Transient stability of power systems and energy functions

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Abstract:- A power system is continually experiencing disturbances. Event disturbances include generator outages, short-circuits caused by lightning or other fault conditions, sudden large load changes, or a combination of such events. Load disturbances, on the other hand, are the small random fluctuations in load demands. Transient stability analysis is concerned with a power system´s ability to reach an acceptable steady-state following an event disturbance. Stability analysis is the study of whether the postfault trajectory will converge to an acceptable steady-state as time passes.

Time-domain approach examines the behavior of the generators, one determines whether stability has been maintained or lost. In contrast to the time-domain approach, direct methods determine system stability based on energy functions.

Key-words:- Nonlinear systems, energy function theory, system stability.

1 Nonlinear systems

We next review some relevant concepts from nonlinear dynamics systems theory. Let

$$\frac{dX(t)}{dt} = f(X(t)) \qquad (1)$$

be the power system model under study, where the state vector X (t) belongs to the Euclidean space R^n, and the function $f : R^n \to R^n$ satisfies the sufficient condition for the existence and uniqueness of solutions. The solution curve of (1) starting from X at t=0 is called a trajectory, denoted by Φ: $R \to R^n$. Note that $\Phi(X,0)=X$.

The concepts of equilibrium point (e.p.), stable and unstable manifolds are important in dynamical system theory. A detailed discussion of these concepts and their implications may be found in [1], [2], [3].

A state vector X is called an equilibrium point of system (1) if $f(X) = 0$. We denote E to be the set the equilibrium points of the system. A state vector X is called a regular point if it is not an equilibrium point. We say that an equilibrium point of (1) is hyperbolic if the Jacobian of $f(X)$ at X, denoted $J_f(X)$, has no eigenvalues with a zero real part. For a hyperbolic equilibrium point, it is a asymptotically stable equilibrium point if all the eigenvalues of its corresponding Jacobian have negative real parts; otherwise it is an unstable equilibrium point. If the Jacobian of the equilibrium point X has exactly one eigenvalue with positive real part, we call it a type-one equilibrium point. Likewise, X is called a type-k equilibrium points if its corresponding Jacobian has exactly k eigenvalues with positive real part. It will be assumed that all the equilibium points of system (1) are hyperbolic.

Let X be a hyperbolic equilibrium point. Its stable and unstable manifolds, $W^s(X)$ and $W^u(X)$, are defined as follows:

$$W^s(X) = \{X \in R^n : \Phi(X,t) \to X \quad as \quad t \to \infty\}$$
$$W^u(X) = \{X \in R^n : \Phi(X,t) \to X \quad as \quad t \to -\infty\} \qquad (2)$$

Both stable and unstable manifolds are invariants set.

For an asymptotically stable equilibrium point, it can be shown that exists a number d >0 such that

$$\|X_0 - X\| < d \quad implies \quad \Phi(X_0,t) \to X \quad as \quad t \to \infty$$

If d is arbitrarily large, then X is called a global state equilibrium point. There are many physical systems containing stable equilibrium points but not global stable equilibrium points. A useful concept for these kinds of systems is that of stability region, also called region of attraction.

The stability region of a stable equilibrium point X_s is defined as

$$A(X_s) = \{X \in R^n : \lim_{t\to\infty} \Phi(X,t) = X_s\} \qquad (3)$$

From a topological point of view, the stability region $A(X_s)$ is a open, invariant, and connected set. The boundary of stability region $A(X_s)$ is called the stability boundary of X_s and will be denoted by $\P A(X_s)$. The stability boundary is topologically an (n-1) dimensional closed and invariant set.

2 Direct methods

Heuristic arguments of the applicability of the direct methods can be derived from the classical equal area criterion. Consider one-machine-infinite-bus system described by the following equations:

$$\frac{d\delta}{dt} = \omega$$

$$M\frac{d\omega}{dt} = -D\omega - P_0 \sin\delta + P_m \qquad (4)$$

There are three equilibrium points lying within the range of $\{(\delta,\omega) = -\pi < \delta < \pi, \omega = 0\}$, and they are $(\delta_s,0) = (\arcsin(P_m/P_o),0)$ which is a stable equilibrium, $(\delta_1,0) = (\pi - \arcsin(P_m/P_0),0)$, $(\delta_2,0) = (-\pi - \arcsin(P_m/P_0),0)$, which are unstable equilibrium points. We consider the following function, termed energy function

$$E(\delta,\omega) = \frac{1}{2}M\omega^2 - P_m\delta - P_0\cos\delta \qquad (5)$$

The energy function can be divided into kinetic energy $K(\omega)$ and potential energy functions $U(\delta)$,

$$E(\delta,\omega) = K(\omega) + U(\delta)$$

where

$K(\omega) = \frac{1}{2}M\omega^2$ and $U(\delta) = -P_m\delta - P_0\cos\delta$.

We notice that function $U(\delta)$ reaches its local maximum at the unstable e.p.'s δ_1 and δ_2.

The system is two-dimensional (2-D). Hence, the stability region of (δ_s,0) is 2-D and the stability boundary $\delta A(\delta_s,0)$ is composed of the stable manifold of the u.e.p. (δ_1,0) and the stable manifold of the u.e.p. (δ_2,0). The u.e.p. (δ_1,0) has the lowest energy function value among all the u.e.p.'s on the stability boundary $\P A(\delta_s,0)$. Hence, (δ_1,0) is termed the closest u.e.p. of (δ_s,0) with respect to the energy function $U(\delta)$. We notice that:

- The intersection between $A(\delta_s,0)$ and the angle space $\{(\delta,\omega): \delta = R, \omega = 0\}$ is $A_\delta = \{(\delta,\omega): \delta \in [\delta_2,\delta_1], \omega = 0\}$.
- The boundary of this one-dimensional region A_δ is composed of two points δ_1 and δ_2, where (δ_1,0) and (δ_2,0) are the u.e.p.'s on the stability boundary $\P A(\delta_s,0)$.
- These two points δ_1 and δ_2 are characterized as being the local maxima of the potential energy function $U(\delta)$.

The stability for this simple system can be directly assessed on the basis of the energy function $U(\delta)$: if a given postfault trajectory (δ,ω), after reaching a local maximum value of $U(\delta)$, δ starts to decrease, then the stability of this postfault trajectory is assured.

The basis of direct methods for the stability assessment of a postfault system is knowledge of the stability region: if the initial condition of the postfault system lies inside the stability region of a desired postfault stable equilibrium point, then one can ensure without performing any numerical integrations that the ensuing postfault trajectory will converge to the desired point. Therefore, knowledge of stability region plays an important role in direct methods.

This section reviews some analytical results associated with energy function theory which enable one to characterize limit sets, stability boundaries and stability regions. A more comprehensive development of energy function theory can be found in [4]. This section also shows how to use energy functions to estimate stability regions.

We say a function $V: R^n \to R$ is an energy function for a system (1) if the following three conditions are satisfied:

1. The derivative of the energy function $V(X)$ along any system trajectory $X(t)$ is nonpositive, i.e., $0 \geq dV(X(t))$.
2. If $X(t)$ is a nontrivial trajectory, i.e. $X(t)$ is not an equilibrium point (e.p.), then along the nontrivial trajectory $X(t)$ the set $\{t \in R : dV(X(t))/dt = 0\}$ has measure zero in R.
3. If a trajectory $X(t)$ has a bounded value of $V(X(t)$ for $t \in R^+$, then the trajectory $X(t)$ is also bounded. Stating this in brief: If $V(X(t))$ is bounded, then $X(t)$ is also bounded.

Property 1. indicates that the energy is nonincreasing along its trajectory, but does not imply that the energy is strictly decreasing along its trajectory. There may exist a time interval $[t_1,t_2]$ such that $dV(X(t))/dt = 0$ for $t \in [t_1, t_2]$. Properties 1. and 2. imply that the energy is strictly decreasing along any system trajectory. Property 3. states that along any system trajectory the energy function is a proper map but its energy need not be a proper map for the entire state space. Obviously, an energy function is not a Lyapunov function.

In general, the behaviors of trajectories of general nonlinear dynamical systems could be very complicate, unless the underlying dynamical system has some special properties. For instance, every trajectory of system (1) having an energy function has only two modes of asymptotic behaviors: it either converges to an equilibrium point or goes to infinity (becomes unbounded) as time increases or decreases.

3 Direct methods for network-preserving models

Traditionally, direct methods have been based on the network-reduction model where all the load representations are expressed in constant impedance and the entire network representation is reduced to the generator internal buses.

There are two advantages of using the network-preserving power system models for direct stability analysis. From a modeling viewpoint, it allows more realistic representations of power system components, especially load behaviors. From a computational viewpoint, it allows the use of the sparse matrix technique for the development of faster solution methods for solving nonlinear algebraic equations involved in direct methods [5]. In this section we discuss direct methods for network-preserving power systems models.

The first network-preserving model was developed by Bergen and Hill [6], who assumed frequency dependent real power demands and constant reactive power demands. Narasimhamurthi and Musavi [7] moved a step further by considering constant real power and voltage dependent reactive power loads. Padiyar and Sastry [8] have included nonlinear voltage dependent loads for both real and reactive powers. Tsolas, Araposthasis and Varaiya [9] developed a network-preserving model with the consideration of flux decay and constant real and reactive power loads. An energy function for a network-preserving model accounting for static var compensators and their operating limits was developed by Hiskens and Hill [10]. For purpose of illustration, we next discuss the Tsolas-Arapostathis-Varaiya model. In this model, each generator is represented by the one-axis-model. The transmission network representation is preserved. The complete dynamic equations are described in the following way.

- Internal Generator Bus: one-axis generator model. For i = 1,...,n,

$$\frac{d\delta_i}{dt} = \omega_i$$

$$M_i \frac{d\omega_i}{dt} = -D_i\omega_i + P_{mi} - P_{ei} \qquad (6)$$

$$T'_{do} \frac{dE'_{qi}}{dt} = -\frac{x_{di}}{x'_{di}} E'_{qi} + \frac{x_{di} - x'_{di}}{x'_{di}} V_i \cos(\delta_i - \theta_i) + E_{fi}$$

where

$$P_{ei} = \frac{1}{x'_{di}} E'_{qi} V_i \sin(\delta_i - \theta_1) + \frac{x'_{di} - x_{qi}}{2x_{qi} - x'_{di}} V_i^2 \sin[2(\delta_i - \theta_i)]$$

- External Generator Bus: For i = 1,...,n,

$$P_{ei} = \sum_{j\neq i}^{n+1} V_i V_j \left(B_{ij} \sin(\theta_i - \theta_j)\right) + G_{ij} \cos(\theta_i - \theta_j) +$$

$$\sum_{l=n+2}^{n+m+1} V_i V_j \left(B_{il} \sin(\theta_i - \theta_l)\right) + G_{il} \cos(\theta_i - \theta_l) \qquad (7)$$

$$0 = \frac{x'_{di} + x_{qi}}{2x_{qi}x'_{di}} V_i^2 - \frac{E'_{qi}V_i}{x'_{di}} \cos(\theta_i - \delta_i) - \frac{x'_{di} - x_{qi}}{2x_{qi}x'_{di}} V_i^2 \cos[2(\theta_i - \delta_i)]$$

$$- \sum_{j=1}^{n+1} V_i V_j \left(B_{ij} \cos(\theta_i - \theta j)\right) + G_{ij} \sin(\theta_i - \theta_j) -$$

$$\sum_{l=n+2}^{n+m+1} V_i V_l \left(B_{il} \cos(\theta_i - \theta_l)\right) + G_{il} \sin(\theta_i - \theta_l) \qquad (8)$$

- Load Bus: For $k = n$+2, ..., n+m+1,

$$P_k^d - D_k \frac{d\theta}{dt} = \sum_{j=1}^{n+m+1} V_k V_j \left(B_{kj} \sin(\theta_k - \theta_j)\right) + G_{kj} \cos(\theta_k - \theta_j)$$

$$Q_k^d(V_k) = -\sum_{j=1}^{n+m+1} V_k V_j \left(B_{kj} \cos(\theta_k - \theta_j)\right) + G_{kj} \sin(\theta_k - \theta_j) \qquad (9)$$

The above model is a network-preserving power system transient stability model. In general, network-preserving models are mathematically described by a set of differential and algebraic equations (DAE's):

$$\frac{dx}{dt} = f(x,y)$$
$$0 = g(x,y) \qquad (10)$$

where $x \in R^n$ and $y \in R^m$. Here differential equations describe generator and/or load dynamics while algebraic equations express the power flow equations at each bus. The above DAE system can be interpreted as an implicitly dynamical system on the algebraic manifold L

$$L = \{(x,y) : g(x,y) = 0\} \qquad (11)$$

It has been shown that a DAE in general can be reduced locally to a ODE. However, despite the strong analogy between DAE's and ODE's, major differences do exist. For instance, once the trajectory intersects the following singular surface S,

$$S = \left\{ \left(x,y : (x,y) \in L, \Delta(x,y) = \det \frac{\partial}{\partial y} g(x,y) = 0 \right) \right\} \qquad (12)$$

a DAE can not be reduced to an ODE. From a dynamical viewpoint, complicated dynamic behaviors will occur in the vicinity of S, most of the trajectories near singular surfaces will not exist beyond singular surfaces; only when the initial conditions of DAE satisfy certain conditions, the trajectories can be extended further. Due to their complicated dynamics near singular surfaces, DAE systems are difficult to analyze and only some phenomena are completely understood. We next discuss some analytical results for DAE systems that are useful for stability analysis of network-preserving power system models.

If the Jacobian $(\P/\P y)/g(x,y)$ is nonsingular, i.e., the system is on the regular part of the DAE (10), then by the Implicit Function Theorem, the system equations (10) are locally equivalent to the following equations:

$$\frac{dx}{dt} = f(x,y)$$
$$\frac{dy}{dt} = -\left(\frac{\partial}{\partial y} g(x,y) \right)^{-1} \frac{\partial}{\partial x} g(x,y) f(x,y) \qquad (13)$$

The existence and uniqueness of solutions of DAE in a neighborhood N of the initial conditions can be guaranteed provided functions f and g are smooth and the Jacobian $(\P/\P y)g(x,y)$ has a full rank on N. An equilibrium point of system is a point such that $f(x,y) = 0$ and $g(x,y) = 0$. A regular equilibrium point is called a type-k equilibrium point if the corresponding equilibrium point of system (13) is a type-k equilibrium point. The stability of an equilibrium of the DAE (10) can be analyzed by using a local energy function.

All existing network-preserving models can be written as a set of general differential-algebraic equations of the following compact form:

$$0 = -\frac{\partial}{\partial u} U(u,w,x,y) + g_1(u,w,x,y)$$
$$0 = -\frac{\partial}{\partial w} U(u,w,x,y) + g_2(u,w,x,y)$$
$$T \frac{\partial x}{\partial t} = -\frac{\partial}{\partial x} U(u,w,x,y) + g_3(u,w,x,y)$$
$$\frac{\partial y}{\partial t} = z \qquad (14)$$
$$M \frac{\partial z}{\partial t} = -Dz - \frac{\partial}{\partial y} U(u,w,x,y) + g_4(u,w,x,y)$$

where $u \in T^k$ and $w \in R^l$ are instantaneously variables while $x \in R^n$, $y \in T^n$ and $z \in R^n$ are state variables. T is a positive definitive matrix and M and D are diagonal positive matrices. $g_1(u,w,x,y)$, $g_2(u,w,x,y)$, $g_3(u,w,x,y)$ and $g_4(u,w,x,y)$ are the vector field representing the effects of the transfer conductance in the network Y-bus matrix.

To avoid an awkward analysis of the DAE representation, the algebraic equations can be treated as the limiting equation of the singularly perturbed differential equations. The compact representation of the network-preserving model thus becomes:

$$e_1 \frac{\partial u}{\partial t} = -\frac{\partial}{\partial u} U(u,w,x,y) + g_1(u,w,x,y)$$
$$e_2 \frac{\partial w}{\partial t} = -\frac{\partial}{\partial w} U(u,w,x,y) + g_2(u,w,x,y)$$
$$T \frac{\partial x}{\partial t} = -\frac{\partial}{\partial x} U(u,w,x,y) + g_3(u,w,x,y)$$
$$\frac{\partial y}{\partial t} = z \qquad (15)$$
$$M \frac{\partial z}{\partial t} = -Dz - \frac{\partial}{\partial y} U(u,w,x,y) + g_4(u,w,x,y)$$

where $\hat{I}_1$ and $\hat{I}_2$ are sufficiently small positive numbers.

If $g_1(u,w,x,y)$, $g_2(u,w,x,y)$, $g_3(u,w,x,y)$ and $g_4(u,w,x,y)$ are zero, the compact representation of the network-preserving model will become:

$$
\begin{aligned}
e_1 \frac{\partial u}{\partial t} &= -\frac{\partial}{\partial u} U(u,w,x,y) \\
e_2 \frac{\partial w}{\partial t} &= -\frac{\partial}{\partial w} U(u,w,x,y) \\
T \frac{\partial x}{\partial t} &= -\frac{\partial}{\partial x} U(u,w,x,y) \\
\frac{\partial y}{\partial t} &= z \\
M \frac{\partial z}{\partial t} &= -Dz - \frac{\partial}{\partial y} U(u,w,x,y)
\end{aligned}
\qquad (16)
$$

If we define

$$W(u,w,x,y,z) = \frac{1}{2} z^T M z + U(u,w,x,y) \qquad (17)$$

then $W(u,w,x,y,z)$ is an energy function. Indeed, by differentiating $W(u,w,x,y,z)$ along the trajectory, one has (18)

$$\frac{\partial W(u,w,x,y,z)}{\partial t} = \frac{\partial W^T}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial W^T}{\partial w}\frac{\partial w}{\partial t} + \frac{\partial W^T}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial W^T}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial W^T}{\partial z}\frac{\partial z}{\partial t}$$

$$= -e_1^{-1}\frac{\partial U^T}{\partial u}\frac{\partial U}{\partial u} - e_2^{-1}\frac{\partial U^T}{\partial w}\partial U \partial w - \frac{\partial U^T}{\partial x} T^{-1}\frac{\partial U}{\partial x} - z^T D z \le 0$$

Therefore, the condition 1. of the energy function is satisfied. Suppose that there is an interval $t \in [t_1,t_2]$ such that $\P W(u(t),w(t),y(t),z(t))/\partial t=0$. It follows from (16) and (18) that $z(t)=0$ and $\P u/\P t=\P w/\P t=\P x/\P t=0$ for $t \in [t_1,t_2]$. This indicates that $y(t)$ is a constant for $t \in [t_1,t_2]$. It then follows that the system state is at an equilibrium point. Thus condition 2. of the energy function also holds. We can employ similar arguments used for the network-reduction model to show that condition 3. is also true; here we omit the detailed proof. Note that the additional terms in $\partial W(u,w,x,y,z)/\partial t$ as compared with those in the network-reduction model, are related to the energy dissipation at load buses. Morever, as $\hat{I}$ approaches zero, these terms become dominant in $\partial W(u,w,x,y,z)/\partial t$. This observation also shows the important role of load models for stability analysis. When the network transfer conductance is not negligible, a general expression of exact energy functions does not exist. In this case, path-dependent numerical energy function may prove adequate. This is similar to the case for network-reduction models. However, the transfer conductances of the network-preserving models are usually much smaller in relative value than that of the network-reduction models. This physical property makes numerical energy functions for network-preserving models 'close' to exact energy functions. This illustrates another advantage for using network-reduction models instead of using network-reduction models.

4 Results

The simulation results presented in this section are on the IEEE 173 bus test system in WSCC format. Dynamic data is needed to run the time domain simulations using the TEFTS program [11].

First, an initial load flow is run based on the power flow data in the IEEE 173-bus test system to compute the system`s stable equilibrium point (s.e.p.). Then, a time domain simulation is carried out using the dynamic data, by perturbing the s.e.p. with a 4 per cent change in the values of some of the system variables. The simulation time is 0.1 s. The previous case is repeated, but the perturbation is produced by applying a solid three-phase fault at bus 127 which is cleared at 0.02 s, and simulation time is reduced to 0.03s.

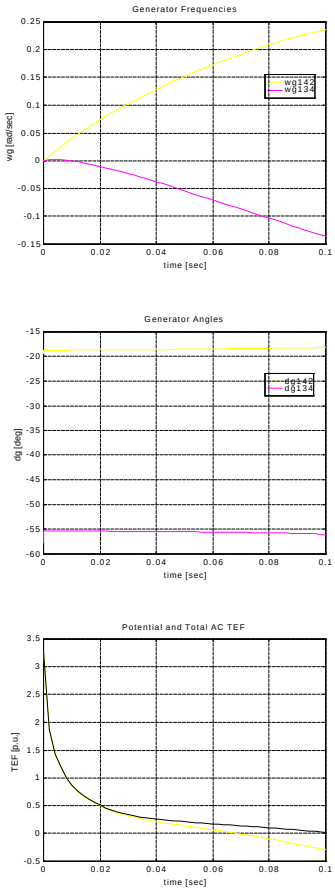

Fig. 1 Perturbing the s.e.p. with a 4 % change

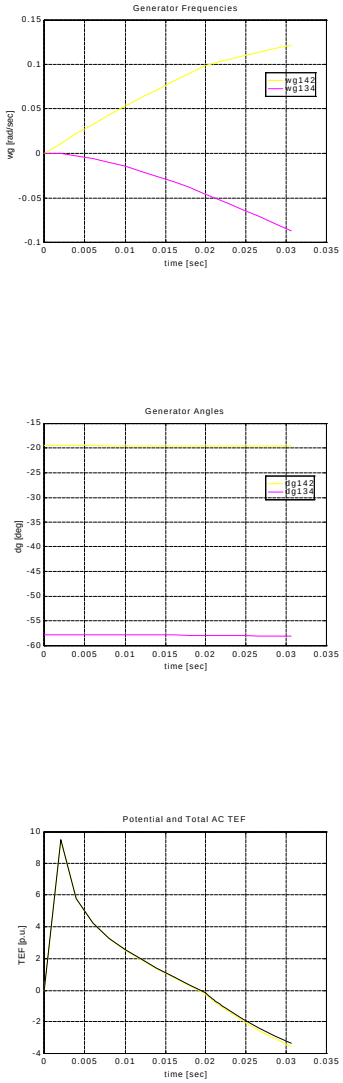

Fig. 2 Perturbation applying a solid three-phase fault

4 Conclusion

Transient stability analysis is concerned with a power system´s ability to reach an acceptable steady-state following an event disturbance. In contrast to the time-domain approach, direct methods determine system stability based on energy functions. There are advantages of using the network-preserving power system models for direct stability analysis. The simulation results presented in this paper are on the IEEE 173 bus test system. A time domain simulation is carried out by perturbing the s.e.p. and by applying a solid three-phase fault.

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