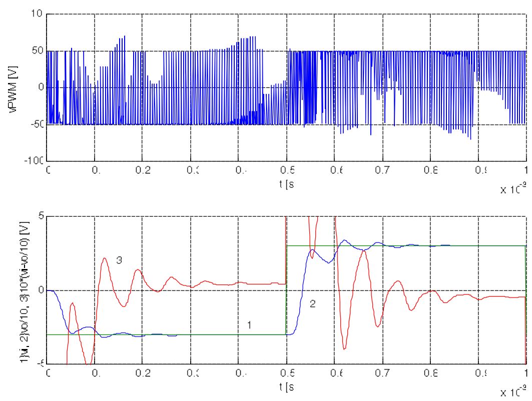

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Fig. 5. Second order Sigma Delta audio amplifier performance; a) v_{PWM}, v_i, and $v_o/10$ voltages for a 20kHz sine input, at 55W output power; b) v_{PWM}, v_i, $v_o/10$, and $10*(v_i - v_o/10)$ voltages for a 1kHz square wave input, at 100W output power.

Figure 5 b) presents the v_{PWM}, v_i, $v_o/10$, and the error $10*(v_i - v_o/10)$ waveforms for a 1kHz square input, showing much less overshoot and oscillations than fig. 3 b). However, the v_{PWM}, v_i, and $v_o/10$ waveforms for a 20kHz sine input presented in fig. 5 a) show increased output voltage loss, compared with the first order Sigma-Delta modulator, since the second order modulator was designed to eliminate the v_o output voltage ringing (therefore reducing the amplifier bandwidth).

Other investigated Sigma Delta second order structures, like the "Cascaded integrators with feedforward", operating near the same switching frequency, present responses comparable with the ones presented in fig. 3. Therefore, to reduce the steady state errors, overshoots and oscillations, the filter model must absolutely be considered in the feedback loop design.

4 Sliding Mode Controlled PWM Audio Amplifier

To design a robust feedback voltage loop for the PWM audio amplifier, independent of semiconductors, power supply, filter and load parameters, the state space model (3) of the amplifier and filter (Fig. 1) will be used together with the application of sliding mode control to the whole system. Since the voltage applied to the loudspeaker is v_o, considering a practical filter, the controlled output must be v_o.

Using the error e_{v_o} of the sampled output variable v_o:

$$e_{v_o} = v_i - \alpha v_o \qquad (411)$$

and the state space model (3), the phase canonical form for the state space model of the PWM amplifier and filter is:

$$\begin{cases} \dfrac{de_{v_o}}{dt} = e_\alpha = \dfrac{dv_i}{dt} - \alpha \dfrac{i_{L2} - i_o}{C_2} \\ \dfrac{de_\alpha}{dt} = e_\beta \\ \dfrac{de_\beta}{dt} = e_\gamma \\ \dfrac{de_\gamma}{dt} = \dfrac{r_1 i_{L1}}{C_1 L_1 C_2 L_2} + \dfrac{v_{C1}}{C_1 L_1 C_2 L_2} + \dfrac{v_{C1}}{C_1 C_2 L_2^2} + \\ + \dfrac{v_{C1}}{C_2^2 L_2^2} - \dfrac{\alpha v_o}{C_1 C_2 L_2^2} + \dfrac{\alpha v_o}{C_2^2 L_2^2} - \\ - \dfrac{1}{C_2^2 L_2} \dfrac{di_o}{dt} - \dfrac{d^4 v_i}{dt^4} - \dfrac{1}{C_2} \dfrac{d^3 i_o}{dt^3} - \dfrac{v_{PWM}}{C_1 L_1 C_2 L_2} \end{cases} \qquad (12)$$

Sliding mode controllers can be designed to reduce the order of this system. To achieve this, the sliding surface $S(E,t)$ (13), where $E=[e_{v_o}, e_\alpha, e_\beta, e_\gamma]^T$, is given by (13) as a linear combination of all the phase canonical state variables E [11].

$$S(E,t) = e_{v_o} + k_\alpha e_\alpha + k_\beta e_\beta + k_\gamma e_\gamma = 0 \qquad (13)$$

Considering (12), the sliding surface (13) can be expressed as a combination of the error voltages and their time derivatives:

$$S(E,t) = v_i - \alpha v_o + k_\alpha \frac{d(v_i - \alpha v_o)}{dt} + k_\beta \frac{d}{dt}\left(\frac{d(v_i - \alpha v_o)}{dt}\right) +$$
$$+ k_\gamma \frac{d}{dt}\left[\frac{d}{dt}\left(\frac{d(v_i - \alpha v_o)}{dt}\right)\right] = 0 \qquad (14)$$

The poles of this expression ensure the required amplifier dynamics. Therefore, they are placed according to a third order Bessel polynomial in order to obtain a stable system, the smallest possible response time t_r, a delay characteristic as flat as possible and almost no overshoot [12]. Hence,

$$S(E,s) = 1 + s + \frac{6}{15} s^2 + \frac{1}{15} s^3 \qquad (15)$$

The response time t_r is taken inversely proportional to a frequency just below the lowest cut-off frequency (ω_1) of the double LC filter. Therefore, t_r 2/ ω_1 1/(ð f_1) 20 µs [12]. Substituting s by st_r, to denormalize the Bessel polynomial, the characteristic polynomial (16), verifying the Routh-Hurwitz criterion is obtained.

$$S(E,s) = 1 + st_r + \frac{6}{15}(st_r)^2 + \frac{1}{15}(st_r)^3 \qquad (16)$$

The existence of the sliding mode implies $S(E,t) = 0$ and $\dot{S}(E,t) = 0$ [8,9]. Given the state space models (3) and (12), and from $\dot{S}(E,t) = 0$, the equivalent average DC input voltage $\overline{v_{PWM}}$, that must be applied at the filter input, in order that the system state slides along the surface (13), imposes the minimum value for the supply voltage V_{dd}. Therefore, the existence of the sliding mode is dependent on the available value of the power supply voltage V_{dd}, which must ensure the $\overline{v_{PWM}}$ average value. However, as the $\overline{v_{PWM}}$ voltage contains the derivatives of the reference voltage, the system will not be able to stay in sliding mode with step inputs.

The stability condition for a system in sliding mode [8], [9], can be written:

$$S(E,t)\,\dot{S}(E,t) < 0 \qquad (17)$$

The fulfilment of this inequality ensures the convergence of the system state trajectories to the sliding surface $S(E,t)$. Again, the power supply voltage V_{dd} must be greather than the maximum required mean value of the output voltage in a switching period V_{dd} $(\overline{v_{PWM}})_{max}$, and the switching law for the control input $\gamma(t_k)$, must be:

$$\gamma(t_k) = \mathrm{SGN}\,\{S(E,t_k) + \varepsilon\,\mathrm{SGN}\,[S(E,t_{k-1})]\} \qquad (18)$$

When $S(E,t)>\varepsilon$, $e_\gamma>0$, and from (2), (13) and (18) $v_{PWM} = +V_{dd}$, which ensures $de_\gamma/dt<0$. This implies $\dot{S}(E,t) < 0$, verifying (17). Therefore, there will be a time instant where $e_\gamma<0$ and, as a consequence $S(E,t)<0$.

This same reasoning can be made for $S(E,t) < -\varepsilon$, to conclude that as long as V_{dd} $(\overline{v_{PWM}})_{max}$ the system is stable, and in the sliding mode.

Almost the same analysis can be made to prove that the system, outside the sliding mode is able to reach it, if V_{dd} is sufficiently high. This means that the power supply voltage values should be chosen high enough to account for the maximum effects of the perturbations. For step inputs, the system looses sliding mode, but it will reach it again, even if V_{dd} is calculated considering only the maximum steady state values for the perturbations.

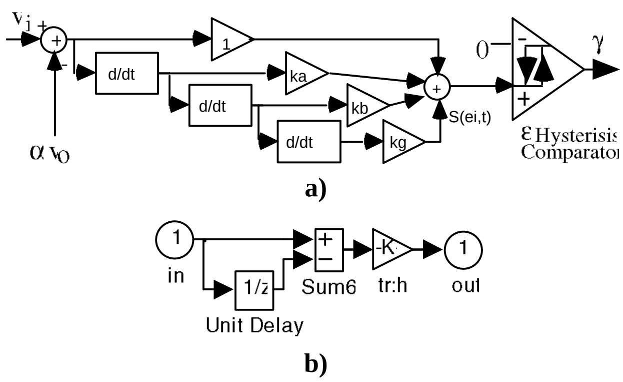
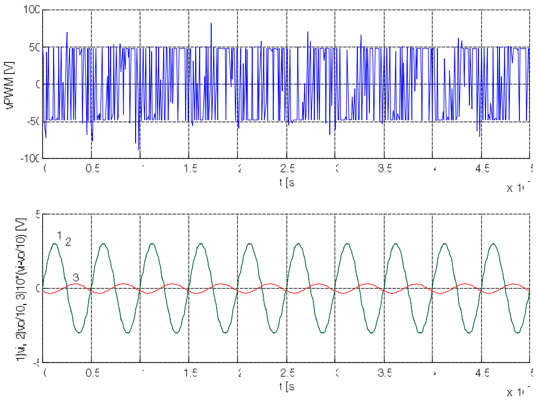


Fig. 6 a) Sliding mode controller for the PWM audio amplifier; b) Practical implementation of the derivative blocks.

Equation (14) and the hysterisis comparator (18) can be implemented with the block diagram of fig. 6 a) (k_a=1, k_b=6/15, k_c=1/15). The derivatives, in practice, can be approximated by the block diagram of fig. 6 b), were h is the over-sampling period.

The outlined approach designs the control and the modulator electronics for the audio power amplifier, using only the sliding mode control framework.



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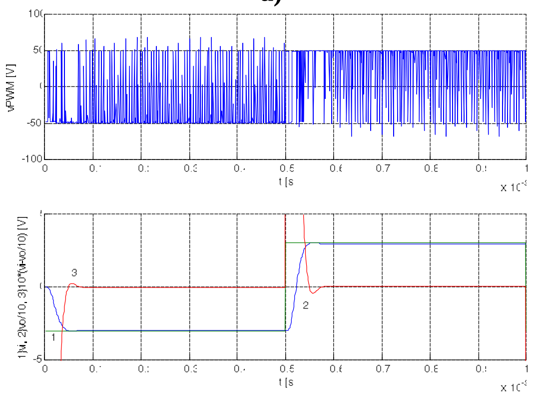

b)

Fig. 7. Sliding mode controlled audio power amplifier performance; a) v_{PWM} , v_i, v_o/10, and 10*(v_i v_o /10) voltages for a 20kHz sine input, at 55W output power; b) v_{PWM} , v_i, v_o/10, and 10*(v_i - v_o /10) voltages for a 1kHz square wave input, at 100W output power.

Figure 7 a) shows the v_{PWM}, v_i, v_o/10, and the error 10*(v_i - v_o /10) waveforms for a 20kHz sine input. The overall behaviour is much better than the obtained with the Sigma Delta controllers (fig. 3 and 5). There is no 0.5dB loss or phase delay over the entire audio band, the Chebyshev filter behaving as a maximally flat filter, with higher stopband attenuation. Figure 7 b) shows almost no steady state error and almost no overshoot on the speaker voltage v_o, and the v_{PWM}, v_i, and 10*(v_i - v_o/10) waveforms for a 1kHz square input. The results obtained attest the speed of response, the stability, the system order reduction, and the usefulness of the sliding mode modulator for the PWM audio amplifier.

5 Conclusion

The sliding mode voltage controller for the PWM audio amplifier, considered as a whole system, was presented and compared with Sigma Delta modulators. These last controllers present steady-state errors and overshoots at the speaker level, due to the output filter. Phase canonical state space models of the whole circuit and Bessel polynomials were used to ensure stability, no overshoot and minimised response time. Sliding mode control concepts were useful for the elimination of the steady state errors, passband losses and delays, for increasing the response speed and to add robustness against load and power supply variations or perturbations. Sliding mode voltage controllers for the PWM audio amplifier do not need to know the speaker model to obtain the desired speaker voltage. The design of regulation and drive electronics, for the power amplifier, is completely integrated using a single theoretical approach, instead of the traditional way of first creating a PWM modulator and then designing a controller for the association modulator/power amplifier and filter.

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