Monitoring the control loop on the basis of a reduced process model

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Abstract: - The control loop usually exhibits some optimal performance in the nominal operating conditions. If these conditions change or a fault occurs in the process, the control system's performance could deteriorate. In this paper, we consider a model-based approach to detecting degradation of the control loop. For this purpose we propose a statistical test on residuals produced by a low-order model.

In general, the discrepancy between the process output and model output can be caused by faults and modelling error. The statistical test on residuals is modified so that it also accounts for the process' unmodelled dynamics. Namely, if the impact of undermodelling on the model output is disregarded, the number of false alarms can significantly increase. It is therefore crucial to have a good estimate of both components of the model error, i.e. the undermodelling and the noise term. In estimating model error, we use the stochastic embedding approach [3], which handles the structural uncertainty of the nominal model by describing it with the FIR model. The coefficients of the FIR model are supposed to be normally distributed with a zero mean and exponentially decaying variance.

Keywords: - inexact model, model-based change detection, test statistics

1 Introduction

Due to changes in the plant, equipment failure or set point change, the control system ceases to work as intended. For example, increasing valve friction results in the hysteresis or stick-slip motion of the valve, resulting in oscillations of the process signals. Due to nonlinearity, a changed operating point results in the change of the process gain and/or time constants. In this case, the model is no longer a reliable description of the process and the model-based controller does not operate as before. If the changes are significant, the model needs to be redesigned and the controller retuned. In industrial applications, many controllers are adjusted and put into operation without further maintenance. It is then left to the operator to observe and make decisions about appropriate actions when an abnormal condition occurs.

The commercial consequences of performance degradation are reflected in increased energy consumption and lower quality final products. The goal of the control system, in the broad sense, is to maintain closed-loop performance at the highest level under all circumstances. This is achieved by monitoring the closed-loop system, diagnosing events that cause performance degradation and, finally, by undertaking the appropriate corrective actions.

In general, performance monitoring methods can be signal-based or model-based [2]. The focus below will be on performance monitoring of the closed-loop on the basis of statistical tests on residuals. The residuals produced by the reduced order model comprise the unmodelled dynamics. Using the traditional test statistics, which is the sum of the squared residuals normalised by the noise variance, could lead to false alarms.

In this work, test statistics on the residuals of the reduced order model is proposed. It makes use of the stochastic embedding description of the unmodelled dynamics [3], which is presented in Section 2. In Section 3, a change detection test based on the inexact ARX model is described, followed by a discussion in Section 4. In Section 5, the proposed change detector is tested on a simulated DC motor-generator. Finally, some concluding remarks are given in Section 6.

2 Process description

We assume that the nominal process behaviour is described by the autoregressive model

$$y(t) = \psi^T(t)\theta + \varphi^T(t)\eta + w(t) \tag{1}$$

where $\psi(t)$ is the regression vector, $\theta \in R^n$ denotes the model parameters corresponding to the current (possibly) faulty data record, y(t) is the process output, w(t) is Gaussian noise $N(0, \sigma_w^2)$ with known variance. The term $\varphi^T(t)\eta$ is FIR model of the unmodelled process dynamics with

$$\varphi^T(t) = [u(t-1), u(t-2), \ldots, u(t-L)] \tag{2}$$

$$\eta = [\eta_1, \eta_2, \ldots, \eta_L]^T \tag{3}$$

while u is input signal and η_i are the coefficients of the finite impulse response model. To avoid estimating L-parameters of the FIR model, the unmodelled dynamics is represented in the statistical framework as a realisation of the random variable. It is assumed that the coefficients of the impulse response associated with the undermodelling is zero mean Gaussian [3]

$$\eta \in N(0, C_\eta(\alpha, \lambda)) \tag{4}$$

with the covariance matrix C_η which is assumed to have an exponentially decaying diagonal structure

$$E[\eta_k \eta_j] = \begin{Bmatrix} \alpha\lambda^k & k = j \\ 0 & k \neq j \end{Bmatrix} \quad 0 < \lambda < 1 \tag{5}$$

This corresponds to the exponentially decaying variance of the undermodelling impulse response [3]

$$E\left[\eta_k{}^2\right] = \alpha\lambda^k \qquad 0 < \lambda < 1 \tag{6}$$

Equation (6) defines the bounds within which the undermodelling impulse response is found with certain probability

The parameters $\sigma_w{}^2$ of the noise and undermodelling α, λ can be estimated from the residuals' vector using the maximum likelihood method.

3 Change-detection test based on the reduced process model

The problem is to detect whether the process parameter vector θ has changed.

To decide whether the process parameters have changed or not, two hypotheses have to be tested (Basseville and Nikiforov, 1993), i.e.:

H_0 *: no change in the process* $\theta = \theta_0$ *versus*

H_1 *: change* $\theta \neq \theta_0$.

In general, the process change effects the mean and variance of the model residuals ε. The change in the time constants only causes change in the residuals' variance. Therefore, any deviation from the nominal values can be revealed by the detection of change in the variance of the model residuals. To detect changes in the control system, the scheme shown in Figure 1 is used.

The nominal model parameters are identified from the available sequence of data, collected during the normal process operation. This can be represented as the mapping

$$(\psi(1), y(1)), \ldots, (\psi(N), y(N)) \rightarrow \theta_0$$

The model residuals are obtained by comparing the model parameterised by θ_0 to the output data in the sliding window of length N, i.e. {y(t), . . . , y(t-N)}

$$\varepsilon(k) = y(k) - \psi^T(k)\theta_0\,., \quad k=t, ..., t-N \tag{7}$$

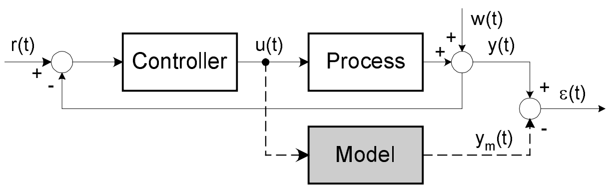

Figure 1: The residuals' generation

The sufficient test statistics for the variance change is derived from the log-likelihood ratio (Rohatgi, 1976) and has the following form

$$\sum_{k=t}^{t-N} \frac{\varepsilon^2(k)}{\sigma^2(k)} \in \chi^2(N). \tag{8}$$

Here, it is supposed that the statistical properties of the measurement noise and the undermodelling do not change with time. $\sigma^2(t)$ is the residuals' variance under hypothesis H_0 (no change in the process). Under the hypothesis of no change $\theta=\theta_0$ the variance of residuals reads

$$\begin{aligned}\sigma^2(k) &= E\left[\left(y(k)-\psi^T(k)\theta_0\right)\left(y(k)-\psi^T(k)\theta_0\right)^T\right] = \\ &= E\left[\left(\varphi^T(k)\eta + w(k)\right)\left(\varphi^T(k)\eta + w(k)\right)^T\right] = \\ &= E\left[\varphi^T(k)\eta\eta^T\varphi(k)\right] + E\left[w^T(k)w(k)\right] =\end{aligned} \tag{9}$$

Since the covariance matrix $E[\eta^T\eta]$ has an exponentially decaying diagonal structure (see 5), the expression (9) becomes

$$\varphi^T(k)E[\eta^T\eta]\varphi^T(k)+\sigma_w^2 =$$

$$= \varphi^T(k)\,\alpha \begin{bmatrix} \lambda & & 0 \\ & \ddots & \\ 0 & & \lambda^L \end{bmatrix} \varphi(k) + \sigma_w^2 =$$

$$= \alpha\lambda\left(u^2(k-1) + \lambda u^2(k-2) + \ldots + \lambda^{L-1}u^2(k-L)\right) + \sigma_w^2 \tag{10}$$

The nominal model parameters θ_0, the noise variance σ_w^2 and the variance of FIR model coefficients $E\left[\eta_i^2\right] = \alpha\lambda^i$, needed in the calculation of the test statistics (8), are estimated from the training data collected during the non-faulty process operation. In the case of "no change" the test statistics (8) is χ^2-distributed with N degrees of freedom.

The hypothesis H_0 is rejected at level α if one of the following conditions holds

$$\sum_{k=t}^{t-N} \frac{\varepsilon^2(k)}{\sigma^2(k)} < c_{\alpha/2} \quad \text{or} \quad \sum_{k=t}^{t-N} \frac{\varepsilon^2(k)}{\sigma^2(k)} > c_{1-\alpha/2} \tag{11}$$

where $c_{1-\alpha/2}$ and $c_{\alpha/2}$ are taken from the χ^2-distribution for the desired degree of confidence α.

4 Discussion

The variance (10) consists of the two summands: the contribution of the process noise σ_w^2 and the undermodelling error (the rest of the expression).

The prediction error variance depends on the input signal at time instants t-1, t-2, ..t-L, because the undermodelling corresponds to the dynamics present in the process but not captured by the model.

The test statistics (8) is mathematically sufficient only for detecting changes in the variance of the residuals. Since a change in the process parameter generally results in a change of the mean and variance of the residuals, it is necessary to test the residuals for variance change as well as for mean value change. Nevertheless, in practice detecting variance changes in the model residuals turns out to be quite an efficient tool for detecting process changes.

5 A simulated example

The performance of the proposed change detector is illustrated on a DC motor-generator model. The transfer function of the simulated process is [6]

$$G_p(q^{-1}) = \frac{0.059q^{-1} - 0.0272q^{-2}}{1 - 1.7558q^{-1} + 0.7923q^{-2}} q^{-2} \tag{12}$$

The measurement noise w(t) is normally distributed with a variance $\sigma_w^2 = 0.01^2$. The PI controller was designed using the pole placement method on the basis of the first order model. The desired characteristic polynomial is $A_C(q^{-1}) = (1 - 0.9608q^{-1})^2$. The nominal model and controller parameters obtained by the iterative identification and controller design [6] are as follows

$$G_m(q^{-1}) = \frac{0.1067q^{-1}}{1 - 0.8930q^{-1}} \tag{13}$$

$$C(q^{-1}) = \frac{0.0308 - 0.000621q^{-1}}{1 - q^{-1}} \tag{14}$$

Figure 2 shows the matching between the process and the model in the Bode diagram.

Figures 3 and 4 show the performance of the closed-loop system in a fault-free and faulty case. The process change is emulated by changing the process poles. The output signal follows the reference before and after the fault occurrence (Figure 4), because the controller compensates for the fault by decreasing the input signal (Figure 3).

The residuals are depicted in Figure 5. For N=1, the term $c_{1-\alpha/2}\sigma^2(t)$ in equation (11) can be regarded as the threshold for the squared residual $\varepsilon^2(t)$. Figure 6 shows the squared residuals and the corresponding threshold $c_{1-\alpha/2}\sigma^2$ for N=1. Since an estimation of the undermodelling was included in the calculation of the residuals' variance, the threshold changes with the input signal. The result is that the number of false alarms is reduced. If undermodelling is disregarded, the threshold for the squared residuals $c_{1-\alpha/2}\sigma_0^2$ depends only on the noise variance (depicted in Figure 6 by the dashed line) and is completely inappropriate. Use of such a threshold would result in false alarms. Figure 7 shows decision-making about hypothesis H_0 and H_1 on the basis of signals in Figure 6. It can be noticed that detecting changes on the basis of one sample of residuals results in diagnostic instability. To be able to track the trend of residuals, we included each time 500 samples (N=500) of residuals in the calculation of the test statistics. The test statistics based on 500 samples strongly increases when the fault is injected into the process (Figure 8). After a short delay, the test statistics exceed the threshold $c_{1-\alpha/2}$ which results in a rejection of hypothesis H_0 (Figure 9).

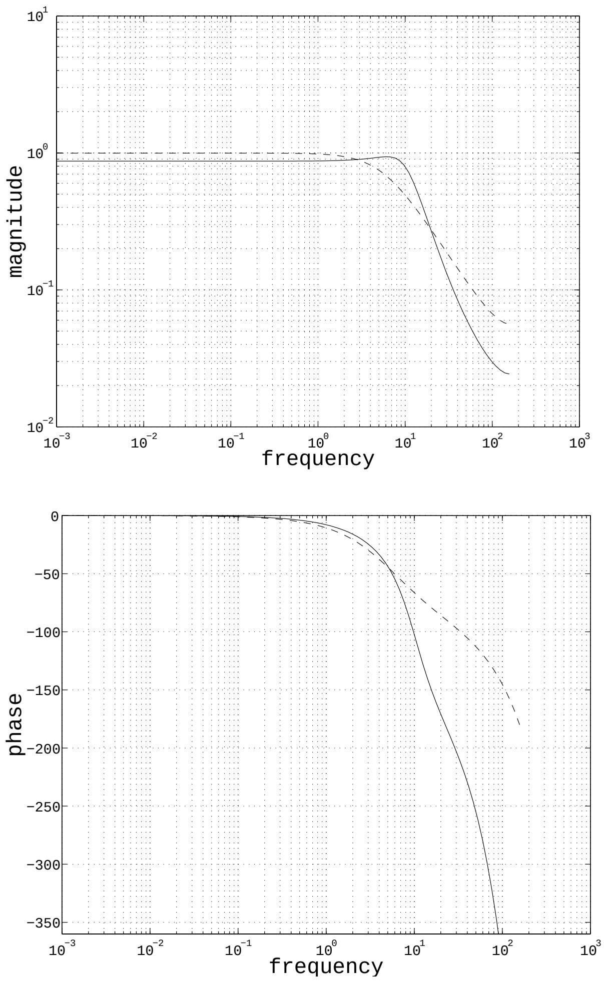

Figure 2: Bode diagram of the process (solid line) and the model (dashed line)

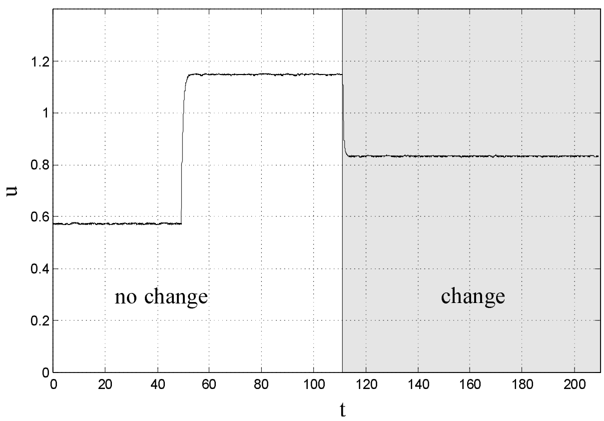

Figure 3: The input signal of the process before and after the parameter change

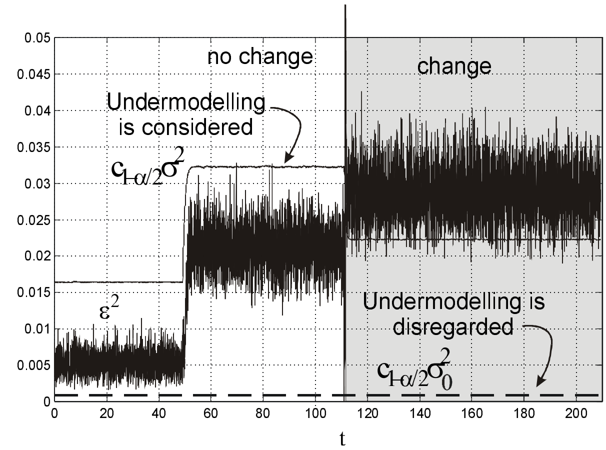

Figure 6: The squared residuals; the threshold in the case where undermodelling is considered (solid line); the threshold in the case where undermodelling is disregarded

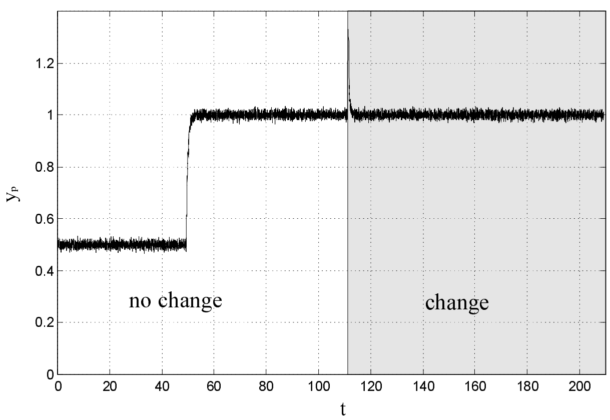

Figure 4: The output signal of the process before and after the parameter change

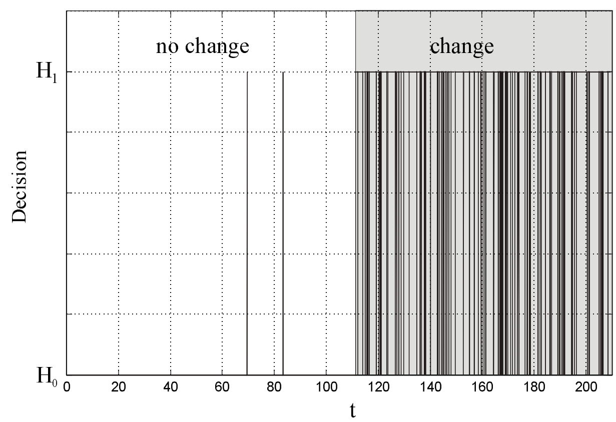

Figure 7: Deciding between H_0 and H_1 for N=1

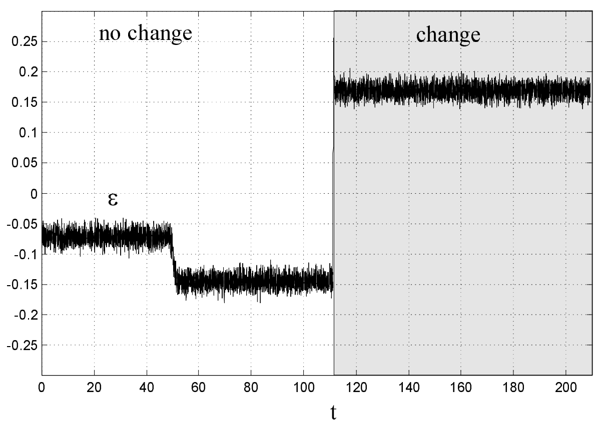

Figure 5: The residuals before and after the parameter change

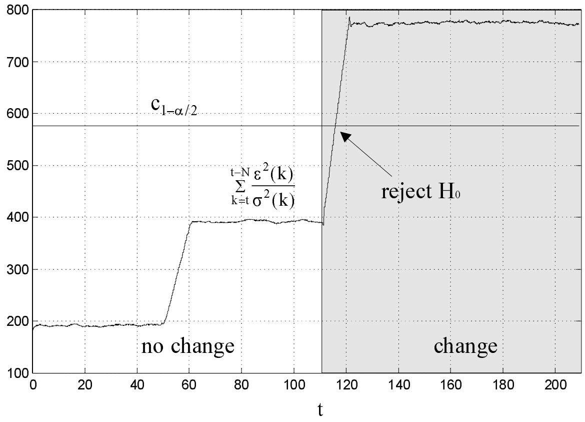

Figure 8: Test statistics S(N) for N=500

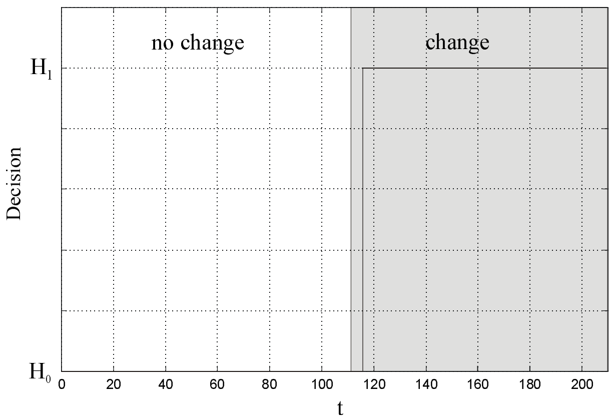

Figure 9: Deciding between H_0 and H_1 for N=1

6 Conclusions

Statistical testing for a change in residuals' variance has been described in significant detail. It is based on checking the test statistics, i.e. the sum of squared residuals normalised by the variance.

The method of testing the variance change was extended to the application of reduced order models by including a more precise estimation of the model error. The unmodelled dynamics is thought of as a realisation of a random process with parameterised second-order statistics (as suggested by Goodwin, 1993). In this way, we gain by reducing the number of false alarms, because the threshold for fault detection which depends on the estimated model uncertainty, is more appropriately chosen. The latter statement has been confirmed by the simulation using a DC motor generator set-up.

The stochastic embedding approach to the estimation of the model error offers a description of the average properties of undermodelling, thereby resulting in a pessimistic estimate of the model error. In the future, we intend to use a more precise description of the undermodelling error in order to increase the sensitivity of the change detector.

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