Fractal Wavelets Image Data Compression

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Abstract: **-** A fractal coding scheme in wavelet transform domain is presented in this paper. The combination and links between these techniques are investigated. We analysed the capability of fractal coders to predict wavelet coefficients where higher frequency subbands of higher levels coefficients of wavelet pyramid are described by filtering, decimating and scaling the coefficients from lower levels higher frequency bands. This paper explains the improved fractal coders in wavelet domain and proposes a new mixed fractal wavelet compression scheme. It has a significant advantage of simplifying decoding by avoiding iterations and a time saving as compared to exhaustive searching. Also, it provides new thought into the concept of fractal-wavelet coding. The resulting coder is a combination of transform and predictive techniques. Simulation results show that the proposed scheme provides a superior performance in terms of PSNR and compression rate and show that it has a potential as a data compression techniques for still images.

Key-words: -Image Compression, fractal coding, wavelet transform, self similarity.
CSCC'99 Proceedings: - Pages 5111-5554

1 Introduction

The fast development of multimedia computing has leed to the demand of using digital images. The manipulation, storage and transmission of these images in their raw form is very expensive, it significantly slows the transmission and makes storage costly. However, digital image processing is exploited in many diverse applications, but the size of these images places excessive demands current storage and transmission technology. Image data compression is required to permit further use of digital image processing, it is the process of reducing the number of bits required to represent these images with lower bit rate, better quality and fast implementation. The wavelet and fractal coding has aroused a lot of attention in both still/video compression. The wavelet representation provides a multiresolution and multi-frequency expression of nonstationary image signals with localisation in both time and frequency domain [1][2]. Such property is desired for image compression because multiresolution subbands become relatively more stationary, and can be coded separately. Hence, the development of fast data compression technique becomes important issue. Various compression methods have been proposed in recent years using different techniques to achieve high compression ratio with acceptable quality [3][4]. All these methods share the same characteristic of being approximate. In this paper, we focus upon examination and improvement of digital images compression methods based on a fractal theory in wavelet domain that incorporates many of the features of both techniques. The objective is that by taking the first level of the pyramid wavelet transform as Domain blocks and the high frequency subbands of the next level as Range blocks and searching for the best set of affine transformation among all possible domain candidates that match the selected range blocks. Before the matching process, the domain blocks are filtered then decimated to get of the same corresponding subband block size, using a kernel filter with a unity sum and mirror coefficients.

This paper is organised as follows: Section 2 describes an overview of fractal compression. Section 3 explains the link between fractal and wavelet transform and provide the proposed algorithm for still images which can obtain a good trade-off between image quality and compression ratio, and then experiments and simulation results explained in Section 4. Finally, Discussion & Conclusion sgiven in Section 5.

Notations:

D_{m_i} : Domain Block Of size 2B

R : Range Block of size B.

x : Spatial contraction function that maps Domain Blocks to the size of Range Blocks (Filtering & Decimation).

s_i : Scalar factor that maps pixel intensity of Domain Blocks to the Range Blocks.

o_i : Scalar offset

2 Overview of fractal image compression

Fractal image coding is based on the theory of Iterated Function system (IFS). It is closely related to certain forms of Vector quantization (VQ), with the distinction that a codebook is not required, it is a comparatively new technique, which gain considerable attention due to its potential high compression rates and fast decoding performance. The basic principle is that an image can be reconstructed by using the self-similarities in the image it self or between the subbands in wavelet domain. These similarities between scales are used for compression. Since Barnsley [5] presented the idea of fractal image coding, various fractal image coding approaches have been developed. Jacquin [6] first introduced the practical image coding algorithm based on the Piecewise Transformation system (PTS), it has been extensively studied for encoding of natural images. The most significant advantages claimed are high reconstruction quality at low coding rates, rapid decoding, and resolution independence. But the encoding process is extremely timed consuming. However, in order to get a high compression, large range blocks are selected whenever corresponding domain blocks can be found. Beaumout [12], who also gives a clear exposition of the basic principle of the technique, presents a modification of Jacquin's method. Minor modification introduced by Oien *et al.* [13] have resulted with higher PSNR (31 dB) and lower bit rate (0.5 bpp). Monor and Dudbridge [14] simplify the coding technique using least squares approach for coefficients determination of contractive transformation, a good compression has been achieved. Pentland and Horowitz [15] have merged the self-similarity of the various levels of decomposition in wavelet domain, they got a rate of 0.125 bpp with PSNR 32 dB. The concept of fractals has been used in many different branches of science including mathematics, physics, chemistry, computer graphic, computer vision, and image processing.

3 Wavelet and Fractal Image Compression

Wavelet transform provides the dyadic self – similarity those fractal coders looking for to exploit. It is a tool for demonstrating the scale invariance of edges; it is obvious that the subbands in successive wavelet levels are similar. Since the wavelet multiresolution subbands consists of coefficients that have the same spatial location with different scaling resolution, intensity and orientation. Such property is desirable in fractal image compression. Therefor, that could exploits the scale invariance of edges to find a form of approximate self – similarity required by IFS. The combination of wavelet with fractal coding in one compression scheme is still a new research topic. The first work of generalising the fractal coding from spatial domain to wavelet domain was done by Davis G. [9][10] and Krupnik H et.al.[11]. In this paper, a fractal – wavelet scheme is proposed, it is designed to overcome some of the major shortcomings of most other types of fractal transforms designed for image coding/ compression. The idea of this proposal is to approximate each range subtree by a domain subtree through fractal transformation. Generally, this process can be explained as a prediction of coefficients of higher frequency subbands in higher levels pyramid from higher frequency of lower levels by using a kernel filter with a unity sums and a mirror reflection. Where the wavelet coefficients have the following relations:

$$\mathrm{Coeff}^{\,d}{}_{L}(R) \approx 2^{-1}\,\mathrm{Coeff}^{\,d}{}_{L+1}(D)\,,\ d = 1, 2, \text{and } 3$$

i.e., the coefficients of range blocks at resolution L approximately equals to the coefficients of domain blocks at lower resolution L+1, multiply by a factor of 2^{-1}. Since, filtering and sub-sampling obtain wavelet coefficients, thus each sample at resolution L corresponding to 4 samples at higher resolution L-1. In the context of coding, this method provides a flexible and powerful generalised method.

3.1 Method Implementation

Step (1). Split the input image X using the wavelet transform into a pyramid structure with L levels that resulted $3*L$ high frequency subbands and one coarse band.

Step (2). Take the L level high subbands ($X_L{}^{LH}$, $X_L{}^{HL}$, $X_L{}^{HH}$) as Range Blocks R_L with a size $2^r * 2^r$ each band of non overlapping sub-squares and $L-1$ level high subbands ($X_{L-1}{}^{LH}$, $X^{HL}{}_{L-1}$, $X^{HH}{}_{L-1}$) as corresponding Domain Blocks D_L with a size $2^{r+1} * 2^{r+1}$

Step (3). For each R_L, search through all of D to find $D_i \in D$ which minimises the root square error by the next steps:

(a) Filter the Domain Blocks by Gaussian kernel filter with a sums unity in horizontal direction

(b) Decimate the resulted from filter by two.

(c) Repeat a, b in vertical direction.

(d) Find the scale s_i and o_i for transformation (w_i)

Step (4) Repeat steps 2, 3 for the other levels pyramid structure.

The transformation code consists of scaling factor s_i, offset o_i and the indices for each block in each level.

To compute the similarities between a given range block and a transformed domain block, we must choose a metric measure which describe the “distance” between the two blocks. The distance represented by the variable mean_square between them.

$$\text{mean\_sq} = \sum_{j=1}^{n} ([s * D_j + o] - R_j)^2$$

The coefficient s is clamped to $[-s_{max}, s_{max}]$ with $0 < s_{max} < 1$ to ensure convergence in the decoding. During the search using traditional method is computationally expensive. Fast nearest neighbour search technique is used to accelerate the search and speeds it up. Once all range blocks have been matched with similar domain blocks, the mapping transformation is written in the code file. The decoding process is straightforward, since it involve the finding of a fixed point of a contractive transformation. Applying the code file over an arbitrary initial image until desired proximity to the fixed point is reached can do this.

4 Experimental Simulation results

We have introduces a technique for applying fractal coding in wavelet domain. It could be used to improve compression systems. The 256x256 gray level cameraman with 8 bits per pixel is used for testing our coding scheme where Daubechies wavelet is employed. The Domain Blocks in each case is filtered with reflection 1 –D separable kernel. The reconstructed image is shown in figure (1) with a compression ratio 62:1. The performance of the proposed algorithm is compared with Efficient Pyramid Image code (EPIC) and Frap-.0.91 (by Matthias Ruhl) in terms of PSNR and compression ratio in Table (1). The experimental results show our method is better than the mentioned methods. In addition, the proposed method is lower computation requirements and its implementation is simple.

5 Discussion & Conclusions

This paper shows that the proposed method is highly efficient and fractal coding in wavelet domain exploit repetition of patterns at different scales. It significantly reduces the block artifacts and permits reconstruction in finite number of iterations, thus it reduces the number of domains to be searched and the number of computations. Using large range blocks ensure high compression ratio to be large as possible. The performance of this coder is comparable to the best results of published algorithms in this area. Our experiments with this code indicate that fractal coders derive of their effectiveness from their ability to efficiently represent wavelet. Our scheme reveals some of the fundamental limitation of current fractal compression schemes.

	EPIC	Frap-0.91	Our method
Compression ratio	62.83:1	60.4:1	62.2:1
PSNR	26.22	31.25	31.45

Table 1: Comparison of coding results for test image Cameraman in terms of PSNR (dB) and compression ratio

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(a) Original Image

(b) Reconstructed Image

Figure (1)