ANALYSIS OF SYNCHRONOUS WDMA PROTOCOLS WITH FINITE NUMBER OF TUNABLE RECEIVERS

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Abstract: This paper examines the problem of receiver collision at destination for synchronous WDMA protocols in a single-hope WDM network using passive star topology. We evaluate the performance reduction due to finite number of tunable receivers at each station assuming Poisson arrivals and finite number of stations. The effect of receiver collision is estimated by the average rejection probability at destination of a packet.

Keywords: wavelength division multiple access WDMA, multichannel, electronic processing bottleneck, receiver collision, rejection probability.

1 Model and Assumptions

The system under consideration as figure 1 shows is a passive star network. The system uses (v+1 wavelengths $\{\lambda_0, \lambda_1, ..., \lambda_v\}$ to serve a finite number M(M>v) of stations [1]. The multichannel system at wavelength λ_0 operates as control channel while the remaining v channels at wavelengths $\{\lambda_1 ... \lambda_v\}$ constitute the data multichannel system. The Network interface unit (NIU) can be described as a $CC\text{-}TT\text{-}FR\text{-}TR^F$ structure [2]. It means that each station has a tunable transmitter tuned for any of channels $\{\lambda_0, \lambda_1, \ldots, \lambda_v\}$. The outcoming traffic from a station is connected to one input of the passive star coupler. Every station also uses one fixed tuned receiver for each control channel and $F(1< F<v)$ tunable receivers to any of data channel $\{\lambda_1 ... \lambda_v\}$. The incoming traffic to a user station is splitted into F+1 portions by a 1x(F+1) WDMA splitter as Figure 1 indicates. A station will hear the result of a transmission of its control and data packets by listening to the star coupler multichannel system since it operates as broadcast medium.

We assume that the total traffic from new generated and retransmitted packets obeys to Poisson statistics according to Bertsekas's assumption [3]. In the proposed protocol the control channel and data channels are slotted. The control channel is slotted with the fixed size of the control packet that is called minislot. Slots on data channels fit to the fixed size of data packets. The transmission time of a fixed size control packet is used as time unit. Thus the data packet transmission time normalized in control minislot time units is L (L>1). In our analysis the access methods to control channel and data multichannel system are based on ALOHA protocol. Both control and data channels use the same time reference, which we call cycle. We define as cycle, the time interval that includes w time units for control packet transmissions followed by a data packet transmission period. Thus the cycle the duration is C=L+w time units as Figure 2 illustrates. Time axis is divided into contiguous cycles of equal length. The stations are synchronized for the transmission on the control and data packet during a cycle.

A station generating a data packet waits the beginning of the next cycle selects randomly one of the w contiguous minislots and sends a control packet on the control channel to compete according to the ALOHA protocol, to gain access. The control packet as Figure 2 displays, is consisting of the transmitter address, the receiver address and the wavelength λ_k of the data channel. If the control packet is transmitted successfully, after the end of the first w time units of the cycle, the station transmits the corresponding data packet over the λ_k data channel. A station will hear the result of the transmission its control and data packet by listening to the star coupler multichannel system since it is operates as a broadcast medium. We assume that the total offered traffic from new generated and retransmitted control packets obeys to Poisson statistics.

In the receiving mode if a station sees its address announced in a control packet, immediately adjusts its receiver to the transmission wavelength channel specified in the control packet for packet reception. We say that the tunable receiver of a station, Z, is active, if it is tuned receiving a data packet from a data channel of a λ_{dk} (k=1,...v). Data channels are slotted, so data packets are coordinated for transmission at the beginning of data slots. It is possible more than F successfully transmitted data packets on different data channels to have as destination the same station during a given cycle. In this case, the station tunable receivers are tuned to F data channels of the incoming successful transmissions and rejects the others as Figure 3 illustrates. This phenomenon is called receiver collision [4]. Depending on the examined protocols there are different policies to select which packet is received correctly at destination. As an example we can say that data packets that are transmitted on the lower data channels number win the competition. If a new generated and successful transmitted data packet involves in receiver collision at destination, the data packet is destroyed and the station becomes backlogged. A backlogged station waits some random time before its new retransmission after a collision, depended upon the particular backoff strategy being used. We consider that at any point in time, each station is capable of transmitting at particular wavelength and simultaneously receiving at F different wavelengths that belong to the set $\{\lambda_1 \ldots \lambda_v\}$. In addition tuning times and propagation delays are assumed negligible.

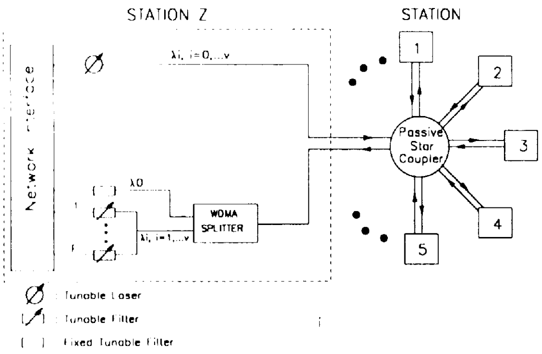

Figure 1 :Passive star architecture

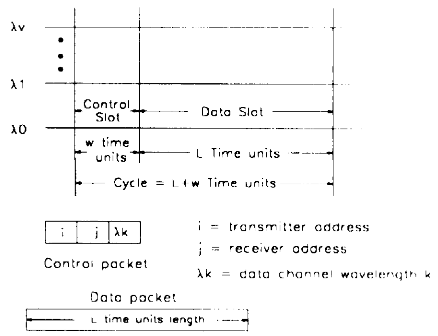

Figure 2 : Cycle and structure of the control packet

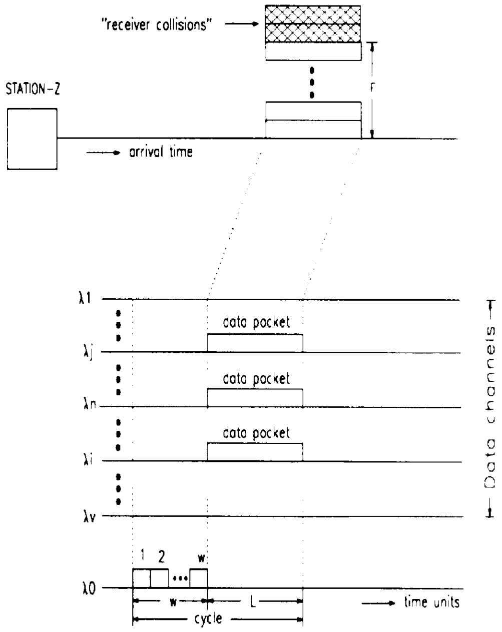

Figure 3 :Receiver collision

2 Analysis

We follow the following notations.

G = The average number of transmitted control packets per minislot in the control channel in steady state.

G_T = The average number of transmitted control packets during a cycle in steady state

T = L(time units), the data packet transmission time.

C = L+w, the cycle duration time.

S_{cT} = the average number of successful transmitted control packets during a cycle in steady state.

P_s = the probability of successful transmission of a control packet during a cycle in steady state.

P_{suc} = the probability of success of a station on both the control channel as well as a data channel during a cycle.

S_{dT} = the average number of successful transmitted data packets during a cycle in steady state.

S_{rej} = the total average rate of rejected data packets at destination due to active tunable receivers

S_{rc} = the average number of correctly received data packets at destination during a cycle in steady state.

2.1 Case I

The described and analyzed protocol uses a simple policy for data packet transmission. In this policy a station transmits a control packet in the first part of a given cycle, and then after the end of the w_{th} minislot transmits the corresponding data packet over one of the data channels. This policy is identified as " tell and go " policy, does not take under consideration any control channel collision in the previous part of the cycle of the accompanying control packet. In high speed protocols this transmission policy is preferable when the round trip delays are more much longer than reservation part of the cycle. We assume Poisson approximations of the overall traffic of finite population.

$$G_T = w\,G \tag{1}$$

The probability of one Poisson arrival in the j_{th} minislot is given by

$$G = G\,e^{-G} \tag{2}$$

So

$$S_{cT} = w\,G\,e^{-G}, \tag{3}$$

And

$$P_s = S_{cT} / G_T = e^{-G} \tag{4}$$

Let $A_w = k$, random variable representing the number of control packets that have transmitted in the i_{th} cycle. We assume that these packets have been uniformly distributed among v data channels. Thus, the random distribution in v channels gives v^k arrangements each with probability v^{-k}.

Let $P_d(k)$ = the probability that only one from k data packets have been destined to a given data channel n, $n \in \{1,2,\ldots,v\}$ in the data slot time of the i_{th} cycle.

The remaining k-1 packets are destined to the remaining (v-1) data channels in $(v-1)^{k-1}$ different ways. Then.

$$P_d(k) = (k/v)\,[1-(1/v)]^{k-1} \tag{5}$$

The approximation for large number of v, gives

$$P_d(k) = (k/v)e^{-(k-1)/v} \tag{6}$$

In steady state,

$$E[A_w = k] = G_T \tag{7}$$

The conditional probability, $P_d(G)$, that a data packet is transmitted without collision in a data channel in steady state regardless of successful or not transmission of the corresponding control packet is given by

$$P_d(G) = E[P_d(k)] \approx (G_T/v)\,e^{-G_T/v} \tag{8}$$

$$P_{suc}(G) = P_s P_d(G) = e^{-G}\,(G_T/v)\,\exp(-G_T/v) \tag{9}$$

$A_v(G)$ = random variable representing the number of successfully transmitted data packets during i_{th} cycle, given that the offered traffic per minislot is G. The conditional probability of finding $A_v(G) = m$, data channels every one with one Poisson arrival, obeys to binomial probability low.

$$P[A_v(G)=m] = [\,v!/(v-m)!m!\,]\,P_{suc}^{\,m}(G)\,[1-P_{suc}(G)]^{v-m}$$

$$S_{dT} = E\{P[A_v(G)=m]\} = \sum_{m=1}^{v} m\,P(A_v(G)=m) =$$

$$= w\,G\,\exp[-G(1+w/v)] \tag{10}$$

Let $U_v(r)$ = the number of transmissions with destination a given station let's say Z, conditional that r packets have been transmitted successfully. The examined problem corresponds to the occupancy problem [5]. For sake of simplicity of the analysis we suppose that station Z transmits itself. We assume that these packets have been uniformly distributed among M stations. The random distribution of r packets in M stations gives M^r

arrangements each with probability $(1/M)^r$. The probability that k data packets are destined to Z station can be found as follows: The k packets can chosen in r!/(r-k)!k! ways and the r-k packets are destined to the remaining (M-1) stations in $(M-1)^{r-k}$ different ways.

$$P[U_v(r) = k] = r!/(r-k)!k!(1/M)^k(1 - 1/M)^{r-k} \qquad (11)$$

Given that the number of tunable receivers of each station is F, we examine the rejection probability of a successful transmitted data packet destined to station Z, in steady state.

The probability Pcol(m), that m packets destined to station Z are aborted due to active tunable receivers is given by:

$$P_{col}(m) = \sum_{I=m}^{\min(M,v-F)} P[A_v(G) = F+I]\ P\ [U_v(F+I) = F+m] \qquad (12)$$

The mean probability a successful transmitted data packet to be aborted by a station Z in steady state, given that the number of tunable receivers of a station is F, is defined as

$$Pcol = \sum_{I=1}^{\min(M,v-F)} m\ Pcol(m) \qquad (13)$$

So

$$Srej = M\ Pcol \qquad (14)$$

and

$$Src = [L/C)][S_{dT} - Srej] \qquad (15)$$

The average rejection probability at destination of a packet, is evaluated as the ratio of the average number of packet rejection at destination in steady state due to active receivers, to the average number of successfully transmitted packets per cycle.

$$Prej = Srej / S_{dT} \qquad (16)$$

2.2 Case II

This protocol case corresponds to an improvement modification of the previous protocol and leads to better performance measures. The main drawback of the aforementioned protocol is the bandwidth waste,

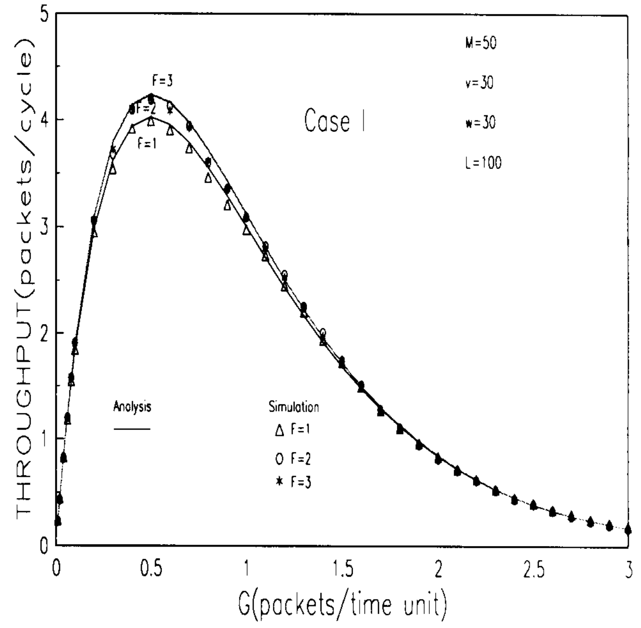

Figure 4: The throughput Src versus the offered traffic G(packets/minislot) for v=30(channel)systems, w=30 minislots and M=50 stations, with F=1,2,3 tuned receivers. Analytical and Simulation results.

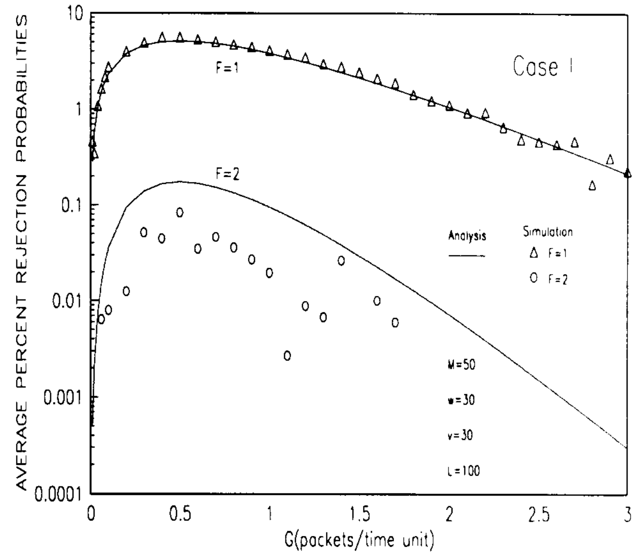

Figure 5 : The average rejection probabilities Prej (Log. Scale) versus G(packets/minislot)f or v=30(channel)systems, w=30 minislots and M=50 stations , with F=1,2 tunable receivers. Analytical and Simulation results

because a data packet is transmitted over the data multichannel system even though the corresponding control packet has been destroyed due to control channel collision. The proposed protocol adopts a different policy for data packet transmission. Therefore a data packet is transmitted if and only if the corresponding control packet has successfully transmitted in the first part of the cycle. Thus if the control channel is transmitted correctly without control channel collision, a station transmits its data packet immediately after the w_{th} minislot. The analysis is analogous of the case I protocol. In this section we modify some equations taking into account the new policy and the introduced transmission restrictions.

We define again the P_d, as the probability of data packet successful transmission in a data slot time of a cycle in steady state, under the condition that the corresponding control packet has been successfully transmitted.

We also redefine $A_w = k$, as the random variable representing the number of successful transmitted control packets in the i_{th} cycle.

In steady state $E[A_w = k] = S_{cT,}$, so using the same methodology as in case I, we take

$$P_d(G) = (S_{cT} /v) \exp (- S_{cT}/v) \qquad (11)$$

and

$$P(A_v(G) =m)= [v!/(v-m)!m!] P_d^{m} (1- P_d)^{v-m} \qquad (12)$$

$$S_{dT} = E[A_v(G) = m] = \sum_{m=1}^{v} m\ P(A_v(G) = m) =$$

$$= w\, G \exp \{ -G[1+(w/v)\, e^{-G}] \} \qquad (13)$$

3 Numerical Results

we apply the above analytical methods to study the effect of tunable receiver variations on the performance of the examined protocols. To supplement the analytical results with the simulation results, we have used an extensive used an extensive simulation model that has one to one correspondence to the actual system. Figures 4 and 6, illustrate the throughput S_{rc} versus the offered traffic G, for Case I

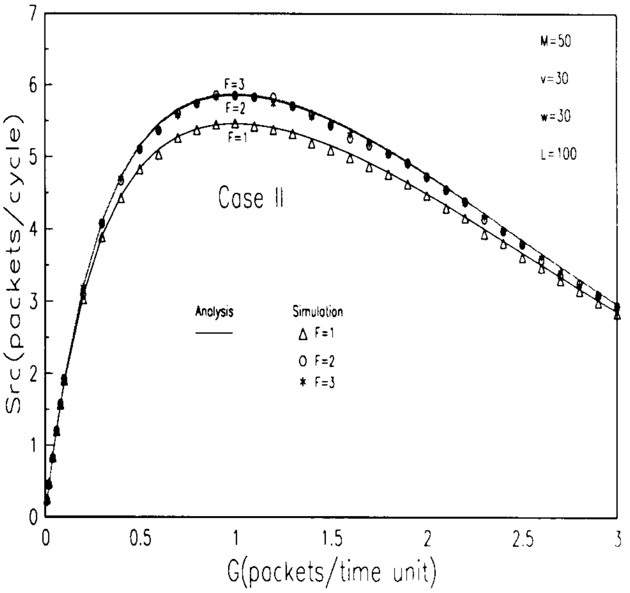

Figure 6 : The throughput Src versus the offered traffic G(packets/minislot) for v=30(channel)systems, w=30 minislots and M=50 stations, with F=1,2,3 tunable receivers. Analytical and Simulation results

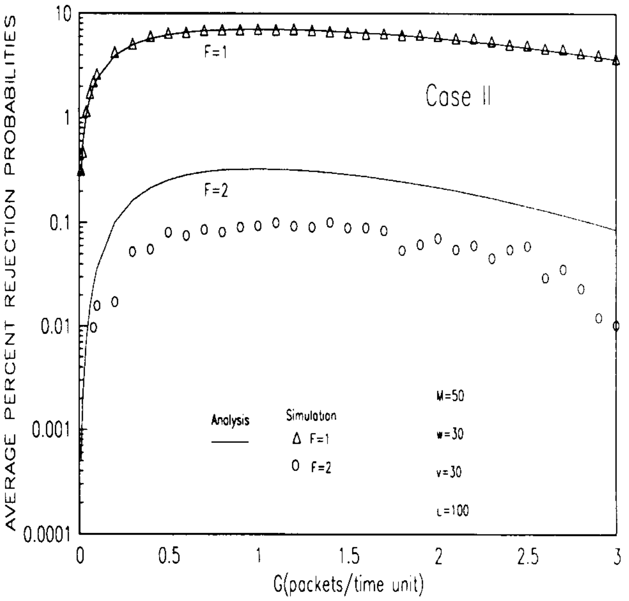

Figure 7 : The average rejection probabilities Prej(Log. Scale) versus the offered traffic G(packets/minislot) for v=30(channel) sysstems, w=30 minislots and M=50 stations, with F=1,2 tunable receivers. Ananlytical and Simulation results

and Case II protocols correspondingly for v=30, w=30, M=50 with F=1,2,3 and L=100. Figures 5 and 7 depict the average rejection probabilities P_{rej} versus the offered traffic G, for Case I and Case II protocols for v=30, w=30, M=50 with F=1,2,3 and L=100.

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