Abstract: - The coupling of electromagnetic fields inside shielded cables is an important issue in electromagnetic compatibility and telecommunication applications, in which the transfer impedance represents one of the main parameters[1].
But if it is easy to define this parameter in the case of coaxial cable, the problem is more difficult in a shielded multiconductor lines.
We will be restricted to simple case of a shielded two-wire cable in which we determine parameters which influence most on the magnitude of differential mode voltages.
We show that the principally origin of these voltages are : dissymetry of shield, dissymetry of interior conductors and dissymetry of the loaded network connected at the both extremities of the cable.
Analysis of some numerical examples show that these dissymetries has very important practical consequences.
The most interesting one is the extension of the practical transfer impedance notion in different voltages.

1 Introduction
Nowadays the use of highly integrated electronic circuits increases more and more. Due to the replacement of mechanical measurement and control devices by computer controlled electronic equipment complex systems, as for example aeroplanes, become vulnerable to interfaces. Especially the interconnections between the electronic devices are very susceptible to incident electromagnetic waves.
In many cases, the shield of transmission cables are important path in the coupling between disturbing sources and control systems[1] & [2].
We will be restricted to simple case of two-wire cable. When a two-wire cable is submitted to electromagnetic field, we decompose the problem in two subproblems that can be solved in a sequential way.
The first, the so-called external problem, is the calculation of induced current and voltage along the shield.
The second way is to apply the previous result to evaluate the differential mode and common mode voltages.
The common mode voltages is like the crosstalk voltages appearing at both ends of the coaxial cable “near-end and far-end crosstalk voltages”.

The origin of the differential mode voltage is very difficult and so that we propose to evaluate its magnitude versus the frequency and the length of the cable.
If the shield of the cable have no revolution dissymetry, we can show that the voltages between the interiors wires and shield are not the same. The result of it is the differential voltage induction which is bounded to the differential transfer impedance, the length of cable and the frequency. A last case concerns the effect of the network of impedance connected to the two extremities of the two-wire cable or the dissymetry of the interior conductors. In this case these dissymetries will create a rejection from the common mode to differential mode.
The practical consequences of the rejection are very important as we will explain later.

2 Mechanism of coupling through shielded cable
We consider a shielded two-wire cable loaded by a network impedances.
The transmission line geometry of interest is shown in figure 1. 
Zd0 and ZdL are the differential mode impedances.
Zc01, Zc02, ZcL1 and ZcL2 are the common mode impedances.
Assuming quasi-TEM propagation, the transmission lines for voltages and currents are:

\[-d[V]/dz = [Z].[I] - [Zt].Ip\]
\[-d[I]/dz = [Y].[V] + [Yt].Vp\]

\([Z]=i.w.[L]\) and \([Y]=i.w.[C]\)

\([L]\) and \([C]\) denote respectively, the per unit-length self inductance and self capacitance of the either lines (1) or (2).

The coupling through to shielded cable occurs by two mechanisms. The transfer impedance \(Zt\) and the transfer admittance \(Yt\).
For a tubular shield with circular apertures, the field penetration inside the sheath is caused by diffusion and aperture penetration. An analytical formula developed by Schelkunoff and Vance can be used for the calculation of the transfer impedance [3] & [4].

\[Zt=Zt_{diffu}+Zt_{diffr}\]
\[Zt_{diffu}=R0.(1+i).(e/\delta)/\sinh[(1+i).(e/\delta)]\]
\[Zt_{diffr}=i.w.Lt\]
\[Lt=2.(1-A).\mu_o.d/(\pi^2.d)\]

\(A\) is the optical coverage, \(\sigma\) is the conductivity of the shield, \(e\) is the wall thickness, and \(\delta\) is the skin depth in the shield given by

\[\delta=1/(\pi.f.\mu.\sigma)^{1/2}\]
\(\mu\) is the permeability of the shield.

\(\mu_0=4.\pi.10^{-7}\).
\(d\) is the diameter of the circular aperture
\(D\) is the diameter of the shield
\(R0=1/(2.\pi.r.e.\sigma)\) is the dc resistance of the tube per unit length.
\(r\) is the radius of the shield
A plot of the magnitude of the transfer impedance for a tubular shield with circular apertures is shown in figure 2.

Figure 2  Variation of the transfer impedance of a tubular shield with a circular apertures

At low frequencies, such that \(e/\delta \ll 1\), the magnitude of the transfer impedance is \(R0\). At high frequencies, such that \(|e/\delta| \gg 1\), the magnitude of the transfer impedance decreases very rapidly, so that very little of the high-frequency spectrum is permitted to penetrate to the interior of the shield. The dominance of the diffusion term well below \(20.10^4 Hz\) and the dominance of the \(w.Lt\) term well above \(20.10^4 Hz\).

The solution of the differential equations system can be obtained numerically. However, if the distribution of the disturbing current \(Ip\) along the shield is assumed to be :

\[Ip(z) = Ip_0.e^{(-\gamma p.z)}\]

the resolution of the differential equation (1) is easy.

\(\gamma p\) is the propagation constant of the disturbing wave
In order to understand the mechanisms mentioned in the introduction, we introduce the common and the differential modes as follow:

\[Vc=(v1+v2)/2\]
\[Ic=(I1+I2)/2\]
\[Vd=(V1-V2)/2\]
The currents \( I_c \) and \( I_d \) are solutions of the following matrix equation:

\[
(-d^2 i/dz) - (y).(z).(i) = -(y).(zt).Ip
\]  

\( I_c \) and \( I_d \) are the elements of the matrix \( i \).

The application of the boundary conditions on the impedance’s connected at both ends of the cable leads to the determination of the near-end and far-end voltages of the two modes.

3 Dissymetries

As described in the introduction, we try to explain in this section all parameters which influence most on the differential voltages.

3.1 Shield dissymetry

This dissymetry concerns the direct action of the shield on the two-wire cable by the differential transfer impedance \( Z_{td} \).

We take \( L_{t1} \neq L_{t2} \).

3.2 Dissymetry of loaded network

This type of dissymetry interest the impedance’s connected between conductors 1 or 2 and the shield.

We explain this dissymetry by the follow inequality:

\[ Z_{c10} \neq Z_{c20} \neq Z_{c1L} \neq Z_{c2L} \]

3.3 Dissymetry of the two-wire cable

If the conductors 1 and 2 are not identical or if their position in relation to shield is different, their series impedance per unit length and their shunt admittance per unit length are different.

\( Z_{11} \neq Z_{22} \) & \( Y_{11} \neq Y_{22} \)

4 Results of the numerical simulation

4.1 Model parameters

The values used in this numerical simulation are:

\[
\begin{align*}
L_{11} &= 520 \text{ nH/m} & Z_{11} &= i.w.L_{11} \\
L_{12} &= 44.6 \text{ nH/m} & Z_{12} &= i.w.L_{12} \\
L_{22} &= L_{11} & Z_{22} &= Z_{11}
\end{align*}
\]

\[
\begin{align*}
C_{11} &= 50 \text{ pF/m} & Y_{11} &= i.w.C_{11} \\
C_{12} &= -4.33 \text{ pF/m} & Y_{12} &= i.w.C_{12} \\
& & Y_{21} &= Y_{22}
\end{align*}
\]

\( C_{22} = C_{11} \)

\( Y_{22} = Y_{11} \)

The length of the cable is \( L = 1 \text{ m} \)

The disturbing current is \( I_p = 1 \text{ A} \) (\( I_{po} = 1 \text{ A} \))

4.2 Shield dissymetry

In order to isolate the action of this dissymetry, we take all parameters symmetrical. The value of \( L_{t2} \) is modified in relation to \( L_{t1} \).

\[
\begin{align*}
L_{t1} &= 0.75 \text{ nH/m} \\
L_{t2} &= 0.65 \text{ nH/m}
\end{align*}
\]

The impedance’s of the common mode are equal to \( 10 \text{ K\Omega} \). The evaluation of the differential and common modes voltages is plotted in the figure (3) as a function of frequency, respectively from \( 10 \text{ KHz} \) to \( 1 \text{ GHz} \).

![Figure 3 Variation of the differential mode and common mode voltages at z=0 and z=L](image)

At low frequencies (\( \lambda >> L \)), the magnitude of the differential mode voltage is proportional to disturbing wave.

\[
V_d(0) = V_d(L) = 0.5.i.w.L_{td}.I_{po}.L
\]

\( L_{td} = L_{t1} - L_{t2} \)

The differential mode voltages are proportional to frequency and to length of the cable.

On the other hand, at high frequencies, this evolution is interrupted by the propagation phenomena. We achieve a maximum of magnitude independently of frequency.

4.3 Interior conductors dissymetry

This dissymetry is related to shield two-wires and concern directly their primary parameters. Here, we
try to look for the effect of these parameters to the magnitude of the differential mode voltages.
To simplify, we take the values as follow

\[ \begin{align*}
L11 &= 550 \text{ nH/m} \\
L22 &= 520 \text{ nH/m} \\
L12 &= L21 = 67.9 \text{ nH/m} \\
C11 &= 48.2 \text{ pF/m} \\
C22 &= 50.2 \text{ pF/m} \\
C12 &= C21 = -6.21 \text{ pF/m}
\end{align*} \]

The impedances of the common mode are equals to \(10 \text{ K}\Omega\).
In order to display the effect of this dissymetry, we take the value of \(Ztd\) equal zero. The origin of the differential voltages is caused by mechanisms of the common mode transfer, since we cancelled the direct coupling by \(Ztd\).
The magnitudes of the common and differential voltages are plotted in the figure (4) as a function of frequency.
We distinguish two frequency ranges in this figure.
At low frequencies, the level of the differential mode voltage is very weak in comparison with the common mode voltage. However, this magnitude is proportional to the frequency.
At high frequencies, the differential mode voltage present the same resonance phenomena like in the common mode voltages.

\[ |Vd(0)| = |Vd(L)| = \frac{Z_{22} |Zc01|}{4 \sqrt{\varepsilon_r \mu_0} Zc10^2} \]  

(17)

The conversion factor associated to this dissymetry is:

\[ Tc1 = \frac{C(L11-L12) |Zc10-Zc20|}{4 \sqrt{\varepsilon_r \mu_0} Zc10^2} \]  

(18)

10
-12
10
-10
10
-8
10
-6
10
-4
10
-2
10
0
10
2

\[ |Vc| \]
\[ |Vd| \]

Figure 4 Variation of the common mode and differential mode voltages “Conductors dissymetry”

At low frequencies, we find :
\[ Vd0 = z_{12}.Zc.L^2.Ipo/(2.Zc01) \]  

(15)

\(Ztc\) is the transfer impedance of the common mode.
If we define a modal conversion coefficient \(Tc\) associated to this dissymetry, we show that:

\[ Tc = |L11-L22|.\pi.L_0/(2.Zc01) \]  

(16)

4.4 Dissymetry of the loaded network

The symmetry condition of the common mode impedance’s fixed in both extremities of the cable is not usually realised. So, it is indispensable to evaluate the effect of this dissymetry on the differential mode voltages.
To simplify, we suppose that \(Ztd\) is equal to zero and the interior conductors are symmetrical.

\[ Zc10 = 11 \text{ K}\Omega \]
\[ Zc20 = Zc1L = Zc2L = 11 \text{ K}\Omega \]

The magnitude of the common mode and differential mode voltages are plotted in the figure (5).
We see that the differential mode voltage is a constant fraction of the common mode voltage. So, we have the same rejection phenomena like in the previous dissymetry.
At low frequencies, the magnitude of the differential voltage is bigger than the magnitude of the common mode voltage.
We find :

\[ |Vd(L)| = |Vc(0)| \]  

(17)

The conversion factor associated to this dissymetry is:

\[ Ti1 = \frac{C(L11-L12).|Zc10-Zc20|}{4 \sqrt{\varepsilon_r \mu_0} Zc10^2} \]  

(18)

\[ |Vc| \]
\[ |Vd| \]

Figure 5 Variation of the common mode and the differential mode voltages “Dissymetry of the loaded network”

\(\varepsilon_r\) is permittivity of the interior medium of the cable.
\(C\) is the velocity of the light.
This factor is independent of the frequency and the length of the cable.
5 Conclusion
The dissymetry of shield is always present in most of usual cables.
If the two-wire cable is perfectly symmetrical and the network impedance's of common mode is perfectly in equilibrium, the differential mode voltage has for origin the shield dissymetry. In this case, the knowledge of the transfer impedance allow us to evaluate the differential mode voltages. On the other hand, if the network of impedance's is not in equilibrium, we will have a transfer from the common mode to differential mode.
When $k << 1$, the transfer mechanisms will have a negligible effect with regard to the dissymetry of shield. This condition is almost satisfied for frequencies lower than the resonance frequency of common mode voltage.
Otherwise when $k >> 1$, the differential mode voltages will be a fraction from the common mode voltages.

References:
[1] THOMAS KASDEPKE and all, "Multiconductor transmission line theory for shielded cables under consideration of the environment, EMC’94 ROMA