Comparing the Inference Capabilities of Three Fuzzy Cognitive Map Systems

ATHANASIOS K. TSADIRAS   KONSTANTINOS MARGARITIS
Department of Applied Infomatics
University of Macedonia
Egnatias 156, P.O. Box 1591
GR-56004 Thessaloniki, GREECE

Abstract: - In this paper the inference capabilities of three Fuzzy Cognitive Map (FCM) systems are compared. The FCMs are a combination of fuzzy logic and artificial neural networks that have been introduced by Kosko [1], [2] and are mainly used for predictions. The conclusions that are drawn from them come from the study of the dynamical behaviour of the systems. The three FCMs systems that are compared here are : a) the trivalent FCM, b) the sigmoid FCM and c) the Certainty Neuron FCM (CNFCM). The comparison is based on the results that came from computer simulations. Two sets of simulations were run and from the results the following conclusions were drawn:

a) Trivalent FCM, Sigmoid FCM and CNFCM qualitatively draw almost the same conclusions.
b) Sigmoid FCM exhibits sudden changes at the activation of the concepts because it does not possess memory capabilities. On the contrary CNFCM possesses memory capabilities and this is the reason it evolves more smoothly and in a continues manner, bringing it close to reality.
c) Sigmoid FCMs have problems with the linguistic background of their transfer function. Although sigmoid function has background on the physiological level of the human brain, there is not similarity with the linguistic lever of human mind. On the contrary, CNFCMs use a transfer function with strong linguistic background since it is used by many systems for the aggregation of qualitative values.
d) Sigmoid FCM tends to find equilibrium at levels close to -1 and 1 which are the saturation levels of the sigmoid function, instead of using the whole interval [-1,1]. CNFCM on the other hand, finds equilibrium at levels from the whole interval [-1,1].
e) CNFCM is much more reliable in the predictions it makes, since it uses a much better transfer function with strong linguistic background. On the other hand the degrees of increase or decrease predicted by a Sigmoid FCM are not reliable due its weak linguistic background and its tension to find equilibrium close to -1 and 1.

From the above, the advantages of using CNFCM are apparent. We conclude that CNFCM has better structure and should be preferred instead of using sigmoid FCM or trivalent FCM.

Key-Words: - Fuzzy Cognitive Maps, Fuzzy Systems, Computer Simulations, Predictions, Neural Networks, Decision Making

1 Introduction

Fuzzy Cognitive Maps (FCMs) are a combination of fuzzy logic and artificial neural networks that have been introduced by Kosko [1], [2] based on Axelord's work on Cognitive Maps [3]. Further developments of FCMs have been discussed by several researchers (see for example [4]-[9]). An example of an FCM, concerning the freeway congestion of a city at rush hours, is given in Figure 1. FCMs are used to create models as collections of concepts and the various causal relations that exist between these concepts. The concepts are represented by nodes and the causal relationships by directed arcs between the nodes. Each arc is accompanied by a weight that defines the type of causal relation between the two nodes. Positive (negative) causal relation between two concepts C_i and C_j means that an increase of the activation level of concept C_i will increase (decrease) C_j and also a decrease of concept C_i will decrease (increase) C_j.

Each concept C_i is accompanied by a number A_i

that represents its level of activation. If n is the number of concepts of an FCM, at time step t the vector $\mathbf{A^t} = [A_1^t, A_2^t, ..., A_n^t]$ gives the state of the FCM, where A_i^t is the activation level of concept C_i at time step t. $\mathbf{W}$ is also defined as an n×n matrix where w_{ij} is the weight of the arc that connects C_i and C_j (it is taken that w_{ii}=0, i=1,...,n because no loop from a concept to itself is allowed). The activation level of all concepts is updated simultaneously (synchronous updating). This means that $\mathbf{A^{t+1}} = [A_1^{t+1}, A_2^{t+1}, \ldots, A_n^{t+1}]$ where A_i^{t+1}, i=1,...,n is calculated by the following formula

$$A_i^{t+1} = f(\sum_{j=1}^{n} A_j^t w_{ji}) \qquad (1)$$

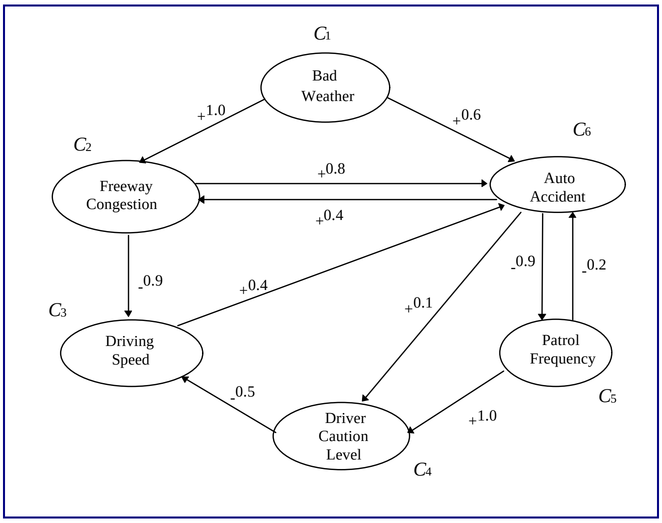

Figure 1: An FCM for the behaviour of freeway congestion at rush hour [10]

The non-linear function f() allows the activation to take a value among the distinct values that are allowed. Using matrix notation, eq. (1) can be written as $\mathbf{A^{t+1}} = f(\mathbf{A^t W})$.

2 The three FCM systems

The dynamical behaviour of the FCM depends heavily on the choice of the transfer function f() that the neurons of the FCM will use. The three most common transfer functions are:

i) The trivalent transfer function

$$f_T(x) = \begin{cases} 1 & \text{if x > 0} \\ 0 & \text{if x = 0} \\ -1 & \text{if x < 0} \end{cases} \qquad (2)$$

Using this transfer function, the trivalent FCM is created.

ii) The sigmoid transfer function

$$f_S(x) = \tanh(x) \qquad (3)$$

having saturation levels at -1 and 1. Using this transfer function, the sigmoid FCM is created.

iii) The transfer function used by Certainty Neurons

$$f_{CN}(x, y) = f_M(x, y) - dx \qquad (4)$$

where d is a decay factor and f_M is given by the following formula:

$$f_M(x,y) = \begin{cases} x + y(1-x) & \text{if } x, y > 0 \\ x + y(1+x) & \text{if } x, y < 0 \\ (x+y)/(1-\min(|x|,|y|)) & \text{else} \end{cases} \qquad |x|,|y| \le 1$$

Function f_M is the function used by MYCIN Expert System for the aggregation of the certainty factors [11]. The use of this transfer function has been proposed and studied in [12] and leads to the creation of the Certainty Neuron FCM (CNFCM).

In FCM systems, the conclusions are drawn by the study of their dynamical behaviour. The dynamical behaviour of each of the three FCM systems described above is different. The simplest is that of the trivalent FCM because the systems can move only at the edges, corners and centers of the sides of the hypercube $[-1,1]^n$ that an n-concept trivalent FCM creates. The system has only 3^n different states. On the other hand, sigmoid FCM can move in the whole space $[-1,1]^n$ having an infinite number of states. It has been also shown that sigmoid FCMs can also exhibits chaotic behaviour [13]. CNFCM systems move also in the whole space $[-1,1]^n$ and have an infinite number of states.

3 Comparing the Inference Capabilities using one Weight Matrix

To compare the inference capabilities of the three FCM systems, the differences in their dynamical behaviour should be compared. To achieve that a number of simulations of the FCMs systems have been made. The FCM model that was used is that of figure 1, concerning the freeway congestion at rush hour.

Different simulations have been run for each of the three FCM systems. In all of them, the "steady values" technique was applied, keeping the value of the concept "Bad Weather" steady. In this way, the consequences of the "bad weather" to the other concepts of the model were predicted, according to the three systems.

The simulation program works in the following way: At the beginning, 1000 random initial states of the FCM system are created. For each of these states, the FCM evolves using the transfer function of the corresponding FCM system. After some initial steps of the evolution that correspond to the transition stage, the system reaches an equilibrium that is recorded. After that the next initial state is introduce to the system and the system evolves once again. The simulation stops only after all 1000 initial states are introduced to the system.

The above simulation program was used for the simulation of the trivalent FCM, the sigmoid FCM and the CNFCM. Only the results from five initial states are presented in Appendix A but the conclusions are drawn from the study of all 1000 random initial states.

In the four of the five cases presented in Appendix A, the system reached an equilibrium point. This actually applies to 822 of the 1000 random initial states. Moreover, when in the trivalent FCM a concept is positive which means that its increase is predicted, the concept is also positive in the sigmoid FCM and the CNFCM. Similarly, when a concept in the trivalent FCM is negative that means that its decrease is predicted, it is also negative in the sigmoid FCM and the CNFCM. We conclude that the inferred conclusions are qualitatively the same.

Sigmoid FCMs and CNFCMs can also predict the size of the increase or decrease of the concepts. The simulations show that the degree of increase/decrease that is predicted by the two types of FCM is different. It is also shown that sigmoid FCM tend to find equilibrium at activation levels close to -1 or 1which are the saturation levels of the sigmoid function. This fact, together with the fact that there is no linguistic background to the use of the sigmoid function for the aggregation of influences, lead us to the conclusion that the sigmoid FCM can not efficiently calculate the degree of the increase or decrease of an FCM concept. On the other hand, we should notice that if the concepts of the FCM are sorted based on their activation level at the equilibrium point, the order of the concepts is the same for sigmoid FCM and CNFCM.

Checking the last 122 cases where no equilibrium point was reached for sigmoid FCM, we found out that the sigmoid FCMs reach a limit cycle behaviour, whereas trivalent FCMs and CNFCMs reach an equilibrium point. Studying these cases we conclude that this happens because the activation level for the concept "Bad Weather" is kept steady at quite low levels. In this way this concept was not strong enough to lead the other concepts to a steady activation but led the systems to a limit cycle behaviour among two opposite states. In the same cases, CNFCM reached an equilibrium point. This was made after a long transition stage as shown in figure 2. CNFCM can still make inference. This better behaviour of CNFCM is due to the memory capabilities that it possesses, remembering its previous activation level.

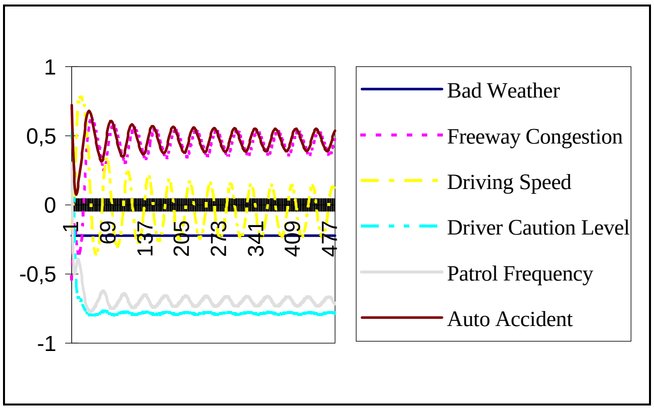

Figure 2 : Long transition stage to equilibrium of an CNFCM

4 Comparing the Inference Capabilities using many Weight Matrices

The conclusion drawn by the above simulations should be check for other weight matrices. To draw conclusions independent from the specific Cognitive Map, another series of simulations was run with the weight matrix of the Cognitive Map to change. The simulation program has the following structure.

Change randomly a weight of the weight matrix.

Create randomly an initial state.

Evolution of trivalent FCM until an equilibrium is reached.

Evolution of sigmoid FCM until an

equilibrium is reached.
Evolution of CNFCM until an equilibrium is reached.
Return for a new random initial state (30 times)
Return for a new change of the weight matrix (30 times).

The above program initially changes a weight from the weight matrix and then initializes the three FCM systems with a random initial state. After that, the three systems are free to evolve and reach an equilibrium. The three equilibrium states are recorded and compared. The comparison between the equilibrium states of the trivalent FCM and the sigmoid FCM or the CNFCM is made based on the signs of the activation levels at equilibrium. Between the sigmoid FCM and the CNFCM two comparisons are made. One based on the signs of the concepts at equilibrium and the other based on the Euclidean distance between the two equilibrium points. After 30 comparisons for 30 different initial states, the weight matrix changes. This happens 30 times giving a total of 30×30=900 different equilibrium points for each system. The results from the comparison are given the following table.

Table 1

	No Difference in signs	One Difference in signs	Two Differences in signs	Three Differences in signs
Trivalent FCM - Sigmoid FCM	776	118	6	0
Trivalent FCM - CNFCM	697	169	34	0
Sigmoid FCM - CNFCM	583	287	30	0

The results are also graphically presented to figure 3. We can see that the conclusions are almost the same for the three systems. The predictions are the same at a level 98% if we assume same two predictions having only one concept with different sign. The cases where two systems have two concepts with different signs are very few.

5 Conclusions

The conclusions that can be drawn from the above simulations are the following:

a) Trivalent FCM, Sigmoid FCM and CNFCM qualitatively draw almost the same conclusions. By qualitatively we mean that when a system predicts the increase (decrease) of a concept, then the other systems will also predict increase (decrease) of the same concept.

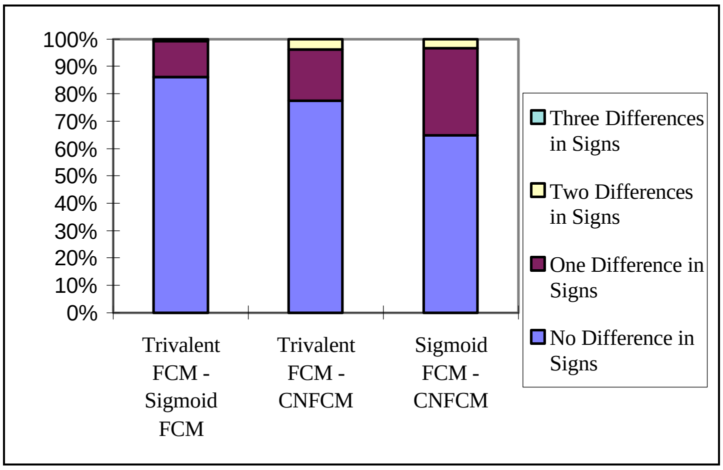

Figure 3 : The results from the comparison of the three FCM systems

b) Sigmoid FCM is based on classical neurons and does not possess memory capabilities. This leads the sigmoid FCM to exhibit sudden changes at the activation of the concepts which is not close to reality. On the contrary CNFCM possess memory capabilities and this is the reason it evolves more smoothly and in a continues manner, bringing it close to reality.

c) Sigmoid FCMs have problems with the linguistic background of their transfer function. Although sigmoid function has background on the physiological level of the human brain, there is not similarity with the linguistic lever of human mind. On the contrary, CNFCMs use a transfer function with strong linguistic background since it is used by many systems for the aggregation of qualitative values.

d) Sigmoid FCM tends to find equilibrium at levels close to -1 and 1 which are the saturation levels of the sigmoid function, instead of using the whole interval [-1,1]. CNFCM on the other hand, finds equilibrium at levels from the whole interval [-1,1].

e) CNFCM is much more reliable in the predictions it makes, since it uses a much better transfer function with strong linguistic background. On the other hand the degrees of increase or decrease predicted by a Sigmoid FCM are not reliable due its weak linguistic background and its tension to find equilibrium close to -1 and 1.

From the above the advantages of using CNFCM are apparent. Details of their implementation can be found in [14].

References:

[1] Kosko B., *Fuzzy Cognitive Maps*, Inter. Jour. of Man-Machine Studies, vol. 24, 1986, pp.65-75.

[2] Kosko B., *Neural Networks and Fuzzy Systems,* Prentice-Hall, 1992.

[3] Axelrod R., *Structure of Design* Princeton: Princeton University Press, 1976.

[4] Zhang W., Bezdek J., Pettus R., *NPN relations and cognitive map development*, Proceeding of the International Symposium on Methodologies for Intelligent Systems, Charlotte 1988.

[5] Zhang W., Chen S., *A logical calculus for cognitive maps*, Proceedings of the International Conference on Neural Networks, San Diego 1988.

[6] Zhang W.R., Chen S.S., Bezdek J.C., *Pool2: A Generic System for Cognitive Map Development and Decision Analysis*, IEEE Trans. on Systems, Man and Cybernetics, vol. 19 no. 1, 1989, pp. 31-39.

[7] Buede D., Ferell D., *Convergence in Problem Solving: A Prelude to Quantitative Analysis*, IEEE Trans. on Systems, Man and Cybernetics, vol. 23, no. 3, 1993, pp. 746-765.

[8] Hagiwara M., *Extended Fuzzy Cognitive Maps*, IEEE Inter. Confer. on Fuzzy Systems, pp. 795-801.

[9] Kosko B., *Fuzzy Thinking: The new science of Fuzzy Logic*, Harper Collins Publishers, 1994, pp. 222-235.

[10] Simpson P., *Artificial Neural Systems*, Pergamon Press, 1990.

[11] B. G. Buchanan and E. H. Shortliffe, *Rule-Based Expert Systems. The MYCIN Experiments of the Stanford Heuristic Programming Project*. Reading, MA: Addison-Wesley, 1984.

[12] Á. Ê. Tsadiras and K. G. Margaritis, “The MYCIN Certainty Factor Handling Function as Uninorm Operator and its Use as Threshold Function in Artificial Neurons,” *Fuzzy Sets and Systems*, vol. 93, pp.263-274, 1998.

[13] Á. Ê. Tsadiras and K. G. Margaritis, “An Experimental Study of Dynamics of the Certainty Neuron Fuzzy Cognitive Maps,” *NeuroComputing*, vol. 24, pp. 95-116,1999.

[14] Á. Ê. Tsadiras and K. G. Margaritis, “Cognitive Mapping and Certainty Neuron Fuzzy Cognitive Maps,” *Information Sciences*, vol. 101, pp.109-130, 1997.

Appendix A

The results from five random initial states (results are drawn from simulations of 1000 initial states)

	Bad Weather	Freeway Congestion	Driving Speed	Driver Caution Level	Patrol Frequency	Auto Accident
Initial State 1	0.89	-0.413	-0.691	0.009	0.017	-0.057
Trivalent FCM	1	1	-1	-1	-1	1
Sigmoid FCM	0.89	1	-0.774	-0.975	-0.975	0.967
CNFCM	0.89	0.895	-0.693	-0.8	-0.802	0.812
Initial State 2	-0.802	-0.47	-0.678	0.341	0.536	-0.028
Trivalent FCM	-1	-1	1	1	1	-1
Sigmoid FCM	-0.802	-0.999	0.773	0.975	0.973	-0.958
CNFCM	-0.802	-0.889	0.69	0.8	0.8	-0.8
Initial State 3	0.976	-0.547	0.352	0.426	-0.403	0.49
Trivalent FCM	1	1	-1	-1	-1	1
Sigmoid FCM	0.976	1	-0.774	-0.975	-0.975	0.975
CNFCM	0.976	0.901	-0.695	-0.8	-0.804	0.822
Initial State 4	-0.468	0.596	0.052	-0.789	-0.976	-0.348
Trivalent FCM	-1	-1	1	1	1	-1
Sigmoid FCM	-0.468	-0.994	0.769	0.975	0.964	-0.887
CNFCM	-0.468	-0.855	0.673	0.798	0.787	-0.739
Initial State 5	-0.117	-0.853	0.73	0.124	-0.507	0.239
Trivalent FCM	-1	-1	1	1	1	-1
Sigmoid FCM Limit Cycle	-0.117	0.931	0.998	0.991	-0.976	-0.99
	-0.117	-0.979	-0.997	-0.991	0.977	0.978
CNFCM	-0.117	0.556	-0.382	-0.78	-0.682	0.429