Multi-target tracking in the framework of possibility theory

Mourad Oussalah
K.U. of Leuven, PMA, Celestjnenlaan 300B
B-3001 Heverlee, Belguim
Tel: +32 16 32 25 28. Fax: +32 16 32 29 87

Abstract: Target tracking is an attempt to estimate parameters that are changing with time. Traditionally, multi-target tracking systems utilize algorithms like Kalman filter and its derivatives Probabilistic Data Association Filter (PDAF) for a single target and Joint Probabilistic Data Association Filter (JPDAF) for multitarget as deeply investigated by by Bar-Shalom and Foortmann [3]. In this paper one considers a possibilistic re-formulation of JPDAF in the setting of possibility theory [11]. The proposed approach is based on both the manipulation of fuzzy quantities and the similarity calculations, which allows a re-formulation of joint association probabilities. A simulation example will be shown.

Introduction:

Target tracking in cluttered environment is a difficult problem because it referees to two distinct level of uncertainty. The first one is inherent to the measurement acquisition which is modelled in the probabilistic setting as a continuous random noise added to the current estimate, and conveniently propagated through the mathematical model of the considered system. The second one refers to the origin of measurement. In other words, given the current measurement, we can not assert that it is originated from the considered target or another target, or it is just a false alarm, i.e., clutter. This second level may lead to a completely wrong result. The earliest work in this topic uses a nearest-neighbour approach. That is, if there is more one contact-to-track association for a given update, the contact nearest the predicted measurement is used. This, of course, makes a "hard" decision and does not account for possibly incorrect decision. Bar-Shalom, among others [2, 3] have deeply investigated a family of algorithms where is ascribed a probability value to each association target-measurement when several measurements fail in the validation gate. This includes PDAF for single target tracking, JPDAF for multiple targets. The latter acts as a natural extension of the former where all the joint events are considered. The process was simplified by considering just the feasible joints events, those for which there is at most one hit per target and no two tracks associated with the same hit. These algorithms, among others have successfully demonstrated their feasibility in high cluttered situations including air traffic management and military applications [3].
In contrast, even if fuzzy logic [4,11]has demonstrated very successful application in control and engineering designs, its use in tracking problems is still quite limited, and only few works may be founded in this topics. Horton and Jones [10] have proposed to use fuzzy logic extended rule set for multi-target tracking. The approach described here uses a fruitful combination of JPDAF and possibility theory.

Joint Probabilistic Data Association

A detailed derivation of JPDA filter can be found in [3, 8] and is just briefly described here. Formally, assumes a linear system with gaussian and zero mean random noise as

$$x(k|k) = A(k).\ x(k|k-1)+n_1(k), \quad (1)$$
$$y(k|k) = C(k).\ x(k|k-1)+n_2(k). \quad (2)$$

The update equation of Kalman filter becomes

$$\hat{x}^t(k \mid k) = \hat{x}^t(k \mid k-1) + W^t(k).\nu^t(k) \quad (3)$$

And,

$$\nu^t(k) = \sum_{i=1}^{n} \beta_i^t(k).\nu_i^t(k) = \sum_{i=1}^{n} \beta_i^t(k).\left[z_i(k) - C.\hat{x}^t(k \mid k-1)\right] \ (4)$$

Where $\beta_i^t(k)$ is the probability that the i-th hit comes from target t in track (the superscript t stands for a target), $W^t(k)$ is the Kalman gain and z_i is the i-th hit at time k.

The update covariance matrix is given by

$$P^t(k \mid k) = \beta_0^t(k).P^t(k \mid k-1) + \left[1 - \beta_0^t(k)\right]P_c^t(k \mid k-1) + \tilde{P}^t(k)$$

(5)

Where

$$\tilde{P}^t(k \mid k) = W^t(k).\left[\sum_{i=1}^{n} \beta_i^t(k).\nu_i^t(k).(\nu_i^t(k))' - \nu^t(k).(\nu^t(k))'\right].(W^t(k))$$

$$P_c^t(k \mid k) = \left[I - W^t(k).C(k)\right]P^t(k \mid k-1) \quad (6)$$

$\beta_0^t(k)$ corresponds to the probability that none of the measurements is originated from the target t.

The probability calculations of the JPDA filter is first done by determining the joint probability of all feasible joint events. This supposes that all the considered hits fail into a validated gate up to some statistical threshold ###. This means that the validated measurement z fails in gate, which is set up based on the predicted measurement from the target $\hat{z}(k \mid k-1)$ and the associated covariance S(k):

$$\left[z(k) - \hat{z}(k \mid k-1)\right]' S^{-1}.\left[z(k) - \hat{z}(k \mid k-1)\right] \leq \gamma \ (7)$$

A feasible joint event is a non-conflicting association of current targets with hits. This assumes that: i) no two tracks are associated with the same hits; ii) there is at most one hit per target; iii) hits which are not associated with hits are supposed clutter. Additional requirements pertains to JPDA construction may be summarized as: i) there exists some detection probability P_D less than one that target will be detected; ii) the measurement-to-target association probabilities are computed jointly across the targets; ii) the association probabilities are computed only for the latest set of measurements; iii) The state of the targets conditioned on the past observations are assumed independent; iv) the past is summarized by an approximate gaussian sufficient statistic -specified by state estimate and covariance for each target-.

Suppose that ###(k) corresponds to the feasible joint event, Z^k is the history of hits up until time k, i.e., Z^k = {Z(1), Z(2),…,Z(k)}. The probability of an event given its hits' history is

Let N[###$_i$(k)] be the normal distribution with zero mean and covariance equals to a covariance matrix of ###$_i$ (k), ### is the number of clutter points, τ_i is a binary hit indicator, which takes a value one if the i-th hit is assigned to a track and zero otherwise. Let δ^t be a target indicator indicating whether there is a hit associated with a target t (δ^t =1), or not (δ^t =0). Let be the volume of the extension gate. Then, the probability of the joint event (see [3,8] for a full proof), is given by

$$P(\theta(k) \mid Z^k) = \frac{1}{c}.\phi!\,V^{-\phi}.\prod_{i=1}^{m}\left[N[\nu_i(k)]\right]^{\tau_i}.\prod_{j=1}^{Nt}\left[P_D^j\right]^{\delta^t}\left[1 - P_D^j\right]^{1-\delta^t}$$

where Nt is the number of targets, P_D^t is the probability of detection of target t.

Thus, if ###(###(k)) indicates whether track t is associated with hit j in ###(k).

$$\beta_j^t(k) = \sum_{\theta(k)} P(\theta(k) \mid Z^k).\omega(\theta(k)) \quad (11)$$

A had hoc JPDAF formulation to get very easily β_j^t's values was pointed out by Fitzgerald [7]as

$$\beta_j^t = \frac{G_{tj}}{S_t + S_j - G_{tj} + B} \quad (12)$$

Where

$$G_{tjj} = N[\nu_j(k)]\ ;\ S_t = \sum_{j=1}^{m} G_{tj}\ ;\ S_j = \sum_{t=1}^{Nt} G_{tj}$$

B is a constant, which depends on clutter density.

Possibilistic evaluation of joint probabilities

The basis idea behind this evaluation is related to the qualification of this probability. Namely, the joint evaluation β_j^t can be viewed as a degree of pattern matching between measurement j and target t, or a confidence attributed to the association (measurement j , target t).

First, let us recall that a possibility distribution π_x may be viewed as the membership function restricting the fuzzy set of possible values of a variable x taking its values in the universe of discourse X (referential set). That is, π_x (s)=1 means that the value s may completely be a candidate estimate for the variable x, whereas π_x (s)=0 means that the assertion "x=s" is completely impossible. This corresponds in some manner to model the imprecision pertaining the value ascribed to a variable x. The extent of s values for which $\pi_x(s)$ takes non zero element represents the support of the distribution. The extent for which π_x (s)=1 represents the core of the possibility distribution. Usually, for the simplicity purpose, the trapezoidal shape was widely used. This defines two levels of imprecision: the support determines extent to which, it is impossible that values outside it are considered in the sequel, while the core defines the region where the most likely values should lie. Possibility distribution is related to ordinary set by the notion of α-cut set determining the set of acceptable values up to degree α. That is,

$$(\pi_x)_\alpha = \{s, s \in X, \pi_x(s) \geq \alpha\}$$

(see for instance [4] for more details about possibility theory and related notions).

Now, in order to handle the tracking problem in the possibility framework, one may notice that the approach described here is not completely separated with standard JPDAF but considers the possibility approach only in the heavy step of JPDAF, i.e., the calculation of the joint probabilities. Thus, a straightforward combination of both probabilistic and possibilistic framework is considered. To this end, one has first to get possibility distributions. For this purpose, considering the probability of measurement according to a given target $N[\nu_j(k)]$, one constructs a possibility distribution using a probability -possibility distribution. The continuous transformation pointed out by Dubois, Prade and Sandri [6] was used.

Let p be a continuous unimodal probability distribution, such that p is non-decreasing on [a,xo] and non-increasing on [xo,b]. Let f be a function from [a,xo] onto [xo,b] such that f(x)=max{y, p(y) ≥ p(x)}. The possibility distribution π, which minimizes the integral of π over [a,b] and dominates p is defined as

$$\pi(x) = \pi(f(x)) = \int_{-\infty}^{x} p(y)dy + \int_{f(x)}^{+\infty} p(y)dy \quad (13)$$

In the particular case of normal distribution, a rational approximation of bounded interval [a,b] is $[\bar{x} - 3\sigma_x, \bar{x} + 3\sigma_x]$ where $\bar{x}$ and σ_x denotes respectively the mean and standard deviation of the gaussian. Also, it is easy to check that $xo = \bar{x}$ and $f(x) = 2\bar{x} - x$. Therefore,

$$\pi(x) = \pi(f(x)) = \int_{-\infty}^{x} p(y)dy + \int_{2\bar{x}-x}^{+\infty} p(y)dy \quad (14)$$

For x=xo, it is clear that π(xo)=1.

Once the distributions pertaining to each pair measurement-target were obtained, we are able to perform a similarity like operation. Many researchers [1, 12] proposed a similarity measure to quantify at what extent we can say that a possibility distribution π_1 is equal to π_2 with regard to some chosen criterion.

In order to take into account the overlapping between distributions in one hand and the separations of distribution centers in other one, one proposes to use a similarity index introduced in [12].

Let π_1 and π_2 be two possibility distributions, $[a_{1i},b_{1i}]$, $[a_{2i},b_{2i}]$ be two intervals corresponding to α_i level. Let $[\beta_1,\beta_2]$ be the narrowest interval encountering π_1 and π_2.

Let $\Delta(\alpha_i)$ be the normalized distance between the two intervals of ###$_i$-cut defined as

$$\Delta(\alpha_i)=\frac{|a_{1i}-a_{2i}|+|b_{1i}-b_{2i}|}{2(\beta_1-\beta_2)}$$

where $\alpha_0=0$, $\alpha_{n-1}=n$ and $\alpha_n=1$

The similarity measure SM is then defined as follows:

$$SM = \chi.\Delta_1 + (1-\chi).\Delta*.$$

Where

$$0 \le \chi \le 1$$

$$\Delta* = \frac{1}{n+1}.\sum_{k=0}^{k=n}\Delta(\alpha_k).$$

$$\Delta* = \Delta(\alpha_1).$$

is a parameter modeling the importance given to the overlap relative to that of the separation between distribution centers.

Δ_1 is a normalized summation of the dissemblance of the two distributions at each α-cut level. $\Delta*$ is a normalized separation of the centers of the two distributions.

The choice of the parameter χ is related to the probabilistic threshold γ in (7). This means that the similarity index may be viewed as selection tool to obtain a set of feasible matrices as JPDAF does. The next step consists to perform Fitzgerald's formulation given in (12) (with simplification B=0). For this end, one uses a parametrized representation of possibility distribution in terms of L-R representation (see [4]). That is, normal distribution leads to point valued possibility after the use of transformation (14) (which is quite pre-expected) since the transformation preserves all the symmetries ascribed to a probability distribution, and the obtained shape is quite close to the initial one. Therefore, any possibility distribution may be represented as $\pi_x = (m,\alpha,\beta)_{LR}$ such that

$$\pi_x(s) = \begin{cases} L(\frac{m-s}{\alpha}) & \text{if } s < m \\ 1 & \text{if } s = m \\ R(\frac{s-m}{\beta}) & \text{if } x > m \end{cases}$$

Where in our case L=R, and provided from (14). One also uses approximated formulas for division of LR representations. That is, for

$$\pi_{x_1} = (m_1,\alpha_1,\beta_1)_{LR} \text{ and } \pi_{x_2} = (m_2,\alpha_2,\beta_2)_{LR}$$

$$\pi_{x_1} + \pi_{x_2} = (m_1+m_2,\alpha_1+\alpha_2,\beta_1+\beta_2)_{LR}$$

$$\pi_{x_1} - \pi_{x_2} = (m_1-m_2,\alpha_1-\beta_2,\alpha_1-\alpha_2)_{LR}$$

$$\pi_{x_1} \div \pi_{x_2} \approx (\frac{m_1}{m_2},\frac{m_1.\beta_2+\alpha_1.m_2}{m_2{}^2},\frac{\alpha_2.m_1+\beta_1.m_2}{m_2{}^2})_{LR}$$

Notice that the performance of the previous formulation is quite fast and leads at each time increment k to a possibilistic quantity pertaining to each joint probability β_j^t. Thus to generate a single value, a defuzzification procedure is required. Moreover, one may also go further such that β_j^t is kept in its possibilistic form and perform the relation (3) and (5) using L-R representation. This is not considered in this paper. Finally, the general scheme is described in fig. 1

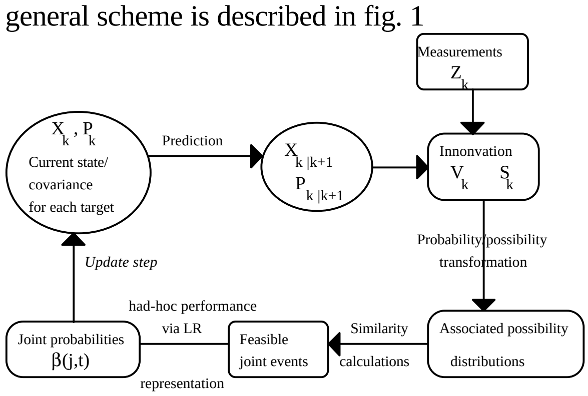

Figure 1. General Scheme

Simulations setting

The simulations were done for two and three targets modelled as constant velocity objects in a plane with process noise that can account for slight changes in the velocities. The two trajectories cross after few seconds. The state vector consists in position and velocity components $X = [x\ \dot{x}\ y\ \dot{y}]'$.

The system corresponds to the constant velocity moving and the measurement to the noised true target positions (x,y) with $R_{ii}=0.0225$ and $Q_{ii}= 4.10^{-4}$, and then adding clutter measurements (supposed Poisson distributed with $\lambda=1$). Initial estimate of the state was obtained by two points differencing of the observations with a corresponding covariance matrix [3]. The state estimations are pictured with circles. The true measurements and true trajectories are also pictured. Notice that the obtained tracking is quite acceptable. Moreover, this result is very close to those obtained by Chang and Bar-Shalom using JPDAF with possibly unresolved measurements.

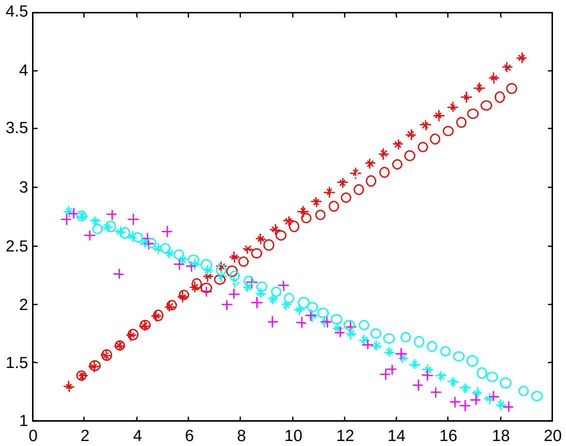

Figure 2. Example of target crossing tracking

In figure 3, one considers errors behaviour over a time for both targets using standard JPDAF and the new one after running several Monte Carlos simulations at each increment time. The result shows that outcomes provided by both algorithms are quite close to each other.

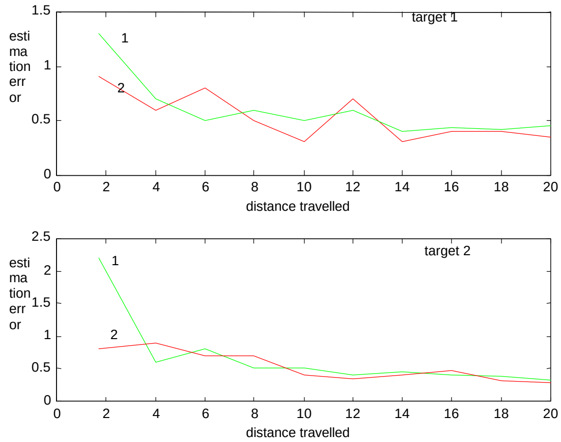

Figure 3. Error behaviour over time for both target. 1stands for possibility-based approach and 2 for JPDAF based approach.

Conclusion

In this paper, we have focused on a *fruitely* combination of probabilistic and possibilistic settings in order to perform a tracking task in the light of Joint Probabilistic Data Association Filter. Our approach considers the modification of joint probabilities evaluations by considering the similarity between possibility distributions obtained via probability transformation for each pair target-measurement. Then feasible events are obtained by means of similarity performances. Next, the joint probabilities are evaluated considering Fitzgerald's had hoc formulation, and finally come back to the standard JPDAF formulation. An example of crossing targets was proposed.

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