Robust PI-Controller Tuning for a Fertigation Process

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Abstract: In this paper, a technique for H^{∞}-PI controller tuning, is applied for controlling an uncertain fertigation process, whose purpose is to supply water with nutrients, having a specific conductivity, to a different number or type of irrigation lines. The effectiveness of the proposed technique is demonstrated by several simulation results, which show that, the PI controller designed in the context of H^{∞}, can effectively face large changes in model parameters, and retains a satisfactory performance in cases of load disturbances as well as set point changes.
Keywords: Mixing processes, fertigation, conductivity, H^{∞} control, robust performance, PI-controllers.
CSCC'99 Proceedings: - Pages 4471-4476

1 Introduction

Simple linear plant models are commonly used to design and analyze process control systems. Such models are obtained through linearization and simplification of the highly nonlinear and complex models, describing the true behavior of the process. Therefore, uncertainties naturally arise in the reduced models. In addition to the simplification aspects, model uncertainty may also arise from the behavior of the plant itself, which changes with time.

Fertigation processes which contain a nutrients mixing process, are very common in agricultural applications [1], [2]. In hydroponics systems, in order to supply water with nutrients, having a specific conductivity, a number of irrigation lines are switched in sequence, each line very often supplying water to a different number of plants. Therefore, different flow rates are a common demand in irrigation systems. This is presented to the feedback control loop as a load (nutrients rate) step change. The control loop has a large time delay (usually defined at the design stage by the maximum demand) which vary with time and depends on the hydraulics system parameters. The mixing process can then adequately be described by a first order plus dead time (FOPDT) transfer function model, relating the electrical conductivity of the water (process output) with the nutrients supply (process input). However, this simple model is appropriate for control purposes, only in cases where high precision analog dosing valves are used. In practice, on/off non-corrosive valves are used instead, operated in PWM mode, to execute the control commands of a closed loop control system. PWM has a bad effect on the process output. To avoid this effect, a hydraulic filter for the valve doses is introduced by adding a pre-filtering tank of small capacity, thus producing a continuous flow of nutrients supply, which however raises the order and the dead time of the system [1], [2]. In particular, in this case, the overall mixing process can be described by a second order plus dead time (SOPDT) model, whose delay time is greater than the delay time of the FOPDT description.

From the above analysis, it becomes clear that the facing of model uncertainties is a common task in fertigation process control. The designer must ultimately insure stability and performance of the actual closed-loop system and the designed controller must be robust to the model uncertainty. This reveals that, in order to control the water conductivity in fertigation processes, robust control techniques must be applied. During the last decade several techniques have been developed to deal with model uncertainty and robust control (for an overview, see [3]-[6]). In this paper, a powerful controller synthesis methodology, known as the H^{∞}-control design method, which is based on unstructured uncertainty description of a plant, is applied to the analysis of a control system, composed either of a FOPDT model or a SOPDT model of the water's electrical conductivity change of the fertigation process and a PI controller. Note that, the reasons for studying such a control scheme is mainly due to the wide use of PI controllers in process control [7], as well as to the wide variation of the fertigation models parameters.

2 The Robust Performance Criterion

Performance as well as robustness objectives can effectively be posed in the context of modern

H^∞ theory. Some of the fundamental results of H^∞ theory, which are useful in the sequel, will be summarized.

<u>Robust Stability</u> [4]. Consider the family **Đ** of plants

$$\mathbf{\Pi} = \left\{ \tilde{G}_P : \left| \frac{\tilde{G}_P(j\omega) - G_P(j\omega)}{G_P(j\omega)} \right| \le \bar{\ell}_m(\omega) \right\} \quad (1)$$

Any member of the family **Đ** satisfies

$$\begin{aligned} \tilde{G}_P(j\omega) &= G_P(j\omega)\big(1+\ell_m(j\omega)\big) \\ \left|\ell_m(j\omega)\right| &\le \bar{\ell}_m(\omega) \end{aligned} \quad (2)$$

where G_P is the nominal plant and the last two equations are referred to as the multiplicative uncertainty description with a given bound. Assume that all plants in the family **Đ** have the same number of right half plane poles and that a particular controller G_C stabilizes the nominal plant G_P. Then the system is «*robustly stable*» with the controller G_C if and only if the complementary sensitivity function (for the nominal plant) T(s), defined by

$$T(s) = \frac{G_C(s)G_P(s)}{1+G_C(s)G_P(s)} \quad (3)$$

satisfies the following bound

$$\left\| T\bar{\ell}_m \right\|_\infty < 1 \quad (4)$$

<u>Robust Performance</u> [4]. Assume that all plants in the family **Đ** have the same number of RHP poles. Then the closed loop system will meet the performance specification

$$\left\| \tilde{S}\, W \right\|_\infty < 1, \quad \forall \tilde{G}_P \in \mathbf{\Pi} \quad (5)$$

where the sensitivity function is defined by

$$\tilde{S}(s) = \frac{1}{1+G_C(s)\tilde{G}_P(s)} \quad (6)$$

and W(s) is the performance weight, if and only if the nominal system is closed loop stable and the sensitivity and complementary sensitivity function of the nominal plant satisfy

$$\left|S\, W\right| + \left|T\, \bar{\ell}_m\right| < 1, \quad \forall\omega \quad (7)$$

It can proven easily that the following is an equivalent condition

$$\frac{|T|}{1-|S\, W|} < \bar{\ell}_m^{-1}, \quad \forall\omega \quad (8)$$

There is a variety of design specifications in the frequency domain, which can be viewed as particular cases of the performance specification (5). In particular, in the case where the design requirements are:

(a) Maximum of the sensitivity function over all frequencies, less than M. (9a)

(b) Bandwidth equal to ω^*_{BW} (9b)

(c) Steady-state error less than A. (9c)

then, these requirements can be incorporated to the following weighting transfer function

$$W(s) = \frac{\left(\frac{s}{M}\right) + \omega^*_{BW}}{s + A\omega^*_{BW}} \quad (10)$$

Clearly, when $s \to 0$, then $1/W(s) \to A$, while when $s \to \infty$, then $1/W(s) \to M$. Finally, as $\omega \to \omega^*_{BW}$, then $|W(\omega)| \to 1$. That is, the satisfaction of (5), subject to the weighting function (10), satisfies also the design requirements (9a)-(9c).

3 Unstructured Uncertainty and Dead Time Models

The parametric uncertainty of FOPDT systems can be described by the following model

$$G(s) = \frac{K}{\tau s + 1}\exp(-t_d s) \quad (11a)$$

where

$$K \in \left[K_{min}, K_{max}\right] \;,\; \tau \in \left[\tau_{min}, \tau_{max}\right] \quad (11b)$$

$$t_d \in \left[t_{d,min}, t_{d,max}\right] \quad (11c)$$

or equivalently

$$G(s) = \frac{(K_0 + \delta K)}{(\tau_0 + \delta\tau)s + 1}\exp\left[-(t_{d,0} + \delta t_d)s\right]$$

where

$$K_0 = \frac{K_{min} + K_{max}}{2}, \quad \tau_0 = \frac{\tau_{min} + \tau_{max}}{2}$$

$$t_{d,0} = \frac{t_{d,min} + t_{d,max}}{2}, \; |\delta K| \le |K_{max} - K_0| = \Delta K$$

$$|\delta\tau| \le |\tau_{max} - \tau_0| = \Delta\tau, \quad |\delta t_d| \le |t_{d,max} - t_{d,0}| = \Delta t_d$$

From the above parametric uncertainty, the respective multiplicative uncertainty can be deduced. This multiplicative uncertainty can be described by the family Đ of transfer functions, of the form (1), where [3]

$$G_P(s) = \frac{K_0}{\tau_0 s + 1}\exp\left[-t_{d,0}s\right]$$

$$\tilde{G}_P(s) = G_P(s)\big(1+\ell_m(s)\big)$$

$$\ell_m(s) = \left(\frac{K_0 + \Delta k}{K_0}\right)\left(\frac{\tau_0 s + 1}{(\tau_0 + \Delta\tau)s + 1}\right)\exp(-\Delta t_d s) - 1$$

It has been proven in [3] that an upper bound for the multiplicative uncertainty of the above FOPDT model can be expressed according to the following relations

$$\bar{\ell}_m(\omega) = \left| \left(\frac{|K_0| + \Delta K}{|K_0|}\right)\left(\frac{\tau_0 s + 1}{(\tau_0 - \Delta\tau)s + 1}\right)\exp(\Delta t_d s) - 1 \right|$$

$\forall\omega < \omega^*$, and

$$\bar{\ell}_m(\omega) = \left| \left(\frac{|K_0| + \Delta K}{|K_0|}\right)\left(\frac{\tau_0 s + 1}{(\tau_0 - \Delta\tau)s + 1}\right) + 1 \right|$$

$\forall\omega \geq \omega^*$, where the frequency ù* is computed as the solution of the equation

$$\Delta t_d \omega^* + \arctan\left[\frac{\Delta\tau\omega^*}{1 + \tau(\tau - \Delta\tau)\omega^{*2}}\right] = \pi$$

$$\frac{\pi}{2} \leq \Delta t_d \omega^* \leq \pi$$

For controller synthesis, the following approximation can effectively be used

$$\bar{\ell}_m(\omega) = \left| \left(\frac{|K_0| + \Delta K}{|K_0|}\right)\left(\frac{\tau_0 s + 1}{(\tau_0 - \Delta\tau)s + 1}\right)\left(\frac{Ts + 1}{-Ts + 1}\right) - 1 \right| \quad (12)$$

where

$$T = \frac{t_{d,max} - t_{d,min}}{4}$$

It is worth noticing that, this approximation can be obtained by using the well-known first order Pade approximant of the term including the dead time, which has the form

$$\exp(\Delta t_d s) = \frac{2 + \Delta t_d s}{2 - \Delta t_d s}$$

Following a similar analysis, in the case of a SOPDT model with parametric uncertainty of the form

$$G(s) = \frac{K}{(\alpha s + 1)(\tau s + 1)} \exp(-t_d s) \quad (13)$$

with a fixed parameter á, and with Ê, ô and t_d as in (11b), (11c), we can easily obtain the following unstructured uncertainty description

$$G_P(s) = \frac{K_0}{(\alpha s + 1)(\tau_0 s + 1)} \exp[-t_{d,0} s]$$

$$\tilde{G}_P(s) = G_P(s)(1 + \ell_m(s))$$

where, $\ell_m(s)$ is the same as in the case of the FOPDT model. In this case, an upper bound for $\ell_m(s)$ is given by (12).

4 Hydroponics System Model

Fertigation processes, whose purpose is to supply water with nutrients, having a specific conductivity, to a different number or type of plants, contain a mixing process, which is schematically depicted in Figure 1. The flow of a liquid at the rate of Q_W into a mixing tank of volume V_T, in which mixing of fertilizers takes place, results in a residence time of $T_r = V_T/Q_W$. Mixing of liquids may appear in two distinct modes; plug flow and perfectly stirred flow. Plug flow produces a zero order system with transportation delay (dead time) equal to the residence time T_r. Perfectly mixed flow produces a first order system of time constant equal to the residence time T_r. In real systems, more or less, the two modes coexist and a mixing tank may be modeled as a first order system with dead time. In such cases, the residence time is apportioned to the two characteristic times, depending on the amount of stirring. In addition, dead times are introduced in real systems by transportation delays in pipe flow. On the basis of the above analysis, a first order plus dead time (FOPDT) model of a single tank mixing process has the form:

$$\frac{C_E(s)}{Q_{NS}(s)} = G_{p,1}(s) = G^*_{p,1}(s)\exp(-t_{d,1}s)$$

$$= \frac{K_1}{T_T s + 1}\exp(-t_{d,1}s) \quad (14)$$

where, $C_E(s)$ is the electrical conductivity of the water corresponding to a particular concentration of nutrients and considered as the process output and $Q_{NS}(s)$ is the nutrient supply, considered as the process input. In (1), K is the process gain defined as $K_1=1/Q_W$, T_T is the process time constant defined as T_T =ãT_r and $t_{d,1}$ is the dead time which, as already mentioned, stems from hydraulics system factors, i.e. transportation delay in pipe flow and imperfect mixing, and has the form t_d=(1-ã)T_r, and ã=0, for plug flow and ã=1, for perfect mixing.

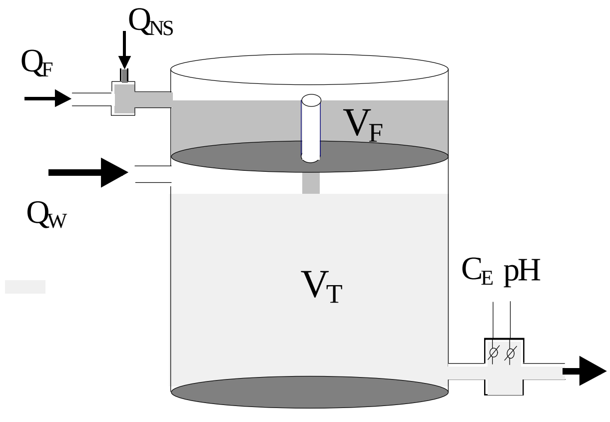

Figure 1. Dual tank mixing process.

On-line mixing of stack solutions with water to prepare a solution of a specific concentration requires high precision analog dosing pumps or valves and a very fast, but robust control system. To avoid associated high cost, on/off non-corrosive valves are used instead, operated in PWM mode, to execute the control commands of a closed loop control system. This requires a mixing tank, for the solution to present an average fertilizer concentration, C_E, at the exit. The tank volume must be big enough to dilute the injected fertilizer doses, without experiencing much variation in concentration. However, big tanks do cause problems when the desired C_E must undergo a step change for a different irrigation line, and also produces a very undesired inflexible device. For example, a minimum cycle of T_C=5s for a dosing valve must be a constraint, to avoid wear and failure. Assume the stack solution has a concentration $100C_E$, and flow rate Q_{NS}, and each injection must be less than 5% of the tank fertilizer to avoid high C_E excursions. The constraint at low Q_W loads is:

Constrain1: $100C_E*Q_{NS}*5s \leq 0.05*C_E*V_T$

At high Q_W loads or low concentrations C_E, the limitation of 5% variation must hold true for T_C =5s valve pause. Therefore an alternate constraint is:

Constrain2: $C_E*Q_w*5s \leq 0.05*C_E*V_T$

To match the two constraints, the hardware design should work around solutions for $Q_{NS} \approx 0.01Q_W$ and $V_T \geq Q_W*100s$. For example, if Q_W =1L/s or Q_{NS} =0.01L/s the tank must have a capacity of 100L. This is the order of the size used by most commercial systems, but is considered an undesirable feature. One way to reduce the size of the tank is to introduce a hydraulic filter for the valve doses by adding a pre-filtering tank of capacity V_F, thus producing a continuous flow of NS, which however raises the order and the dead time of the system. In the case where a pre-filtering tank is used, the overall mixing process can be adequately described by a second order plus dead time (SOPDT) model of the form

$$\frac{C_E(s)}{Q_{NS}(s)} = G_{p,2}(s) = G^*_{p,2}(s)\exp(-t_{d,2}s) = \frac{K_2}{(T_F s+1)(T_T s+1)}\exp(-t_{d,2}s) \qquad (15)$$

where, $K_2=1/(Q_F+Q_W)$, $T_F=V_F/Q_F$ and $t_{d,2}$ is the new dead time (assume $Q_{NS}<<Q_F<Q_W$).

5 H^∞-PI Controller Tuning for the Fertigation Process

Controller tuning in the context of H^∞ for the FOPDT or the SOPDT models of the electrical conductivity variations of the fertigation process will be based on the solution of the problem (7), with a multiplicative uncertainty of the form (12). The particular controller considered here, has the form

$$C(s) = \frac{p_1(1+p_2 s)}{p_2 s} \qquad (16)$$

This form, obviously represent a PI-controller, with proportional gain p_1 and integral time constant p_1/p_2.

The approach that is used for the tuning of the parameters of the H^∞-PI controller consists in the following steps:

- Determination of the parametric uncertainty of the water's electrical conductivity model, for each particular model used (FOPDT or SOPDT).
- Selection, according to the results of Section 3, of an upper bound $\bar{\ell}_m(\omega)$, for the multiplicative uncertainty $\ell_m(s)$ of the water's electrical conductivity model.
- Selection of the performance weighting function W(s), of the form (10).
- Numerical solution of the robust performance problem of the form

$$\sup_{\omega}\left(\left|\tilde{S}(j\omega, p_1, p_2)W(j\omega)\right| + \left|\bar{\ell}_m(\omega)T(j\omega, p_1, p_2)\right|\right) = 1 \qquad (17)$$

It is worth noticing that, for every value of any of the parameters p_1, p_2, it is possible to find a value of the remaining parameter, for which the above equality is satisfied. Therefore, the problem has an infinite number of solutions. A way to simplify its solution is to set $p_2 = \tau_0$, and solve the problem at hand, with respect to p_1. This simplification will be used in the sequel.

In what follows, the technique presented above for H^∞-PI controller tuning will be applied to a particular fertigation process model. To this end, we consider a FOPDT model of the mixing process with parameters V_T=44L, $Q_W \in [1L/s,\ 8L/s]$, $t_{d,1} \in [7s,\ 9s]$, and a SOPDT model with V_F=4L, Q_F=0.5L/s, V_T=40L, $Q_W \in [1L/s,\ 8L/s]$, $t_{d,2} \in [8s,\ 10s]$. It is worth noticing at this point that in all simulations that follow, conductivity of the water is considered as a deviation variable.

In the case of the FOPDT model (Case I), the parametric uncertainty results to a multiplicative uncertainty of the form

$$\ell_m(s) = 1.7778 \times \left(\frac{24.75s+1}{5.5s+1}\right)\left(\frac{0.5s+1}{-0.5s+1}\right) - 1 \qquad (18)$$

Our design requirements for the closed-loop system, consisting of the uncertain process model and the PI-controller, are:

(a) Maximum of the sensitivity function over all frequencies, less than M=3.
(b) Closed-loop system bandwidth equal to $\omega^*_{BW} = 0.002$ rad/sec
(c) Zero steady-state error (A=0).

Therefore, according to (10) the performance weighting function W(s) has the form

$$W(s) = \frac{s + 0.006}{3s}$$

Solution of the problem (17), for $p_2 = \tau_0 = 24.75$ yields p_1=0.6229. The graphical representation of its solution is depicted in Figure 2.

Simulation results, regarding the application of the above designed H^{∞}-PI controller, to the uncertain FOPDT model of the fertigation process are given in Figures 3-5. In these Figures, the output of the controller u(t), the response of the closed-loop system to input changes of the set point and the response of the closed-loop system to step load disturbances, are depicted, when system parameters take several values in the ranges mentioned above. In particular, in Figures 3-5, continuous, dotted and dashed lines are used to depict characteristics of the closed-loop system for the cases where Q_w=1L/s and t_d=9 s, Q_w=4L/s and t_d=8 s, Q_w=6L/s and t_d=7 s, respectively.

In the case of a SOPDT model (Case II), with the same design requirements and with the multiplicative uncertainty of the form (18), the solution of the problem (17), yields p_1=1.1 and p_1/p_2=0.0244. The graphical representation of its solution is depicted in Figure 6. Simulation results for this case are given in Figures 7 and 8, wherein the controller output, as well as the output of the closed-loop system, are depicted for several values of the parameters of the system. It is worth noticing that, in order to obtain the above simulation results, the limits of the control actuator are 0 (off valve) and 10 (fully on valve), while the PWM used has a period 5 sec.

From the above simulation results, it can be easily seen that the designed H^{∞}-PI controllers have a quite satisfactory performance for a wide uncertainty of the fertigation process model.

6 Conclusions

In the present paper, the H^{∞} control design method, has been applied to the analysis of a control system composed either of a FOPDT model or a SOPDT model of the water's electrical conductivity change of a fertigation process and a PI controller. As it can be seen from the simulation results and in particular from Figures 3-5, 7 and 8, the PI controller designed in the context of H^{∞} can effectively face large changes in model parameters, and retains a satisfactory performance in cases of load disturbances as well as set point changes.

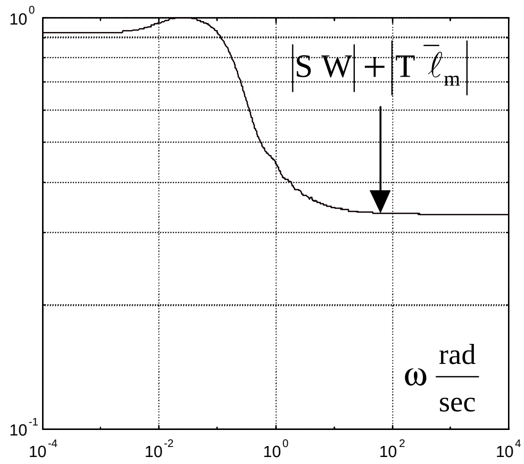

Figure 2. Solution of problem (17) in Case I.

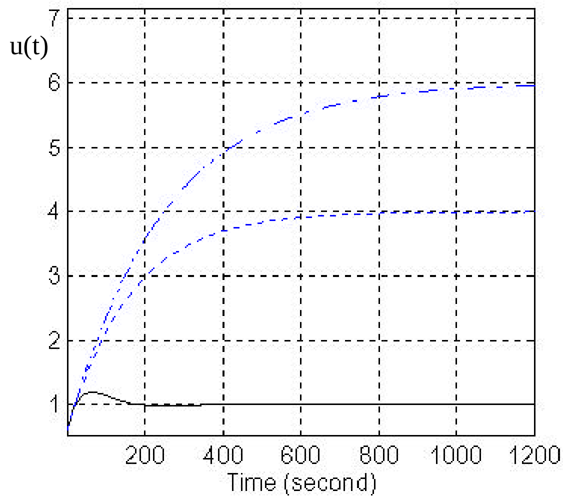

Figure 3. Controller output u(t), in Case I, for several cases of the system parameters.

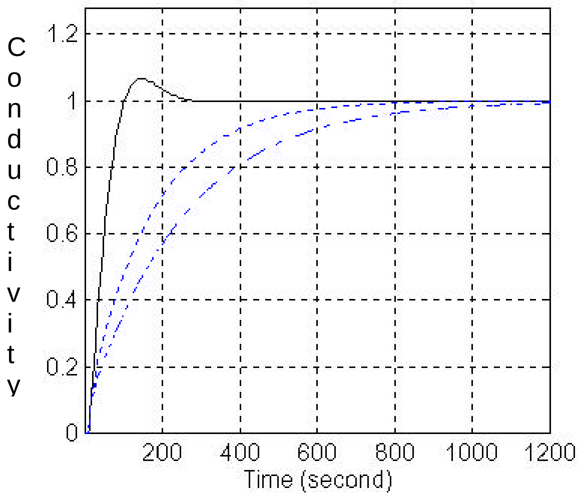

Figure 4. Closed-loop system response to a set point step change, in Case I, for several cases of the system parameters.

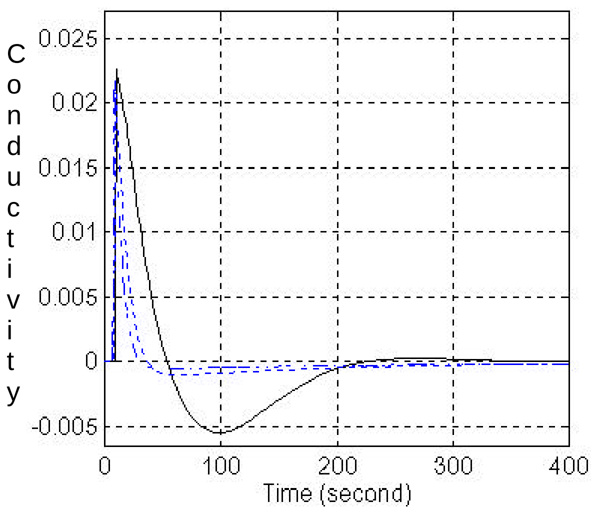

Figure 5. Closed-loop system response to step load disturbances, in Case I, for several cases of system parameters.

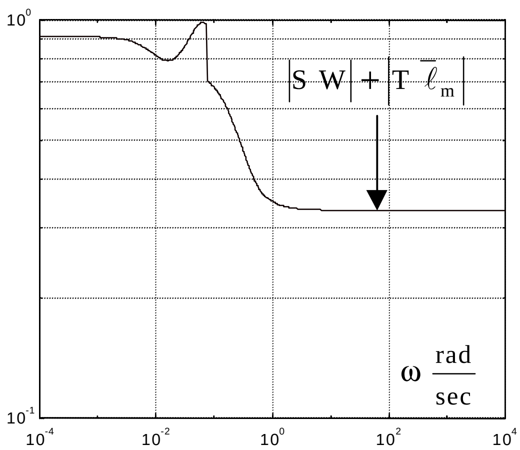

Figure 6. Solution of problem (17) in Case II.

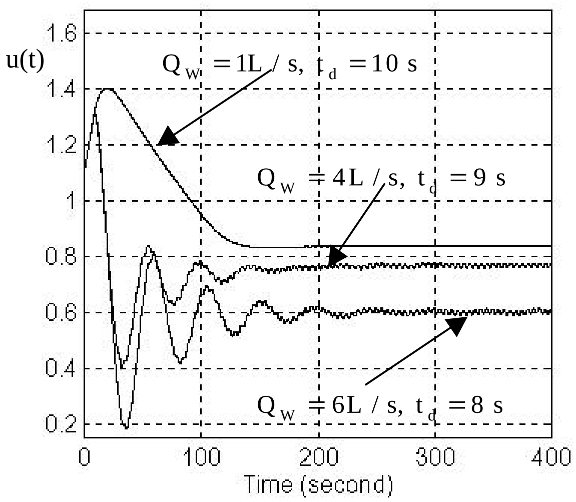

Figure 7. Controller output u(t), in Case II, for several cases of the system parameters.

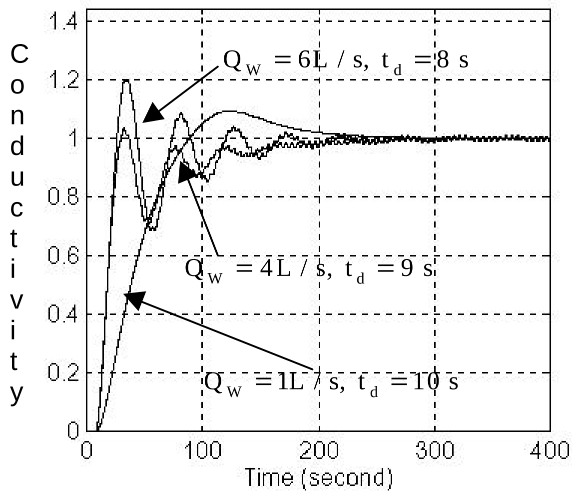

Figure 8. Closed-loop system response to a set point step change, in Case II, for several cases of the system parameters.

The present technique provides a systematic tool for tuning of low level controllers, which is in common use in Agriculture, and which can be seen as part of hybrid systems that combine conventional applications with knowledge based systems. Such hybrid systems are currently a strong demand in agricultural process control.

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