Precedence graphs generation using assembly sequences

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Abstract: - Almost all methods dealing with assembly systems design - for example, assembly line balancing (ALB), tasks-to-workstations assignment algorithms, resource planning - are based on the partition of precedence graphs. These are directed graphs whose nodes represent the assembly tasks and arrows represent the precedence relation between tasks. In the literature there are very efficient and well-known methods to generate assembly sequences and a precedence graph can be obtained by "merging" assembly sequences. Nevertheless, no systematic obtaining method for these graphs was proposed. This paper deals with a property of a given set of assembly sequences, that guarantees the existence of an "equivalent" precedence graph, after suitably defining such an equivalence. This result can be used for the general case, when this property is not met, to find an equivalent set of precedence graphs.

Key-Words: - assembly systems, assembly sequences, precedence graphs

1 Introduction

Precedence graphs are the most used tools in assembly systems design. Assembly line balancing methods (see [1]), that give a first rough layout of an assembly line, have as input data the precedence graph. Other design methods use also precedence graphs. For example, the approach proposed at Draper Laboratory ([8], [9]) is based on assembly task flow graph which are essentially precedence graph with a linear structure. Precedence graphs in different forms are also used in tasks-to-workstations assignment methods ([6]) or resource planning ([7]).

In some papers it is proposed that precedence graphs be obtained by "merging" assembly sequences ([8], [5], [2]). In [5], the authors propose a method for the generation of the precedence graphs from a given set of assembly sequences, which is essentially based on heuristic search.

The precedence graphs have two weak points. The first one is the fact the assembly tasks are generally not well defined. An assembly task is well defined when the base part and the secondary part are specified. The second weak point is the nonexistence of a systematic obtaining method. On the other hand, there are very efficient methods to generate assembly sequences (see [3], [4]), whose assembly tasks are well defined.

This paper presents a theoretical analysis, in order to answer to the question whether there exists a precedence graph "equivalent" to a given set of assembly sequences.

2 Problem statement

In this section, we shall define our problem, that is to find a precedence graph "equivalent" (in a sense that will be stated below) to a given set of assembly sequences for a certain product.

Notation:

S = set of symbols,

N = card (S).

We consider that a symbol represents a part that will be assembled with other parts to make the product. S is the set of symbols corresponding to all parts and N is the number of parts.

An assembly sequence is a total order over the set of parts. Hence, an assembly sequence may be represented by a sequence of symbols:

$$\omega = a_1 a_2 ... a_N, \quad \forall i, a_i \in S \quad , \tag{1}$$

and $a_i \neq a_j$ if $i \neq j$.

From now on, we shall call *complete sequence* a sequence having the form (1) and *partial sequence* a sequence of different symbols whose number of symbols is less than N. So, an assembly sequence is represented by a complete sequence.

Let Ω be a set of complete sequences representing a given set of assembly sequences and n = card(Ω):

$\Omega = \{\omega_i, i = 1,...,n\}$.

Any complete sequence $\omega_i \in \Omega$,

$\omega_i = a_1^i a_2^i ... a_N^i$,

induces an order relation over S, noted O_i, $O_i \subset S \times S$:

$x, y \in S, (x, y) \in O_i \Leftrightarrow x = a^i_j, y = a^i_k, j < k$

We say that x precedes y and we shall note this relation by x < y. Obviously, O_i is a total order relation over S. One can define :

$$O_0 = \bigcap_{i=1}^{n} O_i$$

which is, obviously, a partial order over S. Notice that this order is determined by the initial set of sequences. Ω . The digraph of this relation is G = (S, O_0) and we call it precedence graph. For a given set of symbols S and for a given set of complete sequence Ω, the partial order relation O_0 and its precedence graph G are uniquely defined.

We consider another relation over S:

$I = S \times S - \{(a, b) \in S \times S \mid ((a, b) \in O_0) \vee ((b, a) \in O_0)\}$

We call I indifference relation and we note (x, y) $\in$ I by x?y.

Definition 1: A *complete sequence* $\omega = a_1 a_2 ... a_N$ meets the precedence graph G = (S, O_0) if for any two values $1 \le i < j \le N$ either it exists a path in G from a_i to a_j or a_i ? a_j.

From now on, we note by Θ the set of complete sequences meeting the precedence graph G=(S, O_0). One can say that the graph G is equivalent to the set Θ, because G can be obtained from Θ and vice versa. To simplify the representation of G, any arrow (a_i, a_j) is erased, if there is a path in G from a_i to a_j formed by at least two arrows.

Example 1: Let us consider S_1={a, b, c, d, e, f} and Ω_1={abcdef, abdcef, abdecf}.

The graph G_1 corresponding to the sets S_1 and Ω_1 is presented here-after:

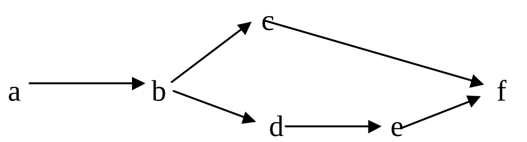

Fig. 1 Precedence graph G_1=(S_1, O_0)

One can verify that the set of sequences meeting the precedence graph is Θ_1={abcdef, abdcef, abdecf}=Ω_1. Hence, the precedence graph G_1 is equivalent to the set Ω_1.

Example 2: Let's consider S_2={a, b, c, d, e, f, g, h} and Ω_2={abcdefgh, acbfdegh}.

The graph G_2 corresponding to the set S_2 and Ω_2 is presented here-after:

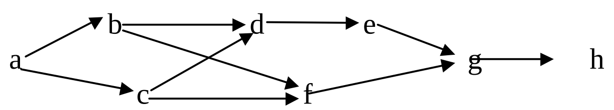

Fig. 2 Precedence graph G_2= (S_2, O_0)

One can verify that the set of sequences meeting the precedence graph G_2 is:

$$\Theta_2 = \begin{Bmatrix} \text{abcdefgh, acbdefgh} \\ \text{abcdfegh, acbdfegh} \\ \text{abcfdegh, acbfdegh} \end{Bmatrix}.$$

It is easy to verify that this time the graph G_2 is not equivalent to the set $\Omega_2 \subset \Theta_2$.

Obviously, from the way we have constructed G, it holds:

$\Omega \subseteq \Theta$ (2)

The favorable situation is when $\Theta = \Omega$, because one can conclude that the graph G= (S, O_0) is equivalent to the set Ω. One can regard the construction

$\Omega \to G \to \Theta$

like a map that defines the set Θ for a given Ω. One can ask the question: does the set Ω have a property that guarantees the equality $\Theta = \Omega$?

In the next sections we shall show that such a property exists.

3 The general form of a complete sequence belonging to Θ

We shall note by

G_I = (S, I)

the undirected graph of the indifference relation, simply called below as indifference graph.

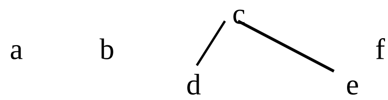

Fig. 3 Indifference graph for example 1.

Generally, G_I is not a connected graph. Let C_i, i=1, 2, …, m be the connected components of G_I that contain more than one symbol. For the example above, it exists only one such connected component: C_1={c, d, e}, while for example 2 there are C_1={b, c} and C_2={d, e, f}. The variability of complete sequences is given only by the symbols belonging to the connected components. The other symbols have fixed positions inside of a complete sequence (for instance, the symbols a, b and f in example 1).

We shall call *segment* a generic partial sequence made up of all symbols belonging to a connected component C_i, i=1,2,…,m.

We shall note by S_i the segment corresponding to C_i. In a given complete sequence ω the segment S_i , i=1,2,…m is instanced by the partial sequence S_i^{ω}, i=1,2,…m.

In example 2, there are two segments, S_1 and S_2, corresponding to the two connected components. A complete sequence has the form aS_1S_2gh, where S_1 is either *bc* or *cb* and S_2 is either *def*, *dfe* or *fde*.

Theorem 1 (the form of a complete sequence meeting a precedence graph G):

$\forall \omega \in \Theta \Rightarrow$

$\omega = a\, b \ldots c . S_1^{(\omega)} . d\, e \ldots f . S_2^{(\omega)} . g\, h \ldots \ldots i . S_m^{(\omega)} . j\, k \ldots m\, n.$

Hence, Θ will appear as the set of all instances of the expression:

$a\, b \ldots c . S_1 . d\, e \ldots f . S_2 . g\, h \ldots \ldots i . S_m . j\, k \ldots m\, n.$

Lemma 1: Let $\omega \in \Theta$.

$(\omega = \sigma_1.a.\sigma_2.b.\sigma_3, a?b, \sigma_2 \neq \phi) \Rightarrow \forall x \in \sigma_2 : (x?a) \vee (x?b)$.

The proof is obvious, because a, b and all the symbols of σ_2 belong to the same connected component of G_I.

4 The case $\Omega=\Theta$. Necessary and sufficient condition

Let consider X a set of complete sequences formed with symbols of S. From X it can be deduced its associated precedence graph G^X, using the same construction as above. Let

$s(X)=\{(a, b) \mid a, b \in S, a ? b\}$

be the set of pairs of indifferent symbols, deduced from G^X.

Notice that $s(\Omega)=s(\Theta)$.

Definition 3 (property P): Let X be a set of complete sequences formed with the symbols of S and $E \subset S\times S$. If

$(\forall\omega\in X, \forall(a,b)\in E \subset S\times S, \omega=\alpha.ab.\beta) \Rightarrow \omega'=\alpha.ba.\beta\in X$

we say that X has the property Π in relation to the set E.

In the case E=s(X), it is simply said that X has the property Π.

Remark: Θ has the property Π.

Notation:

$\omega(i)=a$ - the symbol a has the position i in the complete sequence ω

p(a, b) - the permutation of the symbols a and b

"." - the juxtaposition operator

Lemma 2: If there exist two complete sequences $\omega_1, \omega_2 \in \Theta$, such that $\omega_1(i)=a$, $\omega_2(j)=a$ and i<j, then it exists a complete sequence ω_3 meeting G so that $\omega_3(i+1)=a$ and, moreover, ω_3 can be obtained from ω_1 by applying a sequence of permutations.

Proof:

Because $\omega_1(i)=a$, one can write:

$\omega_1=\sigma_1.a.\sigma_2$,

where σ_1 is a partial sequence containing i-1 symbols.

We shall show that $\exists b \in \sigma_2, a?b$.

Supposing that $\forall x\in \sigma_2$: a<x, it results that $(\forall \omega \in \Theta, \omega(k)=a) \Rightarrow k \leq i$.

But $\exists\omega_2 \in \Theta$ with $\omega_2(j)=a$ and j>i. Therefore, the supposition is false. So, we can note by b the first symbol of σ_2 which is indifferent with a. Hence, we can write:

$\omega_1=\sigma_1.ax_1x_2...x_pb.\sigma_2'$ and $\forall x_i$, i=1...p : a < x_i. (3)

But a ? b, so, applying lemma 1, we obtain:

$(x_i ? a)$ or $(x_i ? b)$, $\forall x_i$, i=1...p. (4)

From (3) and (4), it follows that:

$x_i ? b$, $\forall x_i$, i=1...p.

Using the property Π of Θ, one can apply the sequence of permutations:

$$\omega_1 \xrightarrow{p(x_p,b)} \sigma_1.ax_1x_2...bx_p.\sigma_2'\in\Theta \xrightarrow{p(x_{p-1},b)}$$
$$\xrightarrow{p(x_{p-1},b)} \sigma_1.ax_1x_2...bx_{p-1}x_p.\sigma_2'\in\Theta \rightarrow ... \xrightarrow{p(x_1,b)}$$
$$\xrightarrow{p(x_1,b)} \sigma_1.abx_1x_2...x_p.\sigma_2'\in\Theta \xrightarrow{p(a,b)}$$
$$\xrightarrow{p(a,b)} \sigma_1.bax_1x_2...x_p.\sigma_2'=\omega_3\in\Theta;$$

We note:

$p : p(x_p, b), p(x_{p-1}, b),...., p(x_1, b), p(a, b)$.

It is known that σ_1 contains i-1 symbols, so $\omega_3(i+1)=a$; besides, $\omega_1 \xrightarrow{p} \omega_3$, q.e.d.

Consequence: If there are 2 complete sequences ω_1, ω_2 belonging to Θ, so that $\omega_1(i)=a$, $\omega_2(j)=a$ and i<j, then, for each number k, i<k<j, it exists another complete sequence ω_3 belonging to Θ so that $\omega_3(k)=a$ and, moreover, ω_3 can be obtained from ω_1 by applying a sequence of permutations.

Using the result of theorem 1, we introduce the following

Notation:

$\forall\ \omega \in \Theta$:

$$\omega = \omega(1)\omega(2)...\omega(i_1)\underbrace{\omega(i_1+1)...\omega(i_1+l_1)}_{S_1}\omega(i_1+l_1+1)...$$
$$...\omega(i_2)\underbrace{\omega(i_2+1)...\omega(i_2+l_2)}_{S_2}...$$
$$...\omega(i_m)\underbrace{\omega(i_m+1)...\omega(i_m+l_m)}_{S_m}...\omega(N),$$

$l_i > 1, \forall\ i = 1...m.$

Theorem 2: Let ω_1, $\omega_2 \in \Theta$ be two complete sequences:

$$\omega_1 = \omega_1(1)..S_1^1..S_2^1...S_j^1...S_m^1...\omega_1(N);$$
$$\omega_2 = \omega_2(1)..S_1^2..S_2^2...S_j^2...S_m^2...\omega_2(N) \quad (5)$$

For any j, 1≤j≤m, it exists a sequence of permutations p such that:

$$\omega \xrightarrow{p} \omega_1' = \omega_1(1)..S_1^1..S_2^1...S_j^2...S_m^1...\omega_1(N).$$

Proof:

We shall fix j. To simplify the notation, we shall write:

$$\omega_1 = ...\underbrace{a_1a_2...a_{l_j}}_{S_j^1}... \ ; \ \omega_2 = ...\underbrace{b_1b_2...b_{l_j}}_{S_j^2}...$$

Obviously: $\{a_{l_k}\}_{k=1,2,...j} = \{b_{l_k}\}_{k=1,2,...j}$.

If $a_{l_j} \neq b_{l_j}$, then firstly we shall try to obtain from ω_1 a sequence having the symbol b_{l_j} on the position l_j of S_j. Because $\omega_1(q)=b_{l_j}, \omega_1(r)=a_{l_j}, \omega_2(r)=b_{l_j}$ and q<r, one can apply the consequence of lemma 2; hence, it exists a sequence of permutations, p_1, such that

$$\omega_1 \xrightarrow{p_1} \omega_1^1 = ...c_1c_2...c_{l_j-1}b_{l_j}...$$

In the same way, from ω_1^1 we shall obtain a sequence having the symbol b_{l_j-1} on the position l_j-1 of S_j. Therefore

$\exists p_2 : \omega_1^1 \xrightarrow{p_2} \omega_1^2 = \dots \underbrace{d_1 d_2 \dots d_{1_{j-2}} b_{1_{j-1}} b_{1_j}}_{S_j} \dots$

Remark: To meet exactly the conditions of lemma 2, we must have the guarantee that p_2 does not change the position of b_{1_j} in ω_1^1. This means we must prove that b_{1_j} is not the first symbol indifferent with $b_{1_{j-1}}$. In the opposite situation, we should have

$\omega_1^1 = \dots \underbrace{\dots b_{1_{j-1}} \sigma' b_{1_j}}_{S_j} \dots$ and $\forall x \in \sigma' : b_{1_{j-1}} < x$

So, in every sequence of Θ, all the symbols of σ' should succeed $b_{1_{j-1}}$. But ω_2 is not such a sequence. Hence, our supposition is false.

We proceed in the same way for each symbol of S_j. Finally we obtain

$\omega_1 \xrightarrow{p_1} \omega_1^1 \xrightarrow{p_2} \dots \xrightarrow{p_{1_{j-1}}} \omega_2 = \dots \underbrace{b_1 b_2 \dots b_{1_j}}_{S_j} \dots$

and, so, the sequence of permutations $p : p_1, p_2, \dots, p_{1_{j-1}}$ realises the transfer $\omega_1 \xrightarrow{p} \omega_2$, q.e.d.

Theorem 3: Let ω_1, $\omega_2 \in \Theta$ be two complete sequences. Then it exists a sequence of permutations p such that $\omega_1 \xrightarrow{p} \omega_2$.

Proof:

We write the sequences ω_1 and ω_2 in the form (5) and we use m times the result of theorem 2 as follows:

$\exists p^{(1)} : \omega_1 \xrightarrow{p^{(1)}} \omega_1^{(1)} = \omega_1(1) \dots S_1^2 \dots S_2^1 \dots S_j^1 \dots S_m^1 \dots \omega_1(N);$

$\exists p^{(2)} : \omega_1^{(1)} \xrightarrow{p^{(2)}} \omega_1^{(2)} = \omega_1(1) \dots S_1^2 \dots S_2^2 \dots S_j^1 \dots S_m^1 \dots \omega_1(N);$

…

$\exists p^{(m)} : \omega_1^{(m-1)} \xrightarrow{p^{(m)}} \omega_1^{(m)} = \omega_1(1) \dots S_1^2 \dots S_2^2 \dots S_j^2 \dots S_m^2 \dots \omega_1(N);$

Therefore, the sequence of permutations

$p = p^{(1)} . p^{(2)} \dots . p^{(m)}$.

achieves the transfer:

$$\omega_1 \xrightarrow{p} \omega_1^{(m)} = \omega_1(1) \dots S_1^2 \dots S_2^2 \dots S_j^2 \dots S_m^2 \dots \omega_1(N) =$$
$$= \omega_2(1) \dots S_1^2 \dots S_2^2 \dots S_j^2 \dots S_m^2 \dots \omega_2(N) = \omega_2.$$

In the circumstances of the construction

$\Omega \to G \to \Theta$,

presented before, we give the main result.

Theorem 4 (necessary and sufficient condition for the case Ω=Θ):

Ω=Θ ⇔ Ω has the property Π.

Proof:

Necessity:

We know that Ω=Θ. Because Θ has the property Π and $s(\Theta) = s(\Omega)$, it follows that Ω has the property Π.

Sufficiency:

We know that Ω has the property Π and we shall prove that $\Theta = \Omega$. Supposing that $\Omega \neq \Theta$ and considering the relation (2), it holds

$\Omega \subset \Theta$.

Hence

$\exists \omega$ such that $\omega \in \Theta$ and $\omega \notin \Omega$.

We shall fix such a complete sequence ω.

Let consider ω_0 Î Ω. According to theorem 3, there is a sequence of permutation p_0, such that :

$\omega \xrightarrow{p_0} \omega_0$.

$p_0 = p(a_1, b_1).p(a_2, b_2).\dots.p(a_k, b_k)$, where

$(a_i, b_i) \in s(\Omega) = s(\Theta), \forall i = 1 \dots k$.

$\omega \xrightarrow{p(a_1,b_1)} \omega_1 \xrightarrow{p(a_2,b_2)} \omega_2 \xrightarrow{p(a_3,b_3)} \dots$

$\dots \xrightarrow{p(a_{k-1},b_{k-1})} \omega_{k-1} \xrightarrow{p(a_k,b_k)} \omega_0$.

Figure 4 illustrates this situation.

The last permutation involves by reflexivity the following one:

$\omega_0 \xrightarrow{p(b_k,a_k)} \omega_{k-1}$.

Supposing that $\omega_{\kappa-1} \notin \Omega$, it follows that Ω has not the property Π, fact that contradicts our supposition. Hence, $\omega_{\kappa-1} \in \Omega$.

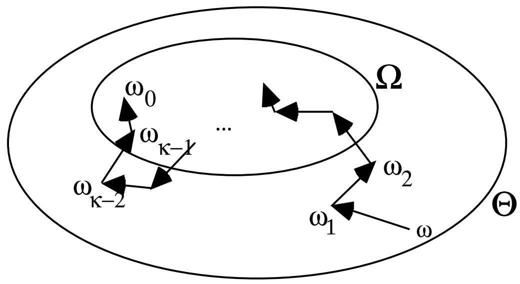

Fig. 4. Sequence of permutations

By applying the same reasoning, it results that ω ∈ Ω, which is a contradiction to our supposition. Therefore, Ω=Θ, q.e.d.

5 Use of the property Π

One can make an algorithm that decides whether a given set Ω of complete sequences has the property Π or not. If the answer is positive, the precedence graph equivalent to the set of sequences is G.

If Ω has not the property Π, the algorithm should generate the partition of Ω into subsets having the property Π, each of them being represented by a precedence graph. The partitioning manner may be subject to two criteria: (a) the minimal number of subsets, and (b) at each iteration choosing the subset of maximal cardinal.

The base idea of the algorithm is to apply sequences of permutations to each complete sequence of Ω, in order to obtain its symmetric sequences. The algorithm can start from any sequence of Ω. When obtaining a symmetric sequence that "overpasses" the set Ω, the algorithm will stop and decide that Ω has not the property Π.

In this case, using the same idea, it must be found the maximal subset of Ω having the property Π in relation with a subset of $s(\Omega)$. One may verify that the problem of partitioning is NP-hard.

In the example below it has been considered as optimal partition that one corresponding to a minimal number of precedence graphs.

Example 3:

Let consider a set of complete sequences:

$$\Omega = \left\{\begin{matrix}\omega_1 = abdcegfh,\ \omega_2 = abcdegfh,\ \omega_3 = abdecgfh \\ \omega_4 = abcdefgh,\ \omega_5 = abcdegfh\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow s(\Omega) = \left\{\underbrace{(b,c)}_{p_1}, \underbrace{(c,d)}_{p_2}, \underbrace{(c,e)}_{p_3}, \underbrace{(f,g)}_{p_4}\right\}$$

$$\Theta = \left\{\begin{matrix}abcdef; acbdef; abcdfe; acbdfe; abcedf; acbedf; \\ abcefd; acbefd; abcfde; acbfde; abcfed; acbfed\end{matrix}\right\}$$

$\Omega \subset \Theta \Leftrightarrow \Omega$ has not the property Π.

The algorithm has been arbitrarily started with ω_1=abdcegfh.

Remark: To obtain symmetric sequences starting with a certain sequence means, in fact, to build the graph of the symmetry relation, whose points are the sequences of Ω and whose edges are labelled by the pairs of $s(\Omega)$. Figure 5 presents this construction.

The optimal partition that has been obtained is:

$X_{opt}^{(1)} = \{\omega_2, \omega_4, \omega_5\}$; $X_{opt}^{(2)} = \{\omega_1, \omega_3\}$.

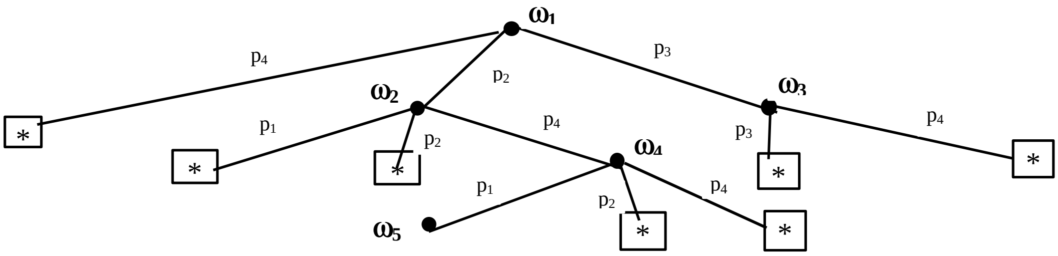

Fig. 5 The graph of the symmetry relation (sequences overpassing Ω are represented by "*")

The obtained partition:

$$\begin{cases}\Omega = X_{opt}^{(1)} \cup X_{opt}^{(2)}; \quad X_{opt}^{(1)} \cap X_{opt}^{(2)} = \varnothing; \\ X_{opt}^{(1)} \text{ has the property } \Pi \text{ in relation with } \{p_2, p_3\} \\ X_{opt}^{(2)} \text{ has the property } \Pi \text{ in relation with } \{p_1\}\end{cases}$$

$$\Omega = \begin{cases}\omega_1 = abdcegfh & \\ \omega_2 = abc\,deg\,fh & X_{opt}^{(1)} \quad \Leftrightarrow \\ \omega_3 = abdecgfh & \\ \text{-------------------------------} & \\ \omega_4 = abcdefgh & X_{opt}^{(2)} \quad \Leftrightarrow \\ \omega_5 = acbdefgh & \end{cases}$$

One can verify that the subset $X_{opt}^{(1)}$ is equivalent to the graph G_1 and the graph G_2 is equivalent to the subset $X_{opt}^{(2)}$ (see figure 6).

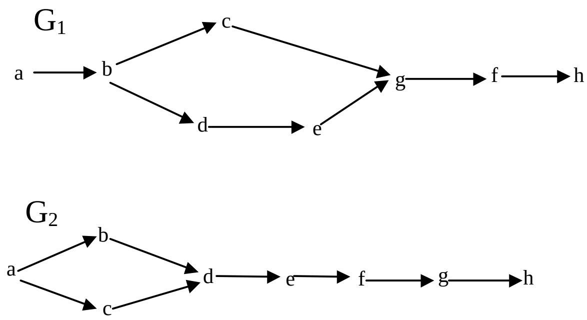

Fig. 6 Precedence graphs representing the set Ω

6 Conclusion

In the field of assembly systems, the representation of a given set of assembly sequences by a precedence graph is a point of interest.

In this paper, we have analyzed the equivalence between a set of assembly sequences and a precedence graph.

When this equivalence exists, we have proved that a property of the initial set of sequences (property Π) is met. The test of this property can be implemented by a simple algorithm. For a given set of assembly sequences that does not meet the property Π, we have suggested a way to obtain an equivalent set of precedence graphs. Obviously, this problem is NP-hard and, so, any algorithm conceived to solve it will be exponentially complex.

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