Vector solitons in KTP: theory and experiments

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Abstract: -We present theoretical and experimental results to outline the conditions for the existence and the main physical properties of vectorial solitary waves in KTP crystals. We demonstrate that the whole family of vector solitons can be generated by a single, linearly polarized, pump frequency and that the properties of the generated soliton depend on the polarization angle of the fundamental frequency at input (i.e. on the imbalance between the fundamental frequency components parallel to the ordinary and extraordinary axes of the birefringent crystal).

We demonstrate that the minimum energy for soliton formation can be at non zero fundamental frequency imbalance or at zero imbalance depending on the sign of the phase mismatch.

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1 Introduction

In the last years, second order nonlinear effects have attracted a great deal of interest in the scientific community; in particular solitons sustained by second order nonlinearity have been extensively studied after the first experimental demonstration of soliton propagation in a KTP crystal [1]. For KTP, in type II geometry for phase matching, soliton self trapping arises from the mutual coupling between two fundamental frequency components and the second harmonic frequency (three waves interaction). The goal of the experimental and theoretical work we present here is to describe the polarization properties and the conditions for the existence of vectorial solitary waves excited by imbalanced fundamental frequency waves (FFs). The properties of these solitary waves are envisaged to be relevant in various applications, ranging from beam shaping in optical parametric amplification and oscillation to all optical addressing of light [2-11].

In the first part of the paper, we will present a simple physical picture describing the interplay between nonlinear effects and diffraction in a second order nonlinear crystal; soliton self-trapping obviously arises when the two competing effects balance each other. We can then prove that this balancing is affected by the following physical terms: transverse size of the beam (that determines the role played by diffraction), phase mismatch among the interacting waves (ruling the relative phase delays among the three waves), intensities of the three waves involved in the process of second harmonic generation (SHG). In particular the role played by the polarization state of the fundamental wave will be outlined. We will show that for negative phase mismatch the minimum energy for soliton formation is always at zero imbalance between the fundamental frequency components (i.e. for the fundamental beam linearly polarized at 45 degrees with respect to the ordinary axis of the birefringent crystal); for positive phase mismatch the situation changes and the minimum energy for soliton formation is found for non zero imbalance. This is a rather unexpected result, since the efficiency of the harmonic conversion is reduced by the imbalancing, due to the reduction of available photons pairs for SHG. We will also show that this rather unexpected result is confirmed by the characteristics of analytical solutions that we have found through a variational approximation of the problem at hand. We will then present the experimental results that demonstrate the soundness of our theory. We will also show that the imbalancing produces a continuous shift of the soliton output position for negative phase mismatch and a discontinuous one for positive phase mismatch.

2 Problem Formulation

As well known the problem of three waves interactions in second order nonlinear media is conveniently described by the following evolution equations for the three slowly varying envelopes of the fields involved in the process:

$$\frac{\partial A_1}{\partial z}+\frac{1}{2j}\nabla^2 A_1 = jA_2{}^* A_3$$
$$\frac{\partial A_2}{\partial z}-d_\omega\frac{\partial A_2}{\partial x}+\frac{1}{2j}\nabla^2 A_2 = jA_1{}^* A_3$$
$$\frac{\partial A_3}{\partial z}-d_{2\omega}\frac{\partial A_3}{\partial x}+\frac{1}{4j}\nabla^2 A_3 = jA_1 A_2 + j\gamma A_3$$

where A_1, A_2, A_3 are the dimensionless envelopes, (x,y) are the dimensionless transverse coordinates (normalized with respect to a transverse scale length r_0, $z_0=r_0^2k_1$ the longitudinal scale length (diffraction length), d_ω, $d_{2\omega}$ are the beam walk-off coefficients, characterizing the different directions of energy and phase fronts in birefringent crystals, $\gamma = z_0\ (k_3\text{-}k_2\text{-}k_1)$ gives the dimensionless mismatch parameter.

The goal of our experiment was to study the conditions for the existence of vectorial solitary waves with imbalanced fundamental frequency waves (FFs); this topic has been already studied theoretically [12,13] and was first experimentally addressed in [14] where the authors reported the dependence of the threshold energy for soliton formation as a function of the phase mismatch for balanced FFs and the threshold energy as a function of the imbalance in the FFs components. The main conclusions were that the minimum energy for soliton formation is at the phase matching condition and, for a given phase mismatch, it is attained for zero imbalance. The result we present in this paper show that this last conclusion is affected by the sign of the phase mismatch; in fact while for negative phase mismatch the minimum energy for soliton formation is always at zero imbalance, for positive phase mismatch the situation changes and the minimum energy for soliton formation can be found for non zero imbalance. This is a rather unexpected result, since the efficiency of the harmonic conversion is reduced by the imbalancing, due to the reduction of available photons pairs for SHG. We will also show that this rather unexpected result is confirmed by the characteristics of analytical solutions that we have found through a variational approximation of the problem at hand and we will also provide a simple qualitative picture to describe the physical reason why this happens.

In order to provide a simple physical picture of the reasons why this behavior is observed let us discuss the phase curvature induced on the beam by diffraction and nonlinearity separately in the case (typically attained in experiments) of negligible walk-off. If diffraction does not play a role, solutions of the governing equations are easily obtained. Let us write $A_1=pU_0exp(j\phi_1)$, $A_2=(U_0/p)exp(j\phi_2)$, $A_3=U_3exp(j\phi_3)$, and $\Phi= \phi_3 - \phi_2 - \phi_1$. Setting the z derivatives of $U_1=pU_0$, $U_2=U_0/p$, U_3 and Φ equal to zero we get the fixed point of the system describing the nonlinear interactions without diffraction. In particular these solutions are characterized by two branches: branch 1 corresponds to $\Phi = 0$ and branch 2 to $\Phi = \pi$. Jumping from branch 1 to branch two entails a change of sign of the curvature of phase front of all three waves involved in the nonlinear interaction. Moreover, as we shall see in a short while, solutions belonging to branch 1 have a curvature of the phase front opposite to the diffraction induced phase front curvature. Thus solutions belonging to branch 2 are of no interest for us here, since they can never compensate diffraction. Note that the factor p fixes the imbalance ($p=1$ is the balanced case) and we are only interested here to discuss the properties of the solutions near the balanced case.

Explicitly, the amplitude of these solutions reads:

$$U_0=\sqrt{\frac{\left(E_0 - 2|U_3|^2\right)p^2}{1+p^4}}$$
$$U_3=\frac{\tilde{a}\,(p^2+p^6)+p^2\sqrt{\tilde{a}^2\left(1+p^4\right)^2+4\left(1+4p^4+p^8\right)E_0}}{2\left(1+4p^4+p^8\right)}$$

where we have defined the total energy function $E_0(R)=(p^2+1/\ p^2)\ U_0^2+2U_3^2$ and $R=\ x^2\ +\ y^2$. Note that non trivial solutions require:

$$\gamma < \frac{1+p^4}{2p^2}(E_0)^{1/2}$$

This last point will be of crucial relevance in the discussion that will follow: in order for the solutions to exist at a given positive phase mismatch, we need to increase E_0 (the total energy) above a critical threshold.

Note that the amplitude of the U_3 component increases monotonically with γ (reflecting the fact that the energy composition of the solutions approaches the case of a dominant second harmonic component as we move towards the boundary of their existence region). Also the phases ϕ_3, ϕ_2, ϕ_1 of the three interacting waves can be easily determined in this approximation. For the sake of clarity and simplicity we can limit our attention to the phase of the second harmonic component ϕ_3; moreover the general expression of the phase is rather cumbersome, but for the problem of interest we can limit ourselves to the expression describing ϕ_3 around $R=0$ and $p=1$.

3 Problem Solution

In the framework described in the previous section we get:

$$\phi_3(R) = \phi_3(0) - \frac{2Q^2}{\alpha^2\left(\gamma^2 + 6Q^6\right)^{1/2}} R^2 + O\left[R^3\right]$$

where, for the sake of simplicity, we have taken a particular total energy function, namely a gaussian function with beam width α:

$$E_0(R) = Q^2 \exp\left(-\left(\frac{R}{\alpha}\right)^2\right)$$

If we now consider also diffraction to play a role in the specific problem, we will have to face the competition between the two effects and diffractive free propagation can result whenever the phase front curvature induced by nonlinearity is compensated by the phase front curvature induced by diffraction. To describe the phase front curvature induced by diffraction we can simply consider a gaussian beam at the second harmonic component with a beam radius w. Therefore, for diffraction free propagation to take place, we must require the linear phase front curvature to be exactly compensated by the nonlinear one, i.e. we must ask the following relation to hold:

$$\pi \frac{\alpha^4}{w^4}\sqrt{\gamma^2 + 6Q^2} = \pi\alpha^2 Q^2 = I$$

where I stands for the total energy carried by the three waves (the integral from minus infinity to plus infinity in the plane x,y of the total energy E_0). Note that the ratio $c = (\alpha/w)$ is the ratio between the width of the total energy function and the width of the second harmonic component. The minimum value of this energy is clearly obtained for Q going toward 0 (and α going to infinity) and it is

$$Ith1 = \pi(c^4)|\gamma|$$

however this limit can be reached only for negative phase mismatches, since for positive phase mismatch we cannot go to arbitrary low Q value: as Q approaches 0 solutions of the nonlinear problem in the positive phase mismatch region tend to disappear.

The minimum attainable Q value for a given γ is

$$Qth = \gamma \frac{2p^2}{1+p^4}$$

which gives a threshold energy

$$Ith2 = \pi(c^4)|\gamma|\sqrt{1+24\left(\frac{p^2}{1+p^4}\right)^2}$$

note that at $p=1$ we have $Ith2 = Ith1(7^{1/2})$ in optimum agreement with the experimental results and that for small p deviations around the condition $p=1$, $Ith1$ does not depend on p. On the contrary small p deviation around 1 (i.e. small deviation from the balanced case) tend to reduce $Ith2$ (as observed in the experimental results). The reason why this happens is that the imbalancing makes it possible to penetrate further in the positive phase mismatch sign (for a given beam energy) or equivalently to maintain the same existence domain for a lower power.

Let us now move to the description of the experimental results and their comparison with the physical picture of soliton self trapping we have just described.

In the experiment we used a *2cm* long KTP crystal cut for type II phase matching; the pump laser (delivering *45ps* pulses at *1064nm* with *10Hz* repetition rate) was focused into the input face of the KTP crystal with a spot size of *22*microns (FWHMI).

Before entering the nonlinear crystal, the linear polarization of the pump beam could be rotated by means of a half wavelength plate, thus allowing to change the relative intensity of the beam along the extraordinary and the ordinary axes of the birefringent crystal (i.e. allowing to change the imbalance). The light at the output of the KTP crystal was then imaged onto a CCD camera where we measured the beam output width at the fundamental frequency (while the second harmonic components was stopped by filtering). For a given phase mismatch and imbalance we then plotted the output width as a function of the input intensity and we defined the threshold for soliton formation as the input intensity where the output beam width is *125%* of the input one. We then repeated this procedure for different imbalances of the input beam. The results are summarized in figure 1 and 2, where the intensity is in GW/cm^2 and $\theta\ (-\pi/4)$ is the oreintation of the linearly polarized input beam with respect to the ordinary axis of the crystal.

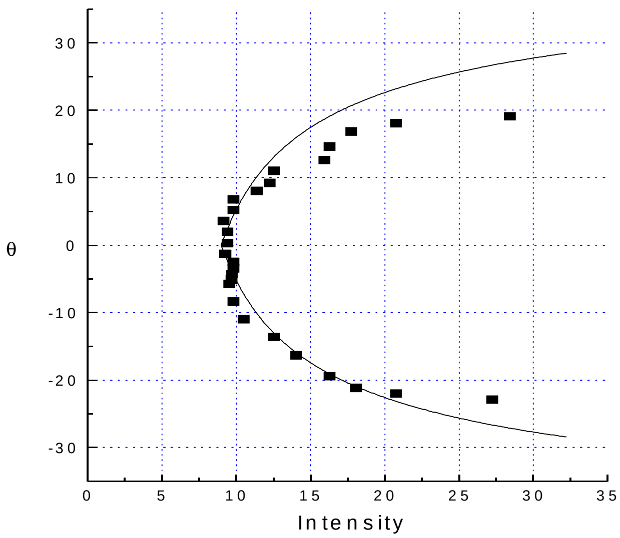

Figure 1: experimental (dots) and analytical (continuous line) threshold curve defining the existence region for solitons. The phase mismatch here is –3 π

Figure 1 refers to a phase mismtach $\Delta k\ L = -3\ \pi$. We can see, as already reported in [14], that the threshold for soliton formation increases with increasing imbalancing. Moreover we note that the change in the imbalance is also responsable for a continuous space displacement of the output soliton beam (the phenomenum known as walking soliton). In the figure the points corresponds to the experimental results and the continuous line refers to the analytical results, obtained in the framework of a variational approximation [13].

Figure 2 refers to a phase mismtach $\Delta k\ L = 3\ \pi$. Contrary to the previous case, we see that the minimum energy for soliton formation is not at zero imbalance; this is due to the existence of a forbidden region for soliton formation near the zero imbalance situation, where the solitary wave requires bigger intensity to be sustained. This is in agreement with the analyses reported in the first part of this paper and also with the exact numerical solution obtained in the framework of a variational approximation [13]. Note also that the shape of this existence curve depends crucially on the beam width at input; reducing the width of the input beam, in fact, the forbidden region observed in figure 2, shrinks and tends to disappear.

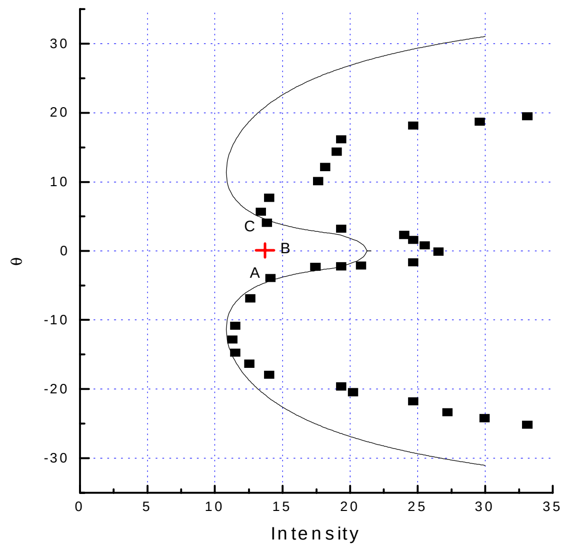

Figure 2: experimental (dots) and analytical (continuous line) threshold curve defining the existence region for solitons. The phase mismatch here is 3 π

If, for a given input intensity (14 GW/cm^2 in figure 2), we change continuously the input imbalance we can move from a region where solitons exist (at non zero positive imbalance) to a region of non existence (near the balanced case) and then again into an existence region.

Correspondingly the soliton beam at output shifts abruptly in space from left (figure 3a, corresponding to point A in figure 2) to right (figure 3c, corresponding to point C in figure 2). In the intermediate region solitons do not exist (figure 3b, corresponding to point B in figure 2).

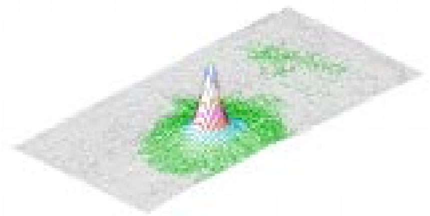

Figure 3a

Figure 3b

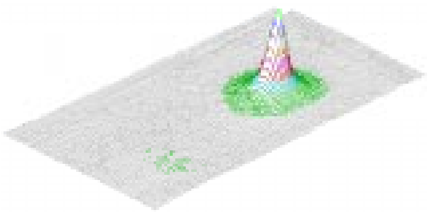

Figure 3c

Figure 3: experimentally observed output beam shape at the fundamental frequency for an input intensity of 14GW/cm^2 at different input imbalances (-4^0 corresponds to fig. 3a, 0^0 corresponds to fig. 3b and 4^0 corresponds to fig. 3c).

4 Conclusion

In conclusion we have presented an experimental and theoretical analysis of the threshold energy for vector soliton formation in a KTP crystal. We have shown that depending on the sign of the phase mismatch the minimum intensity for soliton formation can be at zero or at non zero input imbalance. A good qualitative agreement between theory and experimental results is observed.

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